Intraday Price Discovery in the VIX and VIX Futures Markets

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ABSTRACT

We utilize both the Hasbrouck (1995) information share approach and the Gonzalo and Granger (1995) common factor weight approach to examine the price discovery competition between the VIX index and VIX index futures, and find that under both approaches, as compared to the VIX, VIX futures prices make a greater contribution to price discovery. With an increase in the price difference between the spot VIX and VIX futures, which is commonly referred to as the ‘futures basis’, there is also a tendency for a corresponding increase in the contribution to price discovery made by VIX futures. Our empirical results further reveal that news announcements on macroeconomic issues in the US invariably lead to VIX futures playing a more dominant role in the price discovery process.

Keywords: Price discovery; VIX futures; VIX; Futures basis; Information share.

JEL Classification: C32, G01, G14.
1. INTRODUCTION

Although, theoretically, there should be no lead-lag relationship between the spot and futures markets – in terms of spot and futures prices – in the ‘real world’ financial markets, the futures markets are likely to incorporate information more efficiently than the spot markets, essentially because of their inherent leverage, low transaction costs and the absence of any short-selling constraints.\(^1\) As such, price discovery in the futures markets – which is commonly recognized as the use of futures prices to determine expectations in spot market prices (Schroeder and Goodwin, 1991; Yang, Jang and Zhou, 2012) – is a very important issue which continues to receive considerable attention within the related finance literature.

This study investigates whether the intraday price discovery of VIX futures prices contributes to the efficiency of the Chicago Board Options Exchange (CBOE) VIX index,\(^2\) an index which is compiled from a portfolio of S&P 500 index options to assist in approximating the expected aggregate volatility of the S&P 500 index during the subsequent 30-calendar-day period. Most importantly, our investigation is aimed to find the determinants of price discovery for VIX futures. The traditional


\(^2\) The launch of VIX futures took place on the CBOE on 26 March 2004. Since then, as a result of the increasing demand for practical market risk management, VIX futures have become the most successful new product launch in the history of the CBOE.
view of the determinants of asset prices’ speed of adjustment in response to new information would be differences in the degrees of leverage, transaction cost, asset price volatility, and liquidity across markets in which nearly homogeneous assets are traded, e.g. Chakravarty et al., 2004; Ates and Wang, 2005; Mizrach and Neely, 2008; Chen and Gau, 2009; Chen and Gau, 2010; Fricke and Menkhoff, 2011; Wang et al., 2013. In this study, due to the unique characteristics of volatility asset class, we contribute to the literature by documenting the most important role of VIX futures basis in price discovery of VIX futures, but not liquidity and transaction cost which are both the most common factors influencing price discovery in the various futures markets.

The prior related studies on price discovery in the spot and futures markets have identified at least two unique features of the volatility indices that can have direct impacts on the lead-lag relationships that exist between the VIX and VIX futures prices. As a result of the existence of these two features, our analysis of these lead-lag

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3 In equity index futures market, Ates and Wang (2005) find that, for the S&P 500 and Nasdaq 100 indexes, the joint effects of operational efficiency and relative liquidity determine the greater contribution made towards price discovery by electronic trading relative to open-outcry. Chen and Gau (2009) find that, after the minimum tick size in the stock market decreases, the bid-ask spreads of the component stocks of the stock index get lower, and the contribution of the spot market to price discovery increases. Wang et al. (2013) find that the relative liquidity between regular and mini index TAIFEX futures affects the price discovery of the mini index futures. In stock options market, Chakravarty et al. (2004) find that the price discovery of the options market relates positively to the relative trading volume, relative the bid-ask spread, and underlying volatility. In Treasury futures market, Mizrach and Neely (2008) and Fricke and Menkhoff (2011) find that the relative bid-ask spreads, trading volume, and realized volatility are statistically significant influence on price discovery in the U.S. treasury market. In foreign exchange futures market, Chen and Gau (2010) find that the currency futures market contributes a larger price discovery when the futures market is more liquid.
relationships in the present study is likely to provide results which differ markedly from those reported in the prior related studies. Firstly, investors cannot directly trade the spot VIX, and they also have difficulty in achieving timely replication of the VIX; this implies that no cost-of-carry relationship exists between the price of VIX futures and the VIX.

The studies of both Hibbert, Daigler and Dupoyet (2008) and Da, Engelberg and Gao (2015) provide us with some solid evidence in support of the notion that the behavior of investors tends to lead to excess volatility in the short run. That is to say, during a period of market crisis, the behavioral biases of investors in the VIX are likely to cause significant price deviations from the fundamental value of the assets, and this deviation cannot be easily eliminated by arbitrage activities, essentially because of the non-tradable characteristic of the spot VIX, leading to a reduction in the contribution of the VIX to price discovery.

Secondly, the mean-reversion property of volatility has already been well documented within the finance literature; as noted by Engle and Patton (2001),

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4 Although it is widely known that the value of the VIX is derived from the market prices of a portfolio of S&P 500 index options, it is not simply a weighted sum of the underlying options. The S&P 500 index options from which the VIX is computed sum up to the square of the VIX, not the VIX itself. As such, investors cannot easily buy or sell a basket of index options with expiration prices that are equal to the index, essentially because of the non-linear transformation.

5 Using daily and intraday data, Hibbert et al. (2008) found that the behavioral biases of investors (i.e., their representativeness, effect and extrapolation biases), triggered a major spike in volatility leading to a fall in the asset price, ultimately resulting in the phenomenon of negative return volatility.

6 See, for example, Campbell, Lo and MacKinlay (1997) and Fouque, Papanicolaou and Sircar (2000).
“mean reversion in volatility implies that current information has no effect on the long run forecast”. Accordingly, given that the VIX represents a 30-day measure of the expected volatility of the S&P 500 index, whereas VIX futures prices reflect the expected value of the VIX at the expiration date, investors may tend to trade in VIX futures at a discount (premium) price when the VIX is relatively high (low) as a result of the mean-reversion property of the VIX.

