What affects the dependence of oil and U.S. stock markets?
A mixed-data sampling copula approach

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Abstract
The dependence of oil and stock markets is an important issue for portfolio selection and risk management. Investors tend to find out the economic factors affecting oil and stock dependence so as to improve their portfolio performance. This paper proposes a mixed frequency data sampling copula model with explanatory variables (copula-MIDAS-X) by incorporating low frequency explanatory variables into the high frequency dynamic copula model. The new model enables us to investigate the impacts of economic factors on the dependence of oil and stock returns, regardless of their marginal distributions. In the application of Brent oil and U.S. stock markets, it is found that the oil and stock dependence is influenced by aggregate demand and stock specific negative news. The impacts of aggregate demand will last for two years, while the impacts of bad news specific in stock market will last for one quarter. The implications to market regulators and investors are that the changes in aggregate demand have influential and long-lasting effects on oil and stock markets. Also, for investors who rebalance portfolios daily or weekly, it is suggested to utilize the information of both monthly economic indicators and daily returns in portfolio management.

Keywords: Crude oil, stock, dependence, mixed frequency, copula

1. Introduction
Oil plays an essential role in the development of the world’s economy and financial markets. The relationship between oil and stock markets has attracted considerable attention in the past years because of its implications for portfolio selection and risk management. It has been well recognized that the dependence between oil and U.S. stock markets is not time invariant, and the interactions between these two markets depend on macroeconomic activity, as documented in Huang et al (1996), Sadorsky (1999), Park and Ratti (2008), Kilian and Park (2009), Mollick and Assefa (2013), Sukcharoen et al. (2014), etc. If it is possible to find out some
influential economic factors of oil and stock dependence, portfolio investors and risk managers can utilize the information to improve forecasting the joint distribution of oil and stock returns. To this end, this paper is designed to investigate the economic factors of oil and stock dependence and their economic significance in portfolio management.

However, the sampling frequency of economic indicators is usually lower than the sampling frequency of oil or stock returns. Economic indicators are usually reported on a monthly basis, while oil or stock returns can be observed daily or even more frequently. To see how the economic indicators affect the oil and stock dependence, a traditional way is to aggregate daily returns into monthly and to build a regression model merely based on monthly data. One drawback of this approach is that it ignores the intra-month information by aggregation. Meanwhile, the model is updated once in a month and may not provide enough information for investors who rebalance their oil and stock portfolios within a month.

We address the limitations by proposing a copula model with explanatory variables based upon the idea of mixed frequency data sampling (copula-MIDAS-X). The mixed frequency sampling scheme utilizes both monthly economic indicators and daily returns information, and the copula approach enables us to examine the economic determinants of oil and stock dependence unaffected by the marginal distributions. Hence, the new model offers real-time forecasting of oil and stock dependence on a daily basis, which is more attractive to investors with frequent rebalancing strategies (e.g. daily or weekly). Based upon the new model, the paper is then designed to answer the following two questions. First, what are the most influential economic factors affecting the dependence oil and U.S. stock markets? Second, does understanding the economic factors of oil and stock dependence bring any economic profits to investors?

The copula-MIDAS-X model extends earlier mixed frequency data sampling (MIDAS) models by incorporating low frequency explanatory variables into the dynamic dependence structure. Since the seminal work by Ghysels et al. (2004), a number of MIDAS models have been proposed to extract the long-run and short-run components in returns, volatilities, correlations and copula based dependence structures. Examples can be found in Ghysels et al. (2005), Clements and Galvao (2008), Colacito et al. (2011), Engle et al. (2013), Conrad et al. (2014), Gong et al. (2018), etc. Among them, Gong et al. (2018) use a mixed frequency data sampling copula model without any explanatory variables (Copula-MIDAS) to analyze the dependence of returns and bid-ask spreads in stock index futures. We modify their model by adding low frequency explanatory variables that may affect the dependence structure into the copula model. The new model with explanatory variables is able to find out the most influential factors of the dependence structure, in particular, the factors affecting the tail dependence structure in extreme cases.
Also, our empirical findings about the economic determinants of oil and stock dependence complement the literature on the contemporaneous dependence of oil and stock returns. Existing literature have found that the dependence of oil and stock returns is linked with some macroeconomic variables. Our analysis using copula-MIDAS-X model makes one-step further by investigating the most influential economic factors of oil and stock dependence. It is found that aggregate demand and stock specific negative news are more important than other factors. The impacts of aggregate demand will last for two years, while the impacts of stock specific negative news will last for one quarter. Furthermore, most existing studies only have in-sample analysis. Unlike them we provide both the in-sample and out-of-sample results. And the out-of-sample portfolio performance reveals that investors rebalancing portfolios daily or weekly should use the information of monthly economic indicators and daily returns in portfolio optimization.

The remainder of this paper is organized as follows. Section 2 is a brief review of related literature. Section 3 presents the copula-MIDAS-X model and its estimation. Section 4 is devoted to explaining the data and descriptive statistics. Section 5 is the main results which investigate the economic factors affecting the dependence of oil and stock markets. Section 5 concludes.

2. Literature review

Extant researches have investigated the economic determinants of oil and stock dependence mainly from the following four perspectives.

First, global oil supply is the physical availability of crude oil, which is often measured by percent change in the global production of crude oil as in Hamilton (2003), Kilian (2008).

Second, aggregate demand is the demand for crude oil driven by fluctuations in the global business cycle. It used to be measured by macroeconomic indicators such as industrial production, interest rates, employment (or unemployment) rate in Sadorsky (1999), Papapetrou (2001), Cong et al. (2008), Park and Ratti (2008), Cunado and de Gracia (2014). In recent years, Kilian (2009) starts to use the global index of dry cargo singly voyage freight rates as a proxy for aggregate demand. There has been a number of literature which evaluate and compare the impacts of global oil supply and aggregate demand on oil and stock dependence at the same time, such as Kilian and Park (2009), Filis et al. (2011), Wang et al. (2013), Fang and You (2014). And it is concluded that global oil supply is less important for understanding the changes in oil and stock prices than aggregate demand.

Third, oil specific demand is interpreted as a country’s precautionary demand for oil arising from the uncertainty about oil shortfalls. It is measured by the innovations in oil prices under the econometric models in Kilian and Park (2009). Apergis and Miller (2009) show that oil market idiosyncratic demand shocks contribute to
explaining stock returns in most developed countries. Fayyad and Daly (2011) find that the predictive power of oil returns for stock returns increases after a rise in oil price for Gulf Cooperation Council (GCC) countries, UK and USA. Naifar and Dohaiman (2013) address the importance of oil price fluctuations in explaining the dependence of oil and stock returns in GCC countries.

Fourth, stock specific shocks are also believed to be one of the factors affecting oil and stock dependence. Aloui et al. (2012) point out that the linkage between oil and stock returns in emerging markets depends on the bullish or bearish stock market conditions. Also, stock specific shocks can be related with the global financial crisis as financial crisis usually starts with a crash in stock market. Mollick and Assefa (2013) find that oil and U.S. stock markets are more positively correlated during the 2008 global financial crisis. Aloui et al. (2013) provide evidence that the positive dependence between oil and stock returns in Central and Eastern European (CEE) countries is stronger during global financial crisis. Similar conclusions can be found in Pan (2014) and Zhu et al. (2014) as well.

To summarize, most existing literature use in-sample data to analyze the economic determinants of oil and stock dependence in two different ways: some of them merely based on monthly data investigate the contributions of economic factors quantitatively, while the others utilize daily data to describe oil and stock dependence and then explain the impacts of low frequency economic indicators qualitatively. We contribute to the literature by evaluating four types of economic factors under a unified and quantitative framework, using the copula-MIDAS-X model proposed in this paper. Moreover, instead of modeling the in-sample oil and stock dependence, we also evaluate the potential benefits brought to investors who account for the economic factors in the out-of-sample portfolio risk management.

3. Econometric methodology

The copula-MIDAS-X model in this paper is a natural extension of the copula-MIDAS model without any explanatory variables by Gong et al. (2018). We incorporate low frequency explanatory variables into the copula-MIDAS model so as to explain the evolution of dynamic dependence structure.

3.1 Copula-MIDAS-X model

Suppose there are bivariate daily (high frequency) time series \( r_{1t}, r_{2t}, \) \( t = 1, \ldots, T \) and \( S \) monthly (low frequency) economic variables \( X_\tau = X_{1,\tau}, \ldots, X_{S,\tau}, \) \( \tau = 1, \ldots, \lceil T/N \rceil \) (\( N \) days in a month). We want to examine how \( X_\tau \) affects the long-run dependence of \( r_{1t}, r_{2t} \). Let \( F_1 \) and \( F_2 \) represent the marginal cumulative distribution functions (CDF) of \( r_{1t} \) and \( r_{2t} \) on day \( t \). By the
probability integral transformation, it is $u_i = F_{Y_i}$ for $i = 1, 2$.

The copula-MIDAS-X model is written as follows:

$$
(u_{1t}, u_{2t}) \sim C(u_{1t}, u_{2t}; \theta_t, \eta), \quad \theta_t = \Lambda(\lambda_t),
$$

(1)

$$
\lambda_t = \lambda_t + \alpha \Phi^{-1}(u_{1t-1})\Phi^{-1}(u_{2t-1}) + \beta \lambda_{t-1}.
$$

(2)

$$
\lambda_t = c + \sum_{s=1}^{S} \gamma_s \left[ \sum_{k=0}^{K} \phi_k(\omega_s) X_{s,t-k} \right].
$$

(3)

Equation (1) shows the copula-MIDAS model belongs to the time-varying copula family. $\theta_t$ is a time-varying parameter and $\eta$ is a parsimonious parameter. $\theta_t$ is assumed to be driven by an unobserved dynamic process $\lambda_t$, such that $\theta_t = \Lambda(\lambda_t)$, where $\Lambda(\cdot)$ is an increasing transformation to ensure that $\theta_t$ remains in its domain as in Patton (2006).

Equation (2) decomposes the dynamic dependence $\lambda_t$ into two parts: the long-run component (monthly) $\lambda_t$ and the short-run component (daily) $\alpha \Phi^{-1}(u_{1t-1})\Phi^{-1}(u_{2t-1}) + \beta \lambda_{t-1}$. $\lambda_t$ takes a different value each month or every $N$ days. The specification of short-run component follows Patton (2006) and the short-lived effects are captured by an autoregressive lag $\lambda_{t-1}$ and a data-driven term $\Phi^{-1}(u_{1t-1})\Phi^{-1}(u_{2t-1})$, where $\Phi^{-1}(\cdot)$ is the inverse of standard normal CDF. Hence, $\alpha$ and $\beta$ are the coefficients measuring short-run components in dependence, and $\beta$ is within $-1$ and $1$.

Equation (3) is different from the extant copula-MIDAS model. $\lambda_t$ represents the fundamental or secular causes of time variation in dependence, and it is assumed to be affected by an intercept $c$ and $S$ monthly economic variables $X_{s,t}$. For $s = 1, \cdots, S$, $\gamma_s$ measures how many impacts $X_{s,t}$ has on $\lambda_t$ and $\omega_s$ measures how long the impacts last. $\varphi_k(\omega_s)$ is the Beta weight function in Colacito et al. (2011), which gives higher to lower weights to $X_{s,t-1}, \cdots, X_{s,t-K}$ sequentially, and $\varphi_k(\omega_s) = (1-k/K)^{\omega_s-1}/\sum_{k=1}^{K} (1-k/K)^{\omega_s-1}$. Hence, $c$ and $(\gamma_s, \omega_s)$ for $s = 1, \cdots, S$ are the coefficients measuring long-run components in dependence, in particular, $(\gamma_s, \omega_s)$ measures the impacts of economic indicators on dependence.

The copula-MIDAS-X model differentiates from extant models in two aspects. First, it enables us to examine the determinants of long-run dependence by introducing explanatory variables $X_{s,t}$ into the dynamics of long-run component $\lambda_t$. 


The extant MIDAS models have no exogenous explanatory variables, such as the dynamic conditional correlation (DCC) MIDAS by Colacito et al. (2011) and the copula-MIDAS by Gong et al. (2018). Second, it is flexible enough to describe the dynamics of nonlinear dependence structure, in particular, the dynamics of upper and lower tail dependence. To be specific, the copula-MIDAS-X model with SJC copula (discussed in section 3.2) is able to model the short-run and long-run components in the upper and lower tail dependence simultaneously. However, existing models focus on linear correlation rather than nonlinear tail dependence, such as the dynamic conditional correlation in Engle et al. (2002) and Colacito et al. (2011), and the correlation of skewed t copula in Gong et al. (2018).

3.2 Examples: Gaussian and SJC copula-MIDAS-X

The Gaussian copula-MIDAS-X model has one time-varying correlation $\theta_t$ and has no parsimonious parameter $\eta$ in equation (1). $\Lambda(\cdot)$ is a Fisher transformation $\Lambda(\lambda_t) = \frac{1-\exp(-\lambda_t)}{1+\exp(-\lambda_t)}$ that ensures $\theta_t$ in the domain $(-1,1)$. The dynamics of $\lambda_t$ are given in equation (2) and (3).

The SJC copula-MIDAS-X model $C_{SJC}^{X}(u_{t1,u_{t2};\theta})$ has two time-varying tail dependence coefficients $\theta_t = (\kappa_t^L, \kappa_t^U)'$ and has no parsimonious parameter $\eta$ in equation (1)\(^1\). The lower-tail dependence $\kappa_t^L$ and the upper-tail dependence $\kappa_t^U$ follow the dynamics in equation (4) and (5). For $j = L,U$, $\kappa_t^j = \Lambda(\lambda_t^j)$ is a logistic transformation $\Lambda(\lambda_t^j) = 1/[1+\exp(-\lambda_t^j)]$ that ensures each $\kappa_t^j$ in the domain $(0,1)$.

\[
\lambda_t^L = \lambda_t^L + \alpha^L \Phi^{-1}(u_{t1-1})\Phi^{-1}(u_{t2-1}) + \beta^L \lambda_t^{L-1}, \quad (4)
\]

\[
\lambda_t^U = c^U + \sum_{s=1}^{S} \gamma_t^s \left[ \sum_{k=0}^{K} \varphi_k(\omega_t^{j_k})X_{s,r-t} \right], \quad (5)
\]

3.3 Estimation

The model is estimated by the two-step maximization likelihood method (MLE) in Joe (1997) and Gong et al. (2018). The only difference is that the long-run component in our copula-MIDAS-X model is a linear combination of weighted lagged explanatory variables, while the long-run component in copula-MIDAS by Gong et al. (2018) is a weighted sum of past realized correlations.

\(^1\) There is no parsimonious parameter $\eta$ in our examples of Gaussian and SJC copula-MIDAS-X models. But in t copula-MIDAS-X, $\theta_t$ is the time-varying correlation and $\eta$ is the degree of freedom.
The two-step procedures are as follows. First, estimate the marginal models of daily oil and stock returns and transform them into marginal CDF by probability integral transformation, denoted by \( (\hat{u}_t, \hat{u}_{t_2}) \), \( t=1,\cdots,T \). Second, plug \( (\hat{u}_t, \hat{u}_{t_2}) \) into the copula log-likelihood function and estimate the copula-MIDAS-X parameters by MLE. The optimal MIDAS lag \( K \) is determined by selecting the smallest number of MIDAS lags after which the log-likelihood values seem to reach the plateau. Under certain regularity conditions, the asymptotic properties of the two-step estimator are normally distributed. The variance-covariance matrix is estimated by the block bootstrapping procedure with block length \( \sqrt{T} \).

4. Data description

4.1 Data

This paper investigates the dependence of daily oil and stock returns and the potential monthly economic factors that affect the dependence. The sample is from January 1\(^{st}\), 1997 to February 28\(^{th}\), 2018. We divide the sample into two sub-periods, such that the observations from 1997 to 2016 are used for the in-sample estimation and the remaining observations in 2017 and 2018 are reserved for the out-of-sample forecast.

For the daily data, we use the daily closing prices of Brent crude oil, expressed in dollars per barrel, for the oil market and the daily S&P 500 index for the U.S. stock market. Brent oil is chosen as it is a reference for determining the price of other light crudes in Europe and is closely related to other crude oil markers, such as those for West Texas Intermediate, Maya, Dubai, etc. (Ciner et al. 2013, Reboredo and Rivera-Castro, 2014; Sukcharoen et al., 2014). Brent oil prices are collected from the U.S. Energy Information Administrate (EIA) website. S&P 500 index is obtained from Bloomberg. Let \( P_{i,t} \) be the daily price of oil \( (i = OIL) \) or stock \( (i = STK) \) on day \( t \). The daily return is calculated as \( r_{i,t} = 100 \times \left( \ln P_{i,t} - \ln P_{i,t-1} \right) \).

For the monthly data, we follow Kilian and Park (2009) and consider four economic variables: (1) global oil production in thousands barrel per day \( GOP_t \) as a proxy for global oil supply; (2) global index of dry cargo singly voyage freight rates \( KI_t \) constructed by Kilian (2009) as a proxy for aggregate demand; (3) monthly closing prices of Brent crude oil \( P_{OIL,t} \); (4) monthly S&P 500 index \( P_{STK,t} \). The global oil production data are collected from the U.S. EIA website. The global demand index is collected from Kilian’s website\(^2\).

It should be noted that the four monthly economic variables above are not the explanatory variables \( X_t \) in the copula-MIDAS-X model as they are highly

\(^2\) The personal website of Kilian: http://www-personal.umich.edu/~lkilian.
correlated. To solve the multi-collinearity problem, like Kilian and Park (2009), we decompose them into four orthogonal innovations by a structural vector autoregressive (SVAR) model specified in equation (7).

$$A_{y_{t}} = A_{0} + \sum_{j=1}^{J} B_{j} y_{t-j} + \eta_{t},$$

where $y_{t} = (\Delta \ln G O P_{t}, \Delta K I_{t}, \Delta \ln P_{O I L,t}, \Delta \ln P_{S T K,t})'$ is a vector including monthly changes in log global oil production, Kilian index, log Brent oil price and log S&P 500 index. $A = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$ is a full rank lower-triangular matrix, $A_{0}$ is the intercept vector, and $B_{j}$ is the autoregressive matrix with the maximum lag length $J = 6$ as in Wang et al. (2013).

The orthogonal structural innovations $\eta_{t}$ are used as the explanatory variables $X_{t} = X_{G O P,t}, X_{K I,t}, X_{O I L,t}, X_{S T K,t}$ in the copula-MIDAS-X model. Among them, $X_{G O P,t}$ represents the global oil supply shocks, $X_{K I,t}$ represents the aggregate demand shocks, $X_{O I L,t}$ is the U.S. precautionary oil demand shocks (or U.S. oil specific shocks), and $X_{S T K,t}$ is the U.S. stock specific shocks.

Why do we select the above four economic factors rather than other macroeconomic indicators as the explanatory variables in the copula-MIDAS-X model? The first reason is to avoid multi-collinearity in regression. The four economic factors can be decomposed into four orthogonal shocks with economic meanings by Kilian and Park (2009)’s SVAR model. However, if some other macroeconomic indicators are used, such as industrial production, interest rates, employment rate, consumer price index, it is difficult to evaluate their individual contributions to oil and stock dependence as they are highly correlated. Another reason is that the four economic factors also contain certain information from those macroeconomic indicators. For example, the aggregate demand index that measures real economic activity may share similar information with industrial production and employment rate.

4.2 Descriptive statistics

Figure 1 plots the daily Brent oil and U.S. stock returns. Oil and stock returns tend to co-move in turmoil periods, such as the period 1999-2001, 2008-2009 and 2015-2017. It is also obvious that oil returns are much more fluctuated than stock returns.
Figure 2 plots the monthly global oil production, Kilian index, Brent oil price and S&P 500 index. In panel (a), global oil supply has increased steadily in the past twenty years with a few declines in 1999, 2003 and 2008 as the Organization of Petroleum Exporting Countries (OPEC) cut the quotas. In panel (b), aggregate demand represented by Kilian index reached peaks in 2000 and in early 2008, but dropped dramatically during the subprime crisis. From 2015 to 2017, aggregate demand fell to the trough and then rose due to the increasing demand from large emerging countries like China and India, as mentioned in Fang and You (2014). In panel (c) and (d), the monthly oil price and stock index reached a peak and then declined together due to the 9-11 attacks during 1999 and 2001. Similar pattern can be observed in the period 2008-2009, when both oil and stock prices experienced an extremely bearish performance and recovered afterwards, as discussed in Reboredo and Rivera-Castro (2014). After the subprime crisis, the stock index did not always move in the same directions as oil price. But from 2015 to 2017 the two markets were booming together. The co-movement patterns observed here coincide with the daily oil and stock returns in Figure 1.

Table 1 reports the summary statistics of daily oil and stock returns. For the full sample, oil has slightly lower returns than stock, but its standard deviation is almost twice as much as stock, which is also observed in Figure 1. It implies that Brent oil is more risky than stock. Both oil and stock returns exhibit non-normal features like negative skewness and heavy tails, and they exhibit serial correlation and volatility clustering. Similar features of oil and stock returns can be found in Fayyad and Daly (2011), Mollick and Assefa (2013), Sukcharoen et al. (2014).

Overall, the preliminary analysis above tells us that oil and stock trend to boom or crash together and the co-movement can be linked with some macroeconomic shocks, such as global oil supply, aggregate demand and subprime crisis. It then provides us some hints to employ the copula-MIDAS-X model to further investigate the economic factors of oil and stock dependence.

5. Empirical results

The section has three parts: the in-sample estimates of marginal models for oil and stock returns, the in-sample estimates of copula models for oil and stock dependence and the out-of-sample performance evaluation of oil and stock portfolios.

5.1 In-sample estimates of marginal models

Table 2 shows the in-sample estimates of marginal models for daily oil and stock returns in the period 1997-2016. The ARMA-TGARCH model with student’s t error in equation (8) is employed as the returns are non-normally distributed, serially correlated and volatility clustered. For  \( i = OIL, STK \),
\[ r_{i,t} = \delta_{i,0} + \delta_{i,1} r_{i,t-1} + \sqrt{h_{i,t}} \varepsilon_{i,t} + \sqrt{h_{i,t-1}} \varepsilon_{i,t-1}, \quad \varepsilon_{i,t} \mid \Omega_{i-1} \sim i.i.d. T(\nu_i), \]  
\[ h_{i,t} = \phi_{i,0} + \phi_{i,1} h_{i,t-1} \varepsilon_{i,t-1}^2 + \phi_{i,2} h_{i,t-1} \varepsilon_{i,t-1}^2 1(\varepsilon_{i,t-1} < 0) + \phi_{i,3} h_{i,t-1}. \]  
(8)

The last row of Table 2 indicates that both marginal models are correctly specified. We fail to reject the Kolmogorov–Smirnov tests with the null hypothesis that the model is correctly specified. The result justifies us to estimate copula models by the marginal CDFs of the residuals \( (\hat{\varepsilon}_{OIL,i}, \hat{\varepsilon}_{STK,i}) \). Also, oil and stock returns are heavily tailed \( (\nu_i > 2) \) and serially correlated \( (\delta_{OIL,1} > 0) \). The conditional mean of \( r_{STK,t} \) only has an intercept because the ARMA coefficients are insignificant if added. Large (small) volatilities are likely to be followed by large (small) volatilities \( (\phi_{1,3} > 0) \), and bad news tends to have greater impacts on volatilities than good news \( (\phi_{1,2} > 0) \). The results of marginal models are not only in line with the summary statistics in Table 1, but also found in related literatures, such as Filis et al. (2011), Aloui et al. (2013), Pan (2014), Zhu et al. (2014).

5.2 In-sample estimates of copula models

This section investigates the asymmetric feature of oil and stock dependence in section 5.2.1 and the economic factors of oil and stock dependence in section 5.2.2.

Table 3 provides the in-sample estimates of four copula models. We first transform the standardized residuals \( (\hat{\varepsilon}_{OIL,i}, \hat{\varepsilon}_{STK,i}) \) into marginal CDFs \( (\hat{u}_{OIL,i}, \hat{u}_{STK,i}) \) by probability integral transformation of student’s t distribution and then plug them into copula models. The first two copulas are the copula-MIDAS-X models proposed in this paper: Gaussian copula-MIDAS-X in equation (1)-(3) focuses on the oil and stock correlation without any tail dependence, while SJC copula-MIDAS-X in equation (1), (4), (5) focuses on the tail dependence of oil and stock. The maximum lag is chosen to be 36. Besides, to investigate the importance of mixed frequency data sampling approach, two more copulas with just monthly or daily data are included.

**SJC copula-X (monthly data):**

\[ \left( u_{OIL,t}, u_{STK,t} \right) \sim \mathcal{C}_{SJC} \left( u_{OIL,t}, u_{STK,t} ; k_t^j, k_t^w \right), \quad k_t^j = \Lambda \left( \lambda_t^j \right), \]

\[ \lambda_t^j = c^j + \sum_{s=1}^{S} \gamma_t^j X_{s,t-1}, \quad j = L, U. \]

(9)

Here \( \left( u_{OIL,t}, u_{STK,t} \right) \) are the marginal CDFs of monthly oil and stock returns, \( c^j \) is the intercept, and \( \gamma_t^j \) measures how much impact the 1-month lagged variables \( X_{s,t-1} \) have on the latent tail dependence \( \lambda_t^j \). As discussed in Section 3.2, \( S = 4 \),

\[ X_{1,t-1}, \ldots, X_{S,t-1} = X_{GOP_{1,t-1}}, X_{K1_{1,t-1}}, X_{OIL_{1,t-1}}, X_{STK_{1,t-1}}. \]

This dynamic copula model with explanatory variables is also used in Patton (2004).
SJC copula (daily data):
\[
\left( u_{\text{OIL},t}, u_{\text{STK},t} \right) = C^{\text{SJC}} \left( u_{\text{OIL},t}, u_{\text{STK},t}; \kappa^l_t, \kappa^u_t \right), \quad \kappa^j_t = \Lambda \left( \lambda^j_t \right),
\]
\[
\lambda^j_t = c^j + \alpha^j \Phi^{-1} \left( u_{\text{OIL},t-1} \right) \Phi^{-1} \left( u_{\text{STK},t-1} \right) + \beta^j \lambda^j_{t-1}, \quad j = L, U.
\]

Here \( c^j \) is the intercept, \( \alpha^j \) and \( \beta^j \) are the same as those in equation (2). Since all the variables are daily, we do not include any monthly variables into the model. This dynamic copula model without explanatory variables is used in Patton (2006).

5.2.1 The asymmetry of oil and stock dependence

Table 3 tells us that the dependence of oil and stock returns is asymmetric: they are more likely to decrease together than increase together.

To start with, among the four copulas, the SJC copula-MIDAS-X model has the best in-sample goodness-of-fit performance with the highest log-likelihoods and the lowest HQIC (Hannan-Quinn Information Criterion). When we switch from SJC copula-MIDAS-X to Gaussian copula-MIDAS-X, the HQIC value rises by around 15% (from -205.16 to -175.01), indicating there is nonlinear tail dependence between oil and stock returns captured by SJC copula. Also, the HQIC values are even higher for the SJC copula with just daily data and the SJC copula-X with just monthly data. It addresses the importance of both monthly and daily information in forecasting the oil and stock dependence. Reboredo and Rivera-Castro (2014), by means of wavelet cross-correlation, also find out strong dependence between oil and U.S. stock returns at either daily or monthly level.

Furthermore, the estimates of SJC copula-MIDAS-X show that oil and stock are more likely to decrease together than increase together. The intercept of the long-run component in the lower tail is generally higher than the intercept in the upper tail (\( c^L > c^U \)). By Logistic transformation, the lower tail dependence coefficient on average is 0.4027 while the upper tail dependence coefficient is only 0.3087.

Figure 3 also reveals the asymmetric dependence pattern between oil and stock returns. Panel (a) plots the lower tail dependence ranging from 0.35 to 0.65 and panel (b) plots the upper tail dependence ranging from 0.15 to 0.45. The solid black line is the daily lower or upper tail dependence coefficient \( \kappa^j_t = \Lambda \left( \lambda^j_t \right) \), while the red dotted line is the monthly long-run component of the lower or upper tail dependence \( \kappa^j_t = \Lambda \left( \lambda^j_t \right), \quad j = L, U \). It can be observed that daily tail dependence always fluctuates around the long-run monthly component of tail dependence. More importantly, the lower tail dependence is generally higher than the upper tail dependence, suggesting the oil and stock markets are more likely to crash together than boom together.

The implications of positive and asymmetric tail dependence between oil and stock markets are that the two markets do not provide diversification benefits during
extreme financial conditions, such as the recent subprime crisis. This asymmetric dependence between oil and stock returns has been well documented in the literature, such as Aloui et al. (2013) for the Central and Eastern European (CEE) transition economies, Pan (2014) for the BRIC countries, Reboredo and Rivera-Castro (2014) for the U.S. and European markets and Zhu et al. (2014) for the Asia-Pacific countries. However, Wang et al. (2013) use the SVAR model in Kilian and Park (2009) and find no significant nonlinear Granger causality relationship between monthly oil and stock returns. Unlike them, the copula approach in this paper enables us to uncover the asymmetric tail dependence between daily oil and stock returns. Besides, Sukcharoem et al. (2014) find the oil and stock dependence in U.S. is symmetric and quite low from 1982 to 2007. Our results with much higher asymmetric tail dependence differ from theirs as we account for the sample after the subprime crisis.

5.2.2 The determinants of oil and stock dependence

In Table 3, the result of SJC copula-MIDAS-X indicate that the dependence of oil and stock returns is affected by the aggregate demand and stock specific shocks, but not by the global oil supply and precautionary oil demand shocks.

First, oil and stock tend to decrease dramatically when the aggregate demand declines or when the stock market crashes. In the lower tail part of SJC copula-MIDAS-X, both $\gamma_{KI}^{L}$ and $\gamma_{STK}^{L}$ are significantly negative, while neither $\gamma_{GOPI}^{L}$ nor $\gamma_{OIL}^{L}$ is significant. Specifically, negative $\gamma_{KI}^{L}$ means if aggregate demand declines, both oil and stock markets will experience bearish performance and have extreme decreases in prices, thus leading to an increase in the lower tail dependence. This is exactly what happened during the subprime crisis in 2008-2009. The decline of aggregate demand leads to bullish oil and stock markets so that the lower tail dependence goes up, which is also observed during 2008 and 2009 in panel (a) of Figure 3. The importance of aggregate demand on the relationship between oil and stock markets has been discussed in many literatures, such as Kilian and Park (2009), Filis et al. (2011), Wang et al. (2013), Fang and You (2014). Meanwhile, negative $\gamma_{STK}^{L}$ means if the stock price is driven down by some bad news irrelevant with global oil supply and aggregate demand, both oil and stock prices will decrease dramatically and result in an increase in the lower tail dependence as well. The conclusions are supported by Aloui et al. (2013), Mollick and Assefa (2013) and Zhu et al. (2014), who point out that the oil and stock dependence is closely related with global financial crisis originated from the stock market. Furthermore, it should be noted that $|\gamma_{KI}^{L}|$ is more than twice as much as $|\gamma_{STK}^{L}|$, which implies that the impacts of aggregate demand on the lower tail dependence of oil and stock returns are much larger than the impacts of stock specific shocks.

Second, oil and stock are more likely to increase together when the aggregate
demand is driven up. In the upper tail part of SJC copula-MIDAS-X, only \( \gamma_{KI}^U \) is significantly positive, while the other three factors are insignificant. Positive \( \gamma_{KI}^U \) suggests that an increase in aggregate demand causes bullish performance in oil and stock markets, thus leading to an increase in upper tail dependence between them. The situation occurs in late 2009 due to the global recovery from subprime crisis and in 2016 due to the rising demand from emerging markets. In both cases, rising aggregate demand is followed by booming in oil and stock markets. The findings are rarely discussed in existing studies, as they care about the oil and stock dependence in recessions rather than in booms.

Third, aggregate demand will influence the oil and stock dependence in the next two years, while the impacts of stock specific negative shocks will last for a quarter. Remember that \( \omega_{KI} \) in Gaussian copula-MIDAS-X measures how long the impacts of aggregate demand will have on oil and stock correlation, and \( \omega_{KI}^L \), \( \omega_{STK}^L \) and \( \omega_{KI}^U \) in SJC copula-MIDAS-X measure how long the impacts of aggregate demand or stock specific shocks will have on the lower or upper tail dependence between oil and stock.

Figure 4 visualizes the decaying impacts of past monthly economic variables on the current oil and stock correlation in panel (a) and on the current tail dependence in panel (b), (c) and (d), by plugging \( \omega_{KI} \), \( \omega_{KI}^L \), \( \omega_{STK}^L \) and \( \omega_{KI}^U \) into the weight function \( \varphi_k(\cdot) \) in equation (3). The horizon axis is the number of months the impacts last, and the vertical axis is the weight proportion each lagged month takes on the current correlation or tail dependence. It is found in panel (b) and (d) that aggregate demand will influence the oil and stock lower and upper tail dependence in the next two years, as the weights decay to zero at around 20 to 24 lags. On the contrary, in panel (c), the influence of U.S. stock specific shock on lower tail dependence will last for 3 months, which is much shorter than the impacts of aggregate demand. It indicates that global demand, as one of the most important factors of oil and stock dependence, tends to influence the linkage between oil and stock markets for about two years, while the stock specific bad news also exerts some negative effects on oil and stock markets, but the effects can be quickly absorbed by the markets within a quarter. There are a large number of literatures concerning how long stock price is affected by oil price, oil supply and aggregate demand, such as Sadorsky (1999), Papapetrou (2001), Ewing and Thompson (2007), Park and Ratti (2008), Wang et al. (2013), and so on. However, few of them ever examine their impacts on the cross-market dependence between oil and stock.

Last but not least, the contributions of global oil supply and precautionary oil demand shocks to the oil and stock dependence are limited. Neither global oil production shocks nor the oil specific shocks, regarded as precautionary oil demand
shocks in Kilian (2009), have significant coefficients in Gaussian or SJC copula-MIDAS-X models. The limited contributions of global oil supply are contrary to Cunado and de Gracia (2014), in which the response of European stock price changes to oil price changes are mostly driven by oil supply shocks rather than demand shocks during 1973 to 2001. And the limited contributions of oil specific shocks are also somewhat different from some literatures. Among them, Apergis and Miller (2009) find that the U.S. oil market idiosyncratic demand shocks play an important role in explaining stock returns, but their sample is from 1981 to 2007, much shorter than our analysis. Fayyad and Daly (2011) provide evidence that an increased oil price has predictive power on U.S. stock returns, but they do not account for global oil supply and aggregate demand shocks together with oil price.

Our findings about the economic factors of oil and stock dependence complement the literatures on the contemporaneous dependence of oil and stock returns, such as Filis et al. (2011), Aloui et al. (2013), Naifar and Al Dohaiman (2013), Sukcharoen et al. (2014), Zhu et al. (2014). These copula-based studies usually investigate one economic determinant of oil and stock dependence in a qualitative way. For instance, Aloui et al. (2013) qualitatively relate the contagion effect between oil and stock markets to the 2008-2009 global financial crises. Sukcharoen et al. (2013) document that the relatively strong oil and stock dependence is possibly due to the introduction of Euro in 1999 in a qualitative way as well. However, this paper pushes their results further by examining four potential economic factors of oil and stock dependence in a unified and quantitative framework. We examine how the oil and stock dependence is affected simultaneously by global oil supply, aggregate demand, precautionary oil demand and stock specific shocks. Even though the four economic factors may overlap with each other from an economic point of view, we decompose them into four uncorrelated shocks by means of the SVAR model in Kilian and Park (2009), thus avoiding the multi-collinearity problem. More importantly, the SJC copula-MIDAS-X model enables us to distinguish the different impacts when both markets crash together and when both markets boom together. And the separation of the economic factors in the lower and upper tail dependence is rarely reported in existing literatures.

The investment implications of understanding the economic determinants of oil and stock dependence are twofold. First, regulators and investors are strongly suggested to be concerned with the changes in aggregate demand in the past two years in their portfolio management. Aggregate demand is the most influential factor that predicts the oil and stock dependence and its impacts will last for about two years, in either bear market or bull market. Second, investors should also pay attention to big negative news from the stock market in the past three months, as it may result in crashes in both stock and oil prices. Stock specific bad news has some explanatory power on the downside dependence between oil and stock returns, but the effects may
last no more than a quarter, which is possibly explained by the overreaction of market participants.

5.3 Out-of-sample evaluation of portfolio performance

To explore the economic value of investigating the factors affecting oil and stock dependence, in this section we consider the optimization problem for an investor allocating wealth between Brent oil and S&P 500 index under short sales constraints. Remember that there are 281 daily observations and 14 monthly observations in the out-of-sample period (January 1st, 2017 to February 28th, 2018). A recursive window method is utilized. At the beginning of each day in the out-of-sample period, investors are supposed to use the historical information up to day \( t \) (starting from January 1st, 1997) to forecast the joint distribution of oil and stock returns on day \( t+1 \), and then determine the optimal weights on oil and stock by minimizing the portfolio’s conditional value-at-risk (CVaR). It is applicable to the investors based on Gaussian copula-MIDAS-X, SJC copula-MIDAS-X and SJC copula.. For the investors based on SJC copula-X, the procedure is similar except that they are using monthly information up to month \( \tau \) and rebalance their portfolios once in a month, overall 14 times during the out-of-sample period.

We follow Rockafeller and Uryasev (2000) and solve the mean-CVaR optimization problem in equation (11).

\[
\begin{align*}
\min_{\eta} \text{CVaR}_q &= \text{VaR}_q + \frac{1}{M (1-q)} \sum_{m=1}^{M} [-\eta^T \hat{r}_{t+L} - \text{VaR}_q]^+ , \\
\text{s.t.} & \\
& \eta_{\text{OIL}} + \eta_{\text{STK}} = 1 \\
& \eta^T \bar{\mu} \geq \bar{\mu}_0 \\
& 0 \leq \eta_i \leq 1, \quad i = \text{OIL, STK}.
\end{align*}
\]

\( \eta=(\eta_{\text{OIL}}, \eta_{\text{STK}}) \) is the weights on oil and stock. \( q = 5\% \) is the significance level and \( \text{VaR}_q \) is the 5\% value-at-risk. \( [z]^+ = \max(0,z) \). \( \hat{r}_{t+L}^{m} \) is the predicted portfolio return on day \( t+L \) based on the information up to day \( t \) in the \( m^{th} \) simulated scenario, and we consider \( M =10000 \) times of simulations. \( \bar{\mu} \) is the vector of expected oil and stock returns (one-year historical returns), and \( \mu_0 \) is the expected portfolio return (3-month Treasury Bill rate).

Table 4 provides the portfolio performance measures based on the four copula models. We consider three types of investors who rebalance their portfolios on a daily (\( L=1 \)), weekly (\( L=5 \)) and monthly (\( L=22 \)) basis. In each case, the portfolios’ annualized return (Mean), annualized standard deviation (SD), Sharpe ratio, terminal wealth value (starting value at 100) and 5\% CVaR are reported. Also, the bottom of each panel reports the results of DM test by Diebold and Mariano (2002) with SJC copula-MIDAS-X as the benchmark model. The null hypothesis is that there is no difference in CVaR between the benchmark and alternative copula. It is worth mentioning that the performance of SJC copula-X based strategy is the same for daily,
weekly and monthly rebalance frequency as the investor only uses monthly information to adjust their portfolio weights. Three points can be summarized from Table 4.

First, the strategy based on the SJC copula-MIDAS-X model outperforms the other three strategies during the out-of-sample period. Among the four copula-based strategies, the strategy following SJC copula-MIDAS-X has the highest return, Sharpe ratio, terminal wealth value and the lowest CVaR for either rebalancing frequency. The rejections of pairwise DM tests also indicate that the CVaRs of SJC copula-MIDAS-X based strategy are statistically lower than other strategies. For example, suppose an hypothetical investor who uses the SJC copula-MIDAS-X model to describe the oil and stock dependence and rebalance his/her portfolio on a daily basis, he will gain $122.92 at the end of February 2018 with 5% CVaR at 18.45%, if he initially invests $100 at the beginning of 2017.

Second, utilizing mixed frequency information is generally more important than describing the tail dependence. If an investor switches from SJC copula-MIDAS-X to Gaussian copula-MIDAS-X, which means he ignores the nonlinear tail dependence between oil and stock, the reduction in Sharpe ratio is not too much: from 2.51 to 2.17 on a daily basis, from 1.80 to 1.79 on a weekly basis, from 1.74 to 1.59 on a monthly basis. However, if the investor switches from SJC copula-MIDAS-X to SJC copula-X or SJC copula, which means he uses only monthly or daily information, the reductions in Sharpe ratio are much more noteworthy: from 2.51 to 1.49/1.47 on a daily basis, from 1.80 to 1.49/1.33 on a monthly basis, from 1.74 to 1.49/1.44 on a monthly basis. It implies that using mixed frequency data contributes more than capturing oil and stock tail dependence to forecasting the joint distribution of oil and stock returns and to improving portfolio performance. The advantage of using mixed frequency information over using single frequency information is also illustrated in Gong et al. (2018), who find that mixed frequency information helps investors better predict the liquidity risk in stock index futures market.

Third, investors with daily or weekly rebalance frequency are suggested to follow the mixed frequency copula based strategies. The advantage of copula-MIDAS-X over SJC copula-X and SJC copula becomes fewer and fewer as the investor rebalances the portfolio from every day to every week and to every month. For example, for daily investors, the annualized returns of Gaussian and SJC copula-MIDAS-X based strategies (15.91%, 18.06%) are much higher than the returns of the other two strategies using just monthly or daily information (13.38%, 13.22%). For weekly investors, the difference between mixed frequency copulas (15.89%, 16.89%) and the other two copulas (13.38%, 14.12%) still can’t be ignored. However, for monthly investors, there are no obvious differences in returns among the four copula based strategies (13.39%, 14.80%, 13.38%, 14.29%). Therefore, mixed daily and monthly information is only useful for investors with relatively higher
rebalance frequency (daily or weekly in our case), while it is less valuable for investors with lower rebalance frequency (monthly).

In summary, this section uses the copula-MIDAS-X model to investigate the economic factors affecting the dependence of oil and stock markets. It is found that aggregate demand and stock specific negative shocks are the key economic determinants of oil and stock dependence, while global oil supply and precautionary oil demand have limited contributions. Specifically, aggregate demand decline and large bad news from U.S. stock market tend to result in bearish performance in both oil and stock markets, while aggregate demand growth tends to lead to bullish performance in both oil and stock markets. The impacts of aggregate demand will last for two years, and the impacts of stock specific negative shocks will last for a quarter. Furthermore, the out-of-sample portfolio performance highlights the economic value of capturing the economic determinants in oil and stock dependence. For investors who rebalance their portfolio daily or weekly, the strategy utilizing mixed frequency information is preferred to the strategies only based on monthly or daily information.

6. Conclusion

The dependence between oil and stock markets is an important issue for global portfolio investors and risk managers, as they tend to add crude oil into a stock portfolio for diversification. In this paper, a copula-MIDAS model with explanatory variables (copula-MIDAS-X) is proposed to investigate the economic factors of oil and stock dependence. The new model, which is built upon Colacito et al. (2011) and Gong et al. (2018), not only extends the mixed frequency data sampling scheme from correlation to the more general dependence measure copula, but also incorporates the low frequency economic explanatory variables into the copula model. It facilitates us to describe the oil and stock dependence regardless of their marginal distributions, and also enables us to measure the impacts of economic factors on oil and stock dependence.

In the empirical analysis of Brent oil and U.S. stock markets, it is found that the copula-MIDAS-X model has the better in-sample goodness-of-fit and out-of-sample performance than other copulas. The dependence between oil and stock returns is asymmetric, which implies that the diversification benefit of adding oil into a stock portfolio is limited during financial crisis. More importantly, the oil and stock dependence is influenced by aggregate demand and negative news from the stock market. The impacts of aggregate demand will last for two years, while the impacts of bad news in stock market will last for one quarter. It suggests that market regulators and investors should pay more attention to the changes in aggregate demand as it has influential and long-lasting effects on the oil and stock markets. Also, it advises investors who rebalance portfolios daily or weekly to account for both daily returns
and monthly economic information in portfolio management.
Reference


[32] Reboredo, J.C., Rivera-Castro, M.A. Wavelet-based evidence of the impact of oil


### Table 1 Descriptive statistics of daily Brent oil and S&P 500 returns

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>In-sample</th>
<th>Out-of-sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1997/1/1-2018/2/28</td>
<td>1997/1/1-2016/12/31</td>
<td>2017/1/1-2018/2/28</td>
</tr>
<tr>
<td></td>
<td>OIL</td>
<td>STOCK</td>
<td>OIL</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0213</td>
<td>0.0248</td>
<td>0.0198</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.3074</td>
<td>1.2156</td>
<td>2.3414</td>
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<tr>
<td>Skewness</td>
<td>-0.0232</td>
<td>-0.2207</td>
<td>-0.0179</td>
</tr>
<tr>
<td>Ex. Kurtosis</td>
<td>4.8092</td>
<td>7.8654</td>
<td>4.7437</td>
</tr>
<tr>
<td>JB (×10³)</td>
<td>4.98***</td>
<td>13.38***</td>
<td>4.59***</td>
</tr>
<tr>
<td>Ljung-Box</td>
<td>44.26***</td>
<td>141.15***</td>
<td>42.78***</td>
</tr>
<tr>
<td>ARCH</td>
<td>110.59***</td>
<td>71.49***</td>
<td>106.91***</td>
</tr>
<tr>
<td>ADF</td>
<td>-70.44***</td>
<td>-76.90***</td>
<td>-68.48***</td>
</tr>
</tbody>
</table>

The table reports the summary statistics of daily Brent oil and S&P 500 returns. Std. Dev. is standard deviation. Ex. Kurtosis is excess kurtosis. J-B is the Jarque-Bera test statistic for normality. Ljung-Box and ARCH are the Ljung-Box tests for serial correlation and for GARCH effect with 12 lags. ADF is the augmented Dicky-Fuller unit root test. *** , ** and * denote significance at the 1%, 5% and 10% levels.
Table 2 In-sample estimates of marginal models: Brent oil returns and S&P 500 returns

<table>
<thead>
<tr>
<th></th>
<th>OIL</th>
<th>STOCK</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{i,0}$</td>
<td>0.0096* (0.0050)</td>
<td>$\delta_{i,0}$</td>
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<tr>
<td>$\delta_{i,1}$</td>
<td>0.5709*** (0.1653)</td>
<td>$\delta_{i,1}$</td>
</tr>
<tr>
<td>$\delta_{i,2}$</td>
<td>-0.6434*** (0.1617)</td>
<td>$\phi_{i,0}$</td>
</tr>
<tr>
<td>$\phi_{i,0}$</td>
<td>$10^5$</td>
<td>$\phi_{i,1}$</td>
</tr>
<tr>
<td>$\phi_{i,1}$</td>
<td>$10^5$</td>
<td>$\phi_{i,2}$</td>
</tr>
<tr>
<td>$\phi_{i,2}$</td>
<td>$10^5$</td>
<td>$\phi_{i,3}$</td>
</tr>
<tr>
<td>$\phi_{i,3}$</td>
<td>$10^5$</td>
<td>$\nu_i$</td>
</tr>
<tr>
<td>logL</td>
<td>-2.9498</td>
<td>logL</td>
</tr>
<tr>
<td>K-S</td>
<td>0.0048</td>
<td>K-S</td>
</tr>
</tbody>
</table>

The table reports the in-sample estimates of marginal models for daily Brent oil returns and S&P 500 returns from January 1st, 1997 to December 31st, 2016. ***, ** and * denote significance at the 1%, 5% and 10% levels. logL is the log likelihoods of each marginal model. K-S is the Kolmogorov-Smirnov test statistic with the null hypothesis that the model is correctly specified.
Table 3 In-sample estimates of U.S. oil-stock dependence by copula models

<table>
<thead>
<tr>
<th></th>
<th>Gaussian copula-MIDAS-X</th>
<th>SJC copula-MIDAS-X</th>
<th>SJC copula-X</th>
<th>SJC copula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.0154* (0.0079)</td>
<td>( \alpha^L ) 0.1104*** (0.0245)</td>
<td>( \alpha^L ) -4.9114*** (1.2875)</td>
<td>( \alpha^L ) -4.1601*** (0.0011)</td>
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<tr>
<td>( \beta )</td>
<td>0.9898** (0.0064)</td>
<td>( \beta^L ) 0.9877*** (0.004)</td>
<td>( \gamma_{GOP}^L ) 0.029 (0.018)</td>
<td>( \beta^L ) 0.7894*** (0.003)</td>
</tr>
<tr>
<td>( c )</td>
<td>0.0010 (0.0008)</td>
<td>( c^L ) -0.3941*** (0.1279)</td>
<td>( \gamma_{Ki}^L ) 0.249 (0.039)</td>
<td>( c^L ) -9.1224*** (0.0045)</td>
</tr>
<tr>
<td>( \gamma_{GOP} )</td>
<td>0.0023 (0.0026)</td>
<td>( \gamma_{GOP}^L ) 0.029 (0.018)</td>
<td>( \gamma_{Ki}^L ) 0.249 (0.039)</td>
<td>( \alpha^U ) -2.7463*** (0.0023)</td>
</tr>
<tr>
<td>( \gamma_{Ki} )</td>
<td>-0.0009** (0.0004)</td>
<td>( \gamma_{Ki}^L ) -0.0052*** (0.001)</td>
<td>( \gamma_{STK}^L ) 0.209 (0.07)</td>
<td>( \beta^U ) 0.7498*** (0.0012)</td>
</tr>
<tr>
<td>( \gamma_{OIL} )</td>
<td>-0.0002 (0.0002)</td>
<td>( \gamma_{OIL}^L ) -0.0039 (0.0027)</td>
<td>( c^U ) -3.9634*** (1.3291)</td>
<td>( c^U ) -8.8161*** (0.0176)</td>
</tr>
<tr>
<td>( \gamma_{STK} )</td>
<td>-0.0007 (0.0009)</td>
<td>( \gamma_{STK}^L ) -0.0022* (0.0011)</td>
<td>( \gamma_{GOP}^U ) -1.6662 (2.2945)</td>
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<tr>
<td>( \omega_{GOP} )</td>
<td>8.9286 (7.1104)</td>
<td>( \omega_{GOP}^L ) 143.1754 (137.712)</td>
<td>( \gamma_{Ki}^U ) 0.079 (0.093)</td>
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<tr>
<td>( \omega_{Ki} )</td>
<td>3.5742 (3.9236)</td>
<td>( \omega_{Ki}^L ) 2.5623*** (0.716)</td>
<td>( \gamma_{OIL}^U ) -0.2251 (0.227)</td>
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<tr>
<td>( \omega_{OIL} )</td>
<td>229.0317 (430.7674)</td>
<td>( \omega_{OIL}^L ) 142.5450 (56.3763)</td>
<td>( \gamma_{STK}^U ) -0.1338 (0.139)</td>
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<tr>
<td>( \omega_{STK} )</td>
<td>2.0013 (3.1969)</td>
<td>( \omega_{STK}^L ) 88.7827** (35.426)</td>
<td>( \gamma_{OIL}^U ) -0.2251 (0.227)</td>
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<tr>
<td>( \omega_{STK} )</td>
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<tr>
<td></td>
<td>( \alpha^U ) 0.1324 (0.1567)</td>
<td>( \beta^U ) 0.9888** (0.0088)</td>
<td>( c^U ) -0.806* (0.435)</td>
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<tr>
<td></td>
<td>( \alpha^U ) 0.1324 (0.1567)</td>
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<td>( c^U ) -0.806* (0.435)</td>
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<tr>
<td></td>
<td>( \gamma_{Ki}^U ) -0.0506 (0.0973)</td>
<td>( \gamma_{Ki}^U ) -0.0506 (0.0973)</td>
<td>( \gamma_{Ki}^U ) -0.0506 (0.0973)</td>
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<tr>
<td></td>
<td>( \gamma_{Ki}^U ) 0.0059* (0.0019)</td>
<td>( \gamma_{Ki}^U ) 0.0059* (0.0019)</td>
<td>( \gamma_{Ki}^U ) 0.0059* (0.0019)</td>
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</table>
The table reports the in-sample estimates of four copulas models for oil and U.S. stock dependence: Gaussian copula-MIDAS-X, SJC copula-MIDAS-X, SJC copula-X and SJC copula. The in-sample period is from January 1st, 1997 to December 31st, 2016. logL is the log likelihoods of both copula density and marginal models. HQIC is the Hannan-Quinn information criteria. ***, ** and * denote significance at the 1%, 5% and 10% levels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_U^{oil}$</td>
<td>0.0047</td>
<td>(0.0053)</td>
</tr>
<tr>
<td>$\gamma_U^{STK}$</td>
<td>0.0110</td>
<td>(0.0072)</td>
</tr>
<tr>
<td>$\omega_{GDP}^U$</td>
<td>264.1391</td>
<td>(482.6373)</td>
</tr>
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<td>$\omega_{KPI}^U$</td>
<td>5.5951**</td>
<td>(2.6876)</td>
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<td>$\omega_{oil}^U$</td>
<td>249.6003</td>
<td>(590.9999)</td>
</tr>
<tr>
<td>$\omega_{STK}^U$</td>
<td>2.0030</td>
<td>(2.2869)</td>
</tr>
</tbody>
</table>

| LogL | -175.0100 |
| HQIC | -205.1600 |
| HQIC | 3.4837 |
| HQIC | -44.0820 |
The table reports the out-of-sample performance of oil and stock portfolios from January 1st, 2017 to February 28th, 2018 based on the four copulas models: Gaussian copula-MIDAS-X, SJC copula-MIDAS-X, SJC copula-X and SJC copula. We consider three types of investors who rebalance their portfolios on a daily ($L=1$), weekly ($L=5$) and monthly ($L=22$) basis. The table reports the portfolios’ annualized return (Mean), annualized standard deviation (SD), Sharpe ratio (SR), terminal wealth value (End value starting at 100), 5% CVaR, and the pairwise Diebold and Mariano (2002) test statistic (DM test) with SJC copula-MIDAS-X as the benchmark model. The null hypothesis that there is no difference in CVaR between the benchmark and alternative copula. ***, ** and * denote significance at the 1%, 5% and 10% levels.

<table>
<thead>
<tr>
<th></th>
<th>Daily ($L=1$)</th>
<th>Weekly ($L=5$)</th>
<th>Monthly ($L=22$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gaussian copula-MIDAS-X</td>
<td>SJC copula-MIDAS-X</td>
<td>SJC copula-X</td>
</tr>
<tr>
<td>Mean</td>
<td>15.9081</td>
<td>18.0557</td>
<td>13.3798</td>
</tr>
<tr>
<td>SD</td>
<td>7.3163</td>
<td>7.1811</td>
<td>8.9841</td>
</tr>
<tr>
<td>SR</td>
<td>2.1743</td>
<td>2.5143</td>
<td>1.4893</td>
</tr>
<tr>
<td>End value</td>
<td>119.9042</td>
<td>122.9172</td>
<td>115.6974</td>
</tr>
<tr>
<td>CVaR(5%)</td>
<td>19.0481</td>
<td>18.4508</td>
<td>20.3525</td>
</tr>
<tr>
<td>DM test</td>
<td>2.4622**</td>
<td>12.6975***</td>
<td>14.4550***</td>
</tr>
</tbody>
</table>

|                | Gaussian copula-MIDAS-X | SJC copula-MIDAS-X | SJC copula-X | SJC copula |
| Mean           | 15.8893       | 16.8909         | 13.3798       | 14.1212     |
| SD             | 8.8968        | 9.3792          | 8.9841        | 10.6177     |
| SR             | 1.7860        | 1.8009          | 1.4893        | 1.3300      |
| End value      | 119.0164      | 120.3011        | 115.6974      | 115.8534    |
| CVaR(5%)       | 19.4137       | 17.9212         | 19.3525       | 19.5510     |
| DM test        | 7.5688***     | 6.1269***       | 8.3586***     |

|                | Gaussian copula-MIDAS-X | SJC copula-MIDAS-X | SJC copula-X | SJC copula |
| SD             | 8.4148        | 8.5264          | 8.9841        | 9.9337      |
| SR             | 1.5912        | 1.7355          | 1.4893        | 1.4382      |
| End value      | 116.2750      | 118.0100        | 116.2633      | 117.3803    |
| CVaR(5%)       | 18.4137       | 16.9212         | 18.3525       | 19.4258     |
| DM test        | 6.1269***     | 5.6911***       | 26.8943***    |
Figures

Notes: The figure plots daily Brent oil and S&P 500 index returns from January 1st, 1997 to February 28th, 2018. The returns are calculated as the log difference of daily price multiplied by 100.

Figure 1. Daily returns of Brent oil and S&P 500 index
Notes: The figure plots monthly global oil production in thousands barrel per day, Kilian index, monthly Brent oil price and monthly S&P 500 index from January 1997 to February 2018.

Figure 2. Monthly oil production, Kilian index, Brent oil prices and S&P 500 index
Notes: The figure plots the lower tail dependence of oil and stock returns in panel (a) and the upper tail dependence of oil and stock returns in panel (b). The solid black line is the logistic transformation of daily tail dependence $\kappa^i_t = \Lambda\left(\lambda^i_t\right)$, and the red dotted line is the logistic transformation of monthly long-run dependence $\kappa^j_t = \Lambda\left(\lambda^j_t\right)$, $j = L, U$. The sample is from January 1st, 1997 to December 31st.

Figure 3. Tail dependence of oil and stock returns
Notes: Panel (a) plots the impacts of aggregate demand on oil and stock correlation based on Gaussian copula-MIDAS-X. Panel (b) and (d) plot the impacts of aggregate demand on the lower and upper tail dependence between oil and stock based on SJC copula-MIDAS-X. Panel (c) plots the impacts of stock specific shocks on the lower tail dependence between oil and stock based on SJC copula-MIDAS-X. The horizon axis is the number of months the impacts last, and the vertical axis is the weight proportion each lagged month takes on the current correlation or tail dependence. The sample is from January 1st, 1997 to December 31st.

Figure 4. Impacts of lagged monthly economic factors on current oil and stock dependence