

## **Learning about Profitability Growth and Expected Stock Returns**

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### **Abstract**

This paper develops and tests a dynamic cash-flow model with learning about long-run profitability growth for pricing cross-sectional stock returns. The learning model extended from Pástor and Veronesi (2003; 2005) shows that expected stock returns are associated with two systematic risks: cash-flow beta (i.e., short-run profitability growth co-movement with aggregate consumption) and growth beta (i.e., long-run profitability growth co-movement with aggregate consumption). We then construct a two-factor model consisted of a short-run cash-flow factor and a long-run profitability growth factor. The two-factor model can characterize all the nine pricing factors and explain their risk premiums proposed in the literature and the cross section of average stock returns formed on profitability, growth, momentum, and volatility. The findings imply that the cash-flow beta and the growth beta are two common elements in asset pricing.

Keywords: Learning, profitability, growth, asset pricing, common factors, anomalies

## 1. Introduction

The Fama-French (1993) three-factor model consisted of a market factor, a value factor, and a size factor and the Carhart (1997) momentum factor have been widely used to explain the cross section of stock returns. In particular, as shown by Fama and French (1992, 1993, 1996), several return anomalies associated with the capital asset pricing model (CAPM), except for the continuation of medium-term returns, largely disappear in their three-factor model. However, recent studies have shown several important empirical irregularities that are hardly explained by the Fama-French (1993) three-factor model augmented by the Carhart (1997) momentum factor.

For example, Novy-Marx (2013) shows that profitable firms have significantly low book-to-market ratios but generate significantly higher returns than do unprofitable firms. The result suggests a negative value premium that contradicts the prediction of the value factor in Fama and French (1992, 1993). In addition, he also shows that the Fama-French (1993) and Carhart (1997) four-factor model fails to explain the portfolio returns formed on the prior 11-month returns. The finding is puzzling since these portfolio returns are the main objective that the momentum factor is designed to account for. Moreover, Fama and French (2014) recently propose a new five-factor model with the addition of an investment factor and a profitability factor. Most strikingly, they find that the value factor in their model becomes redundant for describing average returns. These empirical irregularities not only challenge the structure of the factor pricing models, but also call for a better theoretical explanation for the ambiguous role of the value premium (i.e., firms with high book-to-market (B/M) equity earn higher future returns).

Motivated by these empirical irregularities, this paper develops and tests a dynamic cash-flow model with learning about long-run profitability growth for pricing cross-sectional stock returns. In the model, a firm's current profitability moves toward its long-run profitability

growth, which is unobservable and can be inferred from the market conditions. The cash-flow process follows the dynamic valuation models of Brennan and Xia (2001), Pástor and Veronesi (2003), and Bakshi and Chen (2005), while the rational learning mechanism through the aggregate market is built on Pástor and Veronesi (2005). Therefore, in addition to the systematic risk for the exposure of a firm's current profitability to aggregate consumption (e.g., cash-flow beta), the rational learning mechanism induces another systematic risk for the exposure of the firm's long-run profitability growth to aggregate consumption (e.g., growth beta). The expected stock return is determined by these two separate yet equally important systematic risks. In bad time, while firms with positive cash-flow betas are risky and suffer from profitability decline, firms with negative growth betas are less risky because they are able to hedge against the decline in consumption.

This study is also motivated by the recent development of new factor models. Hou, Xue, and Zhang (2014a), inspired by the neoclassical q-theory of investment, construct an empirical q-factor model with four factors, consisted of a market factor, a size factor, an investment factor, and a profitability factor. They show that the performance of their q-factor model is at least comparable to, and in many cases better than that of the Fama-French (1993) and Carhart (1997) four-factor model in capturing anomalies. Hou, Xue, and Zhang (2014b) further show that the q-theory four-factor model empirically outperforms the Fama and French (2014) five-factor model. They therefore raise the concerns about association and motivation of the five-factor model with respect to the underlying valuation theory suggested by Fama and French (2006).

However, as argued by Cochrane (1991), the logic of the production-based asset-pricing model is exactly analogous to that of the consumption-based model. In addition, as suggested by Lin and Zhang (2013), the investment approach is no more and no less causal than the

consumption approach in explaining anomalies. Thus, the structural q-theory model of Liu, Zhang, and Whited (2009) and the empirical q-factor model of Hou, Xue, and Zhang (2014a) encourage a refined fundamental valuation model for the better understanding of the contemporary factors.

In our proposed dynamic cash-flow model, the stock price for a firm is expressed by its earnings times its price-to-earnings ratio, where the latter is solely determined by the long-run profitability growth. The consumption asset-pricing theory suggests that the expected stock return is determined by the covariance between aggregate consumption growth and the percentage change in stock prices. As a result, our model follows that short-run profitability growth (i.e., the percentage change in current earnings) and long-run profitability growth enter the pricing equation through the cash-flow beta and the growth beta, respectively.

Furthermore, our dynamic cash-flow model provides important empirical implications for these two betas. For example, if higher short-run profitability growth is associated with a higher positive cash-flow beta, then an increase in current earnings implies an increase in stock price and also an increase in expected stock return. Thus, the model implies a negative value premium (i.e., higher B/M is associated with lower expected returns) and a positive price momentum for stocks with positive cash-flow betas. On the other hand, if higher long-run profitability growth is associated with a more negative growth beta, then an increase in long-run profitability growth implies an increase in stock price (since long-run profitability growth is positively related to P/E ratios) but a decrease in expected stock returns. In this case, the model implies a positive value premium and a negative price momentum (or reversal) for stocks with negative growth betas.

To test the model, the first step is to appropriately identify the cash flows across firms. We use the operating profitability of Ball, Gerakos, Linnainmaa, and Nikolaev (2014) as the measure

of corporate earnings, and then define the growth rate of which as the operating profitability growth. This measure undoes Compustat's adjustment on the selling, general and administrative expenses and provides more timely alignment between revenues and expenses.<sup>1</sup> In the second step, a firm's operating profitability growth is then decomposed into (i) the short-run component and (ii) the long-run component, since the model implies that each of these two components constitutes systematic risks with respect to the aggregate consumption growth.

An empirical challenge in the second step is the measurement of the unobservable long-run profitability growth. To deal with the learning problem, we reformulate the underlying model in a state-space form and then apply the maximum likelihood estimation with the Kalman filtering procedure. The optimal filtering provides the estimates for the entire latent process of the long-run profitability growth. The short-run component of a firm's operating profitability growth is then identified by the operating profitability growth net of its long-run mean. The short-run component and the long-run component of the operating profitability growth are then projected into aggregate consumption growth to generate the cash-flow beta and the growth beta, respectively.

We construct 20 test portfolios based on 10 portfolios formed on operating profitability growth and another 10 portfolios formed on the earnings-to-price ratio.<sup>2</sup> These test portfolios meet the empirical patterns implied by the model. In the first set of the test portfolios, firms with high operating profitability growth rates tend to have high post-formation stock returns, lower pre-formation book-to-market ratios, and high pre-formation stock returns. These return patterns

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<sup>1</sup> As Ball, Gerakos, Linnainmaa, and Nikolaev (2014) point out, the accounting item of selling, general and administrative expenses (XSGA) in Compustat contains a firm's actual reported expenses of that as well as the research and development expenditures (XRD), but research and development expenditures which might largely be used to generate future revenues rather than current revenues are expensed as incurred due to conservative accounting rules.

<sup>2</sup> While the E/P ratio has been examined by many studies (see, for example, Basu, 1983; Fama and French, 1996; among others), prior literature uses different measures of earnings from that used in this paper.

indicate a negative value premium and a positive price momentum. In the second set of the test portfolios, firms with high earnings-to-price ratios tend to have high post-formation stock returns, higher pre-formation book-to-market ratios, and lower pre-formation stock returns. These return patterns show a positive value premium and a negative price momentum (i.e., reversal). We show that our measured cash flow beta and growth beta can explain more than 80% of the cross-sectional variation in the risk premiums. Furthermore, the estimated risk prices of both betas are statistically significant and positive in all cases. For the return spreads, the cash-flow beta captures 58% of the risk premium explained by the model and the growth beta captures the remaining 42% of that in portfolios formed on operating profitability growth rates. In contrast, the growth beta captures more than 90% of the risk premium for the return spreads explained by the model in portfolios formed on earnings-to-price ratios. Thus, consistent with the model prediction, the P/E ratio is solely determined by the long-run growth profitability and the return premium associated with the P/E ratio is mainly explained by the growth beta.

Inspired by the pricing structure of the dynamic cash-flow model, we construct an empirical asset pricing model consisted of a short-run cash-flow factor and a long-run profitability growth factor using portfolios formed on operating profitability growth and the P/E ratio. We find that these two cash-flow factors can characterize all the 9 pricing factors and explain their risk premiums proposed by Fama and French (1993, 2014), Carhart (1997), and Hou, Xue, and Zhang (2014a). Further, the two cash-flow factors can also explain the cross section of average stock returns formed on profitability, growth, momentum, and volatility. In the 10 sets of decile portfolios examined, all of the alphas as well as the High-Low alphas are insignificant in our cash-flow model. Applying the GMM cross-sectional regression tests, we find that the two factors are significantly priced and the cash-flow model results in the smallest

pricing errors in 9 sets of the decile portfolios compared with the competing models. Moreover, the cash-flow model passes the Gibbons, Ross, and Shanken (1989, GRS) test in 7 sets of decile portfolios. The findings imply that the cash flow beta and the growth beta are two common elements in asset pricing.

This paper contributes to simplifying the multidimensionality of the cross-sectional expected stock returns. In the American Finance Association presidential address, Cochrane (2011) raises the issue of multidimensional challenge for the cross-sectional anomalies and indicates that “now we have a zoo of new factors.” Indeed, Harvey, Liu, and Zhu (2015) report that there are 316 empirical factors published in a selection of journals since 1967, despite that some of them are not very robust. While the empirical q-theory four-factor model of Hou, Xue, and Zhang (2014a) largely explains the cross-sectional anomalies, this study further reduces the required dimensionality specifically to the two cash-flow factors.

This paper is related to the literature of consumption-based asset pricing models (CCAPM, Rubinstein, 1976; Lucas, 1978; Breeden, 1979). The seminal work of Bansal and Yaron (2004) highlights the importance of long-run consumption risk in explaining the equity market premium and subsequently Bansal, Dittmar, and Lundblad (2005) show that the cash flow covariance with respect to the long-run consumption is important for interpreting differences in the risk premiums across assets. Another successful class of models in addressing the equity premium puzzle is the external habit formation model of Campbell and Cochrane (1999) and Menzly, Santos, and Veronesi (2004). Lettau and Wachter (2007) and Da (2009) suggest that the cash-flow duration (as defined by the expected dividend growth) can help explain the value premium. However, Santos and Veronesi (2010) show that with presence of the strong discount effects implied by Campbell and Cochrane (1999) the cash-flow duration should generate a negative value premium.



Furthermore, Santos and Veronesi (2010) document a “cash-flow risk puzzle” in which the cash-flow risk has a small impact on the value premium observed in the data.

The concept of the long-run profitability growth in this paper is different from that of long-run consumption growth in which the former provides an opposite effect to the cash-flow consumption risk from the firm-level, while the latter induces a pervasive cash-flow consumption risk from the pricing kernel. Likewise