For our analysis of the newly-established and rapidly-growing VIX derivatives markets, we adopt the Hasbrouck (1995) information share approach (which requires the estimation of a vector error correction (VEC) model), along with the common factor weight approach of Gonzalo and Granger (1995). These two approaches, which have been widely used to explore the extent of price discovery, will facilitate our exploration of intraday price discovery in the VIX and VIX futures markets. Our sample period in this study runs from May 2004 to August 2011.

The main advantage of the use of the information share and common factor weight approaches is that a better understanding is acquired of the contribution of a

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7 To facilitate the modeling of VIX futures pricing, a common assumption in the studies of Zhang and Zhu (2006), Zhang, Shu and Brenner (2010) and Dupoyet, Daigler and Chen (2011) was that volatility followed a mean reversion process.

8 For example, Mizrach and Neely (2008) and Fricke and Menkhoff (2011) used the information share approach to study the price discovery process in the US Treasury markets. Forte and Peña (2009) examined price discovery in the stock, bond and credit default swap markets using both the information share and common factor component weight approaches. Chen and Gau (2010) and Chang, Chen, Chou and Gau (2013) both analyzed price discovery between the currency futures and spot markets and Chakravarty, Gulen and Mayhew (2004) studied price discovery between stock options and stocks using the two approaches.
market to price discovery; however, an examination of time-varying price discovery by calculating both the daily information share and the common factor weight measures may also prove to be insightful. Our empirical findings reveal that the average information share (common factor component) of VIX futures is approximately 2.70 (2.83) times greater than that of the spot VIX, which provides strong support for the leading role of VIX futures in the price discovery process.

Furthermore, based upon our reliance on the mean-reversion property of volatility, we can calculate a measure of the VIX futures basis, which is defined as the level of the VIX index minus the price of the nearest VIX futures contract; this enables us to study whether the VIX futures basis, which reflects the expected risk-neutral path of volatility, plays an important role in price discovery. Our empirical results show that the VIX futures basis is positively related to both the information share and the common factor weight of VIX futures, at a significance level of at least 5 percent.

Mean reversion in volatility can help to explain why the VIX futures basis has a statistically significant effect on price discovery, in terms of both the information share and the common factor weight. As noted earlier, the spot VIX index reflects the implied volatility for a 30-day period, whereas VIX futures indicate the expected volatility for the subsequent 30-day period. According to the ‘investor behavior’
arguments of Hibbert et al. (2008) and Da et al. (2015), if there is a short-term increase in economic uncertainty, then we would expect to find the VIX index instantly spiking upwards; for example, following the announcement of the Lehman Brothers default on 24 October 2008, the VIX reached an intraday high of 89.53.

However, due to the mean reversion property of volatility, increases in VIX futures prices are not usually found to be of the same magnitude as increases in the VIX; in other words, the content of the VIX futures basis can be intuitively viewed as the degree of variance in future volatility between short-term and long-term expectations. If there is a sharp rise in the VIX in the short term, as a result of the overreaction of traders, then we would expect to find the VIX futures basis being higher, leading to a reduction (increase) in price discovery in VIX spot (futures).

More interestingly, motivated by the seminal works of Nikkinen and Sahlström (2007), Chen and Clements (2007) and Vähämää and Åijö (2011), in which it was demonstrated that market volatility is significantly affected by announcements of macroeconomic issues, we also find an increase in the contribution to price discovery made by VIX futures prices during periods of macroeconomic announcements in the US, particularly with regard to the consumer price index.

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9 This is why the VIX is commonly referred to as the ‘investor fear gauge’ (Whaley, 2000).
In other words, it is clearly evident that the release of macroeconomic information is accompanied by a decline in the contribution to price discovery by the spot VIX, which is consistent with the notion that announcements on macroeconomic issues create economic uncertainty, ultimately leading to a short-term jump in the spot VIX index; as a result, the VIX prices temporarily move away from the equilibrium price.

Our study adds to the extant literature in the following ways. Firstly, we go on to develop the recent works of Shu and Zhang (2012) and Frijns, Tourani-Rad and Webb (2013) who carried out Granger causality tests between the VIX and the VIX futures; we complement these works by using the Hasbrouck (1995) information share approach and the Gonzalo and Granger (1995) common factor weight approach to investigate intraday price discovery in the VIX and VIX futures markets.10

Secondly, in contrast to the empirical findings reported in the extant literature on price discovery, we find that the VIX futures basis is more pronounced with

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10 There is no consensus for the stationary process of VIX index series and the long-run relations between VIX index and VIX futures in the previous studies. Shu and Zhang (2012) find both VIX index and VIX futures series are nonstationary and one cointegration between the two VIX series. Mean reverting process does not imply that VIX is stationary. In contrast, Frijns, Tourani-Rad and Webb (2013) find VIX index and VIX futures are stationary process. The stationarity of volatility (or VIX series) seems to depend on the historical period and the different data sampling frequency. In our study, VIX index and VIX futures are non-stationary series and there is one cointegration between them. Hence, information share and common factor weight approaches are suitable to examine the price discovery in VIX index.
regard to the price discovery of market volatility, as compared to the usual liquidity measures, such as trading volume and bid-ask spreads.

Thirdly, to the best of our knowledge, our study is the first to empirically examine the effect of US macroeconomic announcements on the price discovery of VIX futures based upon the Hasbrouck information share and Gonzalo and Granger common factor weight approaches. Our empirical results reveal that economic uncertainty relating to the content of macroeconomic announcements is an important factor affecting the extent of price discovery in market volatility.

The remainder of this paper is organized as follows. An explanation of the data and methodology adopted for this study is provided in Section 2, followed in Section 3 by a description of our measures of price discovery in the VIX and VIX futures prices. Our empirical results are presented and discussed in Section 4. Finally, the conclusions drawn from this study are presented in Section 5.

2. DATA AND METHODOLOGY

The CBOE volatility index (VIX) is compiled from the market prices of S&P 500 index options as a means of approximating the expected aggregate volatility of the S&P 500 index during the subsequent 30-calendar-day period; the VIX also essentially represents the aggregate information implied from the S&P 500 options
VIX futures contracts were introduced in March 2004 as a means of offering investors a much more simple and direct channel for trading market volatility without dealing with all of the other associated risk factors that would otherwise have significant impacts on the overall performance of volatility strategies.

The primary data adopted for this study, obtained from CQG Market Data, are the intraday VIX levels and VIX futures tick information, with the sample period running from 3 May 2004 to 30 August 2011. The dataset provides only one-minute spot VIX values, whereas complete information is available for each trade quote and transaction in VIX futures, including the formation date, formation time (in seconds), expiration date and the bid and ask prices for a quote, or the trading price and transaction volume for a transaction. We examine price discovery by matching five-minute trading prices from the VIX index and VIX futures.

We use the VIX futures prices from the nearby contracts up until eight trading days prior to the expiration date in order to avoid the issue of liquidity being imposed upon shorter maturity futures. When the time-to-maturity of the nearest VIX futures contracts is less than eight days, we use the second-nearest contracts. The VIX index, VIX index futures and futures basis (compiled from the VIX index – VIX futures price) series are illustrated in Figure 1.

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11 The methodology currently used for computing the VIX index was introduced in 2003.
As we can see from Figure 1, the VIX index and VIX index futures move in the same direction, whilst also exhibiting a mean-reversion property. During the financial crisis period (September 2008 to June 2009), the VIX index clearly shifted to a much higher level than that of VIX futures; thus, we would expect to find VIX futures prices being more stable than the spot VIX index, particularly during a period of financial crisis. This provides support for the finding of Shu and Zhang (2012) who indicated that volatility tends to follow a mean-reversion process, with higher current volatility being associated with lower future volatility. The futures basis is clearly relatively higher during a period of financial crisis, thereby reflecting the different perspectives on market condition, from short-term to long-term horizons.

3. MEASURING PRICE DISCOVERY

To facilitate our examination in the present study of the price discovery competition between the VIX index and VIX index futures, we use the information share approach of Hasbrouck (1995) and the common factor components approach of Gonzalo and Granger (1995). As the VIX index and VIX futures prices are driven by the same fundamental information, they should be closely related to the common factors.
Shu and Zhang (2012) found that the spot VIX and VIX futures prices were cointegrated, and that there was a long-term (or equilibrium) relationship between the spot VIX and VIX futures. In the Hasbrouck (1995) approach, the contribution of a market to price discovery is measured in terms of the information share, which is defined as the proportion of the efficient price innovation variance attributable to that market. If \( N \) market prices are driven by one common stochastic trend, then a total of \( N-1 \) cointegrating or independent linear relationships will be found to exist between them. Following Engle and Granger (1987), we use a vector error correction (VEC) model to capture the dynamics in returns across \( N \) markets, as follows:

\[
\Delta p_t = \alpha \Delta p_{t-1} + A_1 \Delta p_{t-1} + A_2 \Delta p_{t-2} \cdots A_k \Delta p_{t-k} + \varepsilon_t, \tag{1}
\]

where \( p_t \) is the \( N \times 1 \) vector of prices; the \( N \times N \) matrices \( A_i \) \( (i = 1, 2, \ldots, k) \) are the autoregressive coefficient matrices; \( z_{t-1} = \beta' p_{t-1} \) is the \((N-1) \times 1\) error correction vector relating to the cointegrating vector \( \beta \); \( \alpha \) is the \( N \times (N-1) \) adjustment coefficient matrix which measures the ways in which prices react to the deviation from long-run equilibrium; and \( \varepsilon_t \) represents the vector of innovations as an \( N \times 1 \) vector of zero mean disturbances with the covariance matrix, \( \Omega \).

We can then rewrite the VEC model as a vector moving average (VMA) model, which is expressed as:
where \( I \) denotes the \( N \times N \) identity matrix. The sum of the VMA coefficients is then defined as \( \Psi(1) \).

As noted by Hasbrouck (1995), when the \( N \) prices are cointegrated with a rank of \( N-1 \), all of the rows of \( \Psi(1) \) will be identical. Let \( \psi \) be the common row of \( \Psi(1) \), then, as defined by Hasbrouck (1995), the information share of one market is:

\[
IS^i = \frac{\psi_i^2 \Omega_{ii}}{\psi \Omega \psi'},
\]

(3)

where \( \psi_i \) is the \( i^{th} \) element of \( \psi \), and \( \Omega_{ii} \) is the \((i, i)\) element of \( \Omega \).

However, if the matrix \( \Omega \) is not diagonal, then we are only able to calculate a range of information shares, as opposed to a unique estimate. Let us suppose that \( \Omega = FF' \), where \( F \) is a lower triangular matrix. The information share of market \( i \) can then be calculated as:

\[
IS^i = \frac{[(\psi F)_i]^2}{\psi \Omega \psi'},
\]

(4)

where \((\psi F)_i\) is the \( i^{th} \) element of \( \psi F \).

As noted in the above discussion on the information share measure, we are only able to calculate the upper and lower bounds of the information share; however, Baillie, Booth, Tse and Zabotina (2002) provided various analytical examples to demonstrate that the average of the information shares given by the two permutations is a reasonable estimate of the contribution of a market to price
discovery. We therefore use the average value of the information shares to compare the relative contribution to price discovery made by the spot VIX and VIX futures markets.

Gonzalo and Granger (1995) used an alternative approach, decomposing the vector of market prices into permanent \( (f_t = \alpha'_i p_t) \) and transitory \( (Z_t = \beta' p_{t-1}) \) components. The \( \alpha'_i \) vector is the common factor coefficient vector which is orthogonal to \( \alpha \); this is the vector of the coefficients on the error-correction terms in the VEC model shown in Equation (1). We can use the \( i^{th} \) element of \( \alpha'_i \) (denoted as \( CC^i \)) to measure the overall contribution of market \( i \) to price discovery, since this can effectively gauge the contribution of each market price to the common permanent component.\(^{12}\)

We use five-minute daily data on the VIX spot and futures prices for our calculation of the daily \( IS^i (CC^i) \), for \( i = S \) or \( F \), in order to determine the time-varying contributions of the VIX spot and VIX futures to price discovery. Based on the daily \( IS^i (CC^i) \) of the VIX futures market, we can examine the ways in which daily price discovery varies with market quality and the futures basis.

4. **EMPIRICAL RESULTS**

4.1 **Summary Statistics on the VIX and VIX Futures**

\(^{12}\) For practical purposes, we can normalize this to make the elements add up to 1.
The summary statistics on the five-minute changes in the VIX, VIX futures and futures basis from 3 May 2004 to 30 August 2011 are presented in Table 1. The VIX and VIX futures returns are calculated by the difference in the log index level, and it is quite clear from Table 1 that although the average VIX and VIX futures returns are very close, the standard deviation in the VIX returns is over twice the size of the standard deviation in the VIX futures return; in other words, VIX futures prices are more stable than the spot VIX index.

<Table 1 is inserted about here>

We can observe from Table 1 that the mean of the futures basis is negative and in contango, which is consistent with the finding of Simon and Campasano (2012). Furthermore, as expected, each of the three series is found to have positive skewness, excess kurtosis and significant autocorrelation. Although the VIX index and VIX futures return series display excess kurtosis, the excess kurtosis in the spot VIX (2,516.86) is considerably larger than that in VIX futures (348.16), thereby indicating that extreme fluctuations usually occur more in the former than the latter. Table 1 also reports the results of the augmented Dickey-Fuller unit root test, with the test indicating that all of the series are stationary.

4.2 VIX Price Discovery and the VEC Model

Prior to going on to explore price discovery between the VIX and VIX futures, we
carry out both Johansen cointegration and Granger causality tests. The results of the Johansen maximum-eigenvalue test and trace test reported in Table 2 indicate that the VIX index level and VIX index futures prices are both cointegrated, with cointegrating rank $r=1$; this implies one common stochastic trend ($k = N - r = 1$), which is consistent with the finding of Shu and Zhang (2012).

<Table 2 is inserted about here>

In order to deal with the cointegrating relationship in the VIX index and VIX index futures system, we estimate a bivariate VEC base model for the entire sample, at five-minute frequency, as follows:

$$
\Delta P_i^S = c^S + \alpha^S (P_{i-1}^S - P_{i-1}^F) + \sum_{j=1}^k \gamma_{1j} \cdot \Delta P_{i-1}^S + \sum_{j=1}^k \gamma_{2j} \cdot \Delta P_{i-1}^F + \epsilon_i^S
$$

$$
\Delta P_i^F = c^F + \alpha^F (P_{i-1}^S - P_{i-1}^F) + \sum_{j=1}^k \gamma_{3j} \cdot \Delta P_{i-1}^S + \sum_{j=1}^k \gamma_{4j} \cdot \Delta P_{i-1}^F + \epsilon_i^F
$$

(5)

where $P_i^S$ and $P_i^F$ denote the log prices of the VIX and VIX index futures at time $t$.

We follow several of the prior studies to use the pre-specified cointegrating vector $(1-1)^T$, with $\alpha^i$ representing the coefficients of the error correction terms of the VIX index ($i = S$) and VIX index futures ($i = F$) in the model. The models are estimated using the OLS method with Newey-West standard errors and an autoregressive lag $k$ of 4, according to the Akaike information criterion (AIC).

The results of the Granger causality tests, based upon Equation (5), are reported

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13 See, for example, Booth et al. (1999), Tse (2001) and Theissen (2012).
in Table 3, from which we can see that, consistent with the bi-directional causality reported by Shu and Zhang (2012), \( \Delta P_t^F(\Delta P_t^S) \) is found to Granger cause \( \Delta P_t^S(\Delta P_t^F) \). Although the Granger test indicates bi-directional causality between the VIX and VIX futures prices, the effect of \( \Delta P_t^F \) on \( \Delta P_t^S \) is found to be more significant than the effect of \( \Delta P_t^S \) on \( \Delta P_t^F \); this therefore motivates our further exploration of the dynamic price discovery process in the VIX and VIX futures markets.

<Table 3 is inserted about here>

The coefficient estimates of the VEC model, information shares and common factor weight components from Equation (5) are reported in Table 4, where \( \alpha^S \) and \( \alpha^F \) respectively represent the deviation from the long-run equilibrium for the VIX and VIX futures series. Martens (1998) argued that the greater the propensity for one market to follow the other market, the larger the coefficient \( \alpha \) will be. Hence, a smaller \( \alpha \) (in absolute value) indicates that this is the information leading market. Table 4 clearly demonstrates that \( \alpha^S \) is larger than \( \alpha^F \) (in absolute value), thereby implying that VIX futures play a leading informational role in the price discovery process.

<Table 4 is inserted about here>

According to the results reported in Table 4, in the VEC base model, the average information shares (common factor components) of VIX futures, at 0.73
(0.78) dominate the information shares (common factor components) of the spot VIX, at 0.27 (0.22). This evidence provides support for the leading informational role of VIX futures within the overall price discovery process.

We also employ the VEC model with dummy variables to capture the role of the futures basis in the price discovery process between the VIX and VIX futures. Given that the assumption in the VEC base model in Equation (5) is that the speed of adjustment to deviations in the price levels from their long-run equilibrium relationship will be independent of the futures basis, we can include dummy variables to modify the VEC base model as follows:

\[
\Delta P_i^S = \alpha_i^S (P_{i-1}^S - P_{i-1}^F) + \alpha_2^S D_{t-1} (P_{i-1}^S - P_{i-1}^F) \\
+ \sum_{j=1}^{k} \gamma_{1j} \cdot \Delta P_{i-1}^S + \sum_{j=1}^{k} \gamma_{2j} \cdot \Delta P_{i-1}^F + \epsilon_t^S
\]

\[
\Delta P_i^F = \alpha_i^F (P_{i-1}^S - P_{i-1}^F) + \alpha_2^F D_{t-1} (P_{i-1}^S - P_{i-1}^F) \\
+ \sum_{j=1}^{k} \gamma_{3j} \cdot \Delta P_{i-1}^S + \sum_{j=1}^{k} \gamma_{4j} \cdot \Delta P_{i-1}^F + \epsilon_t^F
\]

where \(\alpha_i^S\) and \(\alpha_i^F\) denote the respective coefficients on the error correction terms of the VIX index \((i = S)\) and VIX index futures \((i = F)\) in the VEC model with dummy variables.

The dummy variable, \(D_{t-1}\), takes the value of 1 if the VIX futures basis is positive as defined above, otherwise 0. The \(\alpha_i^S\) and \(\alpha_i^F\) coefficients measure the adjustment speed of the VIX index and VIX index futures towards equilibrium if the
VIX futures basis is negative. The $\alpha_2^S$ and $\alpha_2^F$ coefficients measure whether the adjustment speed differs if the futures basis changes from negative to positive. We expect to find that the two coefficients will have the same respective signs as $\alpha_1^S$ and $\alpha_1^F$.

The coefficient estimates, information shares and common factor components of the VEC model with dummy variables, based upon Equation (6), are also reported in Table 4, from which we can see that $|\alpha_1^S|$ is larger than $|\alpha_1^F|$ (that is, $|\alpha_1^S| - |\alpha_1^F| > 0$). Similar to the results obtained from the VEC base model, we find that VIX futures continue to dominate within the price discovery process when a dummy variable for the futures basis is included in the VEC base model.

Furthermore, if the futures basis is positive, the estimates of the coefficient on the error correction term ($\alpha_1^i + \alpha_2^i$) are still found to have the same sign and the same result; that is, $|\alpha_1^S + \alpha_2^S|$ is larger than $|\alpha_1^F + \alpha_2^F|$. However, the absolute difference between the coefficient estimates on the error correction term ($= |\alpha_1^S + \alpha_2^S| - |\alpha_1^F + \alpha_2^F|$) is obviously greater in magnitude than $|\alpha_1^S| - |\alpha_1^F|$, thereby providing support for the notion that if the futures basis is positive, then the VIX futures market will have a positive influence on price discovery.

We can also compute the information shares and common factor components of the VIX and VIX futures using Equation (6), from which we find that, for both
positive and negative futures basis, the information shares and common factor components of VIX index futures are larger than those of the VIX index. If the futures basis is positive, the information shares (common factor components) of VIX futures are 0.91 (0.90), whilst those for the VIX are 0.09 (0.10). If the futures basis becomes negative, then the information shares (common factor components) of VIX futures are 0.65 (0.67), whilst those for the VIX are 0.35 (0.33).

A comparison of the contribution to price discovery in the cases of positive and negative futures basis is clearly simplified when using the information share and common factor component approaches. Overall, our empirical results show that the contribution to price discovery made by the VIX futures market is enhanced when the level of the spot VIX exceeds that of VIX futures.

4.3 Daily Information Share and Common Factor Component Measures

We go on in this section to analyze time-varying price discovery between the VIX index and VIX index futures, adopting five-minute data on the VIX and VIX futures to obtain the daily measures of the information shares and common factor components; this enables us to test our hypothesis on the contribution made by the futures market to price discovery. We divided the full sample into those days with positive and negative average futures basis; the daily average of the information shares and common factor components of the VIX index and VIX index futures for
the full sample and the sub-sample are reported in Table 5.

<Table 5 is inserted about here>

As indicated in Table 5, in both the full sample and the sub-sample, the average information shares of VIX index futures dominate those of the VIX index. According to the information share measure on the full sample, VIX index futures provide a greater contribution to price discovery (71.9 per cent) than the VIX index (28.1 per cent), with the common factor component measure providing results that are consistent with those of the information share measure. Since the $t$ statistic rejects the null hypothesis of zero difference in price discovery between the VIX index and VIX index futures, this confirms that VIX index futures account for a greater contribution than the VIX index to price discovery, regardless of whether price discovery is measured using the information share or the common factor component.

Furthermore, if we compare the information shares or common factor components based upon the full sample, the positive futures basis sample and the negative futures basis sample, we find that for VIX index futures, both the information shares and the common factor components are at their largest in the positive futures basis sample, and at their smallest in the negative futures basis sample. This confirms our earlier finding that the contribution to the price discovery
process made by the VIX futures market is enhanced when the spot VIX index exceeds the VIX futures prices.

4.4 Determinants of Information Shares and Common Factor Components

Numerous studies have focused on the determinants of price discovery between the futures market and the corresponding VIX index or spot market. Based upon their examinations of the determinants of price discovery between the stock index and the associated derivative securities, several studies have concluded that the price discovery process is associated with the transaction costs or the bid-ask spread.\(^\text{14}\)

As regards the determinants of price discovery between stock prices and the corresponding options prices, Chakravarty et al. (2004) noted that price discovery in the options market was found to have significant associations with the relative trading volume, relative bid-ask spread and underlying volatility. As for the determinants of price discovery between foreign exchange (FX) and the corresponding futures, Chen and Gau (2010) and Chang et al. (2013) found that price discovery in the FX futures market had significant associations with macroeconomic announcements, trading volume, the bid-ask spread and the structure of the investors.

In contrast to the findings in the stock and FX markets, from their examination of the determinants of price discovery between the US Treasury spot and futures markets, Mizrach and Neely (2008) and Fricke and Menkhoff (2011) found that price discovery was influenced by the relative bid-ask spreads, trading volume and realized volatility, all with statistical significance.

In summary, price discovery between asset spot and futures prices appears to be largely dependent upon bid-ask spreads, trading volume, market conditions and macroeconomic announcements; thus, any market which provides greater liquidity and lower trading costs should be found to have an increased information share and to be playing a more important role in the price discovery process. However, in contrast to the results reported in the prior studies, the unique feature of the volatility index (the tendency to follow a mean-reversion process) may significantly affect the price discovery relationship between the VIX and VIX futures prices.

It is already well known that the spot VIX index reflects 30-day implied volatility, whereas the corresponding VIX futures reflects forward implied volatility (the expected volatility for the subsequent 30-day period). If the behavioral biases of traders cause a temporary rise in the VIX index, VIX futures prices would not increase at the same rate as the spot VIX due to the mean-reversion property of volatility. Thus, the futures basis is expected to play a more important role in the
price discovery process.

We therefore estimate the following regression in order to determine the relative contributions to price discovery made by the futures basis, bid-ask spreads, trading volume and underlying volatility:

\[ IS_i^F = \alpha + \beta \cdot X_i + \varepsilon_i \]  \hspace{1cm} (7)

\[ CC_i^F = \alpha + \beta \cdot X_i + \varepsilon_i \]  \hspace{1cm} (8)

where \( IS_i^F \) and \( CC_i^F \) denote the contributions of VIX index futures to price discovery, respectively measured by the information share (IS) and common factor weight (CC). The explanatory variables include the futures basis (\( Basis_i \)), the S&P 500 index (\( SP_i \)) and two dummy variable; \( Dummy_{iL} \) takes the value of 1 on days when the S&P 500 index is below its 25\(^{th}\) percentile, otherwise 0, and \( Dummy_{iH} \) takes the value of 1 on days when the S&P 500 index is above its 75\(^{th}\) percentile, otherwise 0.

\( Volume_i \) is the trading volume in VIX futures; \( Spread_i \) are the bid-ask spreads in VIX futures and \( CPI_i \), \( GDP_i \) and \( UMP_i \) are respective dummy variables for the consumer price index, gross domestic product and the unemployment rate. The macroeconomic dummy variables take the value of 1 if there are announcements of macroeconomic issues on day \( t \), otherwise 0. We use the Newey and West (1987) covariance estimator to adjust for the presence of heteroskedasticity and autocorrelation in the regression errors. The regression results from Equations (7) and
(8) are reported in Table 6.

<Table 6 is inserted about here>

To the best of our knowledge, ours is the first study to identify the role of the futures basis on price discovery in VIX index futures. As shown in Models (1) to (4) of Table 6, the futures basis has a positive impact on the price discovery contribution made by VIX futures, thereby providing support for the argument that a larger spread between VIX futures and the spot index – essentially caused by the short-term overreaction among investors to new information which is subsequently revised in line with long-term expectations – increases the contribution made by VIX futures to price discovery. A larger short-term deviation in the spot VIX from the mean level is accompanied by a reduction in the relative contribution of the spot VIX to price discovery.

Changes in the S&P 500 index in Models (1) and (3) have significantly positive impacts on the price discovery of VIX futures; however, Shu and Zhang (2012) also found that the lead-lag relationship between the VIX index and VIX futures was unstable and varying with the S&P 500 index. They noted that whilst the VIX index led VIX futures in Q1 of 2005, VIX futures led the VIX index between Q1 and Q3 of 2008. Therefore, in order to further examine the relationship between VIX index price discovery and the S&P 500 index, we add the dummy variables, $Dummy_{i}^H$ and
$Dummy_t^L$, into the regression models. The inclusion of these dummies facilitates joint estimation of the price discovery relationships between three sub-samples, comprising of normal days, bull market days and bear market days.

Models (2) and (4) in Table 6 reveal that the dummy variable, $Dummy_t^L$, has a significantly positive impact on the contribution to price discovery made by VIX futures. This indicates that when there is any significant fall in the stock market, the role of VIX futures in the price discovery process is enhanced. As argued by Shu and Zhang (2012), during bear market periods, investors often exhibit significant overreaction to information which can push the spot VIX far from VIX futures prices.

More interestingly, announcements relating to the CPI are found to have significantly positive impacts on the price discovery contribution of VIX futures, whereas the regression coefficients on the traditional determinants of price discovery, including trading volume and bid-ask spread, are found to be insignificant. Only the results in Models (1) and (3) reveal any significantly positive impacts of trading volume on the contribution made by VIX futures to price discovery. Overall, our empirical results suggest that economic uncertainty with regard to the content of macroeconomic information releases is an important factor affecting the degree of price discovery in market volatility.
5. CONCLUSIONS

We set out in this study to examine the intraday price discovery relationship between the VIX and VIX futures. Following the approaches of Hasbrouck (1995) and Gonzalo and Granger (1995), our empirical results provide support for the leading informational role of VIX futures in the price discovery process; indeed, we find that the average information share (common factor component) in VIX futures is approximately 2.70 (2.83) times that of the spot VIX. We also find that VIX futures prices contribute more to price discovery with an increase in the VIX futures basis, which is defined as the difference between the VIX and VIX futures.

Focusing on the positive futures basis sub-sample, we observe that the average information share (common factor component) in VIX futures rises to about 2.75 (3.04) times that of the spot VIX. A higher futures basis clearly leads to a reduction in the contribution made by the spot VIX to price discovery, which is consistent with the view that the mean-reversion property of volatility leads to VIX futures providing more information on the future volatility level, whilst short-term investor sentiment leads to the VIX deviating from the equilibrium price.

To the best of our knowledge, our study is the first to empirically examine intraday price discovery in the VIX and VIX futures based upon the adoption of the Hasbrouck (1995) information share and Gonzalo and Granger (1995) common factor
component approaches. Our aim is not only to augment the recent works of Shu and Zhang (2012) and Frijns, Tourani-Rad and Webb (2013) – who carried out Granger causality analysis between the VIX and VIX futures – but also to demonstrate the important role played by futures basis in the price discovery of market volatility, as opposed to the usual measures of liquidity, such as trading volume and bid-ask spreads.
REFERENCES


Chen, Y.L. and Y.F. Gau (2009), ‘Tick Sizes and Relative Rates of Price Discovery in Stock, Futures and Options Markets: Evidence from the Taiwan Stock


Iihara, Y., K. Kato and T. Tokunaga (1996), ‘Intraday Return Dynamics between the
Cash and the Futures Markets in Japan’, *Journal of Futures Markets*, **16**: 147-62.


Figure 1 VIX index, VIX futures and futures basis, 2004-2011
<table>
<thead>
<tr>
<th>Variables</th>
<th>VIX Index Returns</th>
<th>VIX Futures Returns</th>
<th>Futures Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.041</td>
<td>0.038</td>
<td>-0.434</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>92.685</td>
<td>44.896</td>
<td>2.679</td>
</tr>
<tr>
<td>Skewness</td>
<td>3.521</td>
<td>3.907</td>
<td>4.379</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2516.860</td>
<td>348.160</td>
<td>34.290</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.105***</td>
<td>-0.021***</td>
<td>0.966***</td>
</tr>
<tr>
<td>$Q(15)$</td>
<td>2879***</td>
<td>105***</td>
<td>890214***</td>
</tr>
<tr>
<td>ADF test</td>
<td>-193.800***</td>
<td>-396.770***</td>
<td>-9.957***</td>
</tr>
<tr>
<td>No. of Obs.</td>
<td>151,043</td>
<td>151,043</td>
<td>151,043</td>
</tr>
</tbody>
</table>

Notes:

- $\rho$ denotes the first-order autocorrelation coefficient; $Q(15)$ refers to the Ljung-Box test statistics for serial correlation in the series up to the fifteenth order; the augmented Dickey-Fuller (ADF) statistics test the null hypothesis that an examined series has a unit root, with a corresponding lag for this test of 14, according to Schwarz information criterion.
- The five-minute VIX index and VIX futures returns are calculated by the difference in the log price multiplied by 10,000.
- The futures basis is obtained by subtracting the futures price from the cash price.
- *** indicates significance at the 1% level.
Table 2  Johansen cointegration tests\(^a\)

<table>
<thead>
<tr>
<th>(H_0) (^b)</th>
<th>(\lambda_{\max}) (^c)</th>
<th>Critical Value (5% level)</th>
<th>(\lambda_{\text{trace}}) (^c)</th>
<th>Critical Value (5% level)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r \leq 0)**</td>
<td>197.33</td>
<td>11.22</td>
<td>198.21</td>
<td>12.32</td>
</tr>
<tr>
<td>(r \leq 1)</td>
<td>0.87</td>
<td>4.12</td>
<td>0.87</td>
<td>4.12</td>
</tr>
</tbody>
</table>

Notes:
\(^a\) This table reports the results of the Johansen cointegration tests on the VIX index and VIX futures for the full sample, with the sample period running from 3 May 2004 to 30 August 2011.

\(^b\) The null hypothesis \(H_0\) states that the system contains at most \(r\) cointegrating vectors, as determined by the SIC (Schwarz information criterion). The conclusion of the cointegration test is, however, robust with regard to the number of lags. **indicates the rejection of the null hypothesis at the 5% level.

\(^c\) \(\lambda_{\max}\) is the Johansen maximum eigenvalue test statistic, and \(\lambda_{\text{trace}}\) is the Johansen trace test statistic; with both tests using four lags.
Table 3  Granger causality test

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>$F$-stat.</th>
<th>$p$-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$: $\Delta P^F_t$ does not Granger-cause $\Delta P^S_t$</td>
<td>1568.39</td>
<td>0.00</td>
</tr>
<tr>
<td>$H_0$: $\Delta P^S_t$ does not Granger-cause $\Delta P^F_t$</td>
<td>66.88</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: This table reports the results of the Granger causality test on the VIX index and VIX futures for the full sample using four lags, with the sample period running from 3 May 2004 to 30 August 2011.
Table 4  VEC model, information shares and common factor components of the VIX index and VIX index futures

<table>
<thead>
<tr>
<th>Variables</th>
<th>VEC Base Model</th>
<th>VEC Model with Dummies</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta P_{t-1}^S$</td>
<td>-0.18***</td>
<td>-0.18***</td>
</tr>
<tr>
<td>$\Delta P_{t-2}^S$</td>
<td>-0.02***</td>
<td>-0.03***</td>
</tr>
<tr>
<td>$\Delta P_{t-3}^S$</td>
<td>-0.07***</td>
<td>-0.07***</td>
</tr>
<tr>
<td>$\Delta P_{t-4}^S$</td>
<td>-0.10***</td>
<td>-0.10***</td>
</tr>
<tr>
<td>$\Delta P_{t-1}^F$</td>
<td>0.54***</td>
<td>0.54***</td>
</tr>
<tr>
<td>$\Delta P_{t-2}^F$</td>
<td>0.17***</td>
<td>0.17***</td>
</tr>
<tr>
<td>$\Delta P_{t-3}^F$</td>
<td>0.10***</td>
<td>0.10***</td>
</tr>
<tr>
<td>$\Delta P_{t-4}^F$</td>
<td>0.09***</td>
<td>0.09***</td>
</tr>
<tr>
<td>$Z_{t-1}$</td>
<td>-0.25***</td>
<td>-0.12***</td>
</tr>
<tr>
<td>$D_{t-1}Z_{t-1}$</td>
<td>-</td>
<td>-0.42***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>IS$^i$</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>CC$^i$</td>
<td>0.22</td>
<td>0.33</td>
</tr>
<tr>
<td>IS$^1$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>CC$^1$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>IS$^2$</td>
<td>-</td>
<td>0.09</td>
</tr>
<tr>
<td>CC$^2$</td>
<td>-</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Notes:

a This table reports the VEC base model with four lags, as defined in Equation (5), along with the VEC model with dummies variables with four lags, as defined in Equation (6). The models are estimated using OLS with Newey-West standard errors. The autoregressive lag $k$ is chosen as 4 according to Akaike information criterion (AIC).

b IS$^i$ and CC$^i$ are the information shares and common factor components of the VIX index and VIX futures calculated from the VEC base model.

c ** indicates significance at the 5% level; and *** indicates significance at the 1% level.
Table 5  Daily information shares and common factor components of the VIX index and VIX index futures\(^a\)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Full Sample(^c)</th>
<th>Basis(<em>t)(</em>&gt;0) Sample(^c)</th>
<th>Basis(<em>t)(</em>&lt;0) Sample(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX Index</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information shares</td>
<td>0.281</td>
<td>0.267</td>
<td>0.312</td>
</tr>
<tr>
<td>Common factor components</td>
<td>0.261</td>
<td>0.247</td>
<td>0.290</td>
</tr>
<tr>
<td>VIX Futures(^b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Information shares</td>
<td>0.719</td>
<td>0.733</td>
<td>0.687</td>
</tr>
<tr>
<td>Common factor components</td>
<td>0.739</td>
<td>0.752</td>
<td>0.710</td>
</tr>
<tr>
<td>t-statistic for IS(_t)(_S) and IS(_t)(_F)</td>
<td>38.320***</td>
<td>34.670***</td>
<td>17.440***</td>
</tr>
<tr>
<td>t-statistic for CC(_t)(_S) and CC(_t)(_F)</td>
<td>38.450***</td>
<td>34.880***</td>
<td>17.500***</td>
</tr>
</tbody>
</table>

Notes:
\(^a\) This table reports the averages of the daily information shares and common factor components for both the full sample and the sub-samples.
\(^b\) The t-statistics test the null hypotheses that the differences between IS\(_t\)\(_S\) and IS\(_t\)\(_F\) (CC\(_t\)\(_S\) and CC\(_t\)\(_F\)) will be equal to zero across the different sample periods.
\(^c\) *** indicates significance at the 1% level.
Table 6  Determinants of VIX futures contributions to price discovery\(^a\)

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Dependent Variable: IS(^t) (^F)</th>
<th>Dependent Variable: CC(^t) (^F)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model (1)</td>
<td>Model (2)</td>
</tr>
<tr>
<td></td>
<td>Coeff.  S.E.</td>
<td>Coeff.  S.E.</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.26*** 0.35</td>
<td>-1.25*** 0.33</td>
</tr>
<tr>
<td>Basis(_t)</td>
<td>0.03** 0.01</td>
<td>0.02** 0.01</td>
</tr>
<tr>
<td>d(SP(_t))</td>
<td>3.81*** 1.46</td>
<td></td>
</tr>
<tr>
<td>Dummy(_t) (^L)</td>
<td></td>
<td>0.18** 0.06</td>
</tr>
<tr>
<td>Dummy(_t) (^H)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln(Volume(_t))</td>
<td>0.04* 0.02</td>
<td>0.04 0.02</td>
</tr>
<tr>
<td>ln(Spread(_t))</td>
<td>-0.09 0.08</td>
<td>-0.08 0.08</td>
</tr>
<tr>
<td>News Announcements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>0.31*** 0.07</td>
<td>0.30* 0.17</td>
</tr>
<tr>
<td>GDP</td>
<td>0.09 0.17</td>
<td>0.10 0.17</td>
</tr>
<tr>
<td>UMP</td>
<td>0.15 0.17</td>
<td>0.14 0.16</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.17*** 0.04</td>
<td>0.16*** 0.04</td>
</tr>
<tr>
<td>Adj. (R^2)</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes:
\(a\) This table reports the regression results of the following model:

\[
IS_t^F = \alpha + \beta \cdot X_t + \epsilon_t \quad \text{and} \quad CC_t^F = \alpha + \beta \cdot X_t + \epsilon_t,
\]

where \(IS_t^F (CC_t^F)\) denotes the contribution of VIX index futures to price discovery, as measured by the information share, IS (common factor weight, CC).

\(b\) The explanatory variables include futures basis (Basis\(_t\)), the S&P 500 index (SP\(_t\)), the dummy variable (Dummy\(_t\) \(^L\)) which is equal to 1 on days when the S&P 500 index is below its 25th percentile, otherwise 0, and the dummy variable (Dummy\(_t\) \(^H\)) which is equal to 1 on days when the S&P 500 index is below its 75th percentile, otherwise 0. Volume\(_t\) refers to the volume of VIX futures, and Spread\(_t\) refers to the bid-ask spread of VIX futures. Finally, CPI, GDP and UMP are dummy variables representing the consumer price index, gross domestic product and unemployment rate. News dummy variables take the value of 1 for news announcement on day \(t\), otherwise 0. We use the Newey and West (1987) covariance estimator to adjust for the presence of heteroskedasticity and autocorrelation in the regression errors.

\(c\) ***indicate significance at the 1% level; ** indicate significance at the 5% level; and * indicate significance at the 10% level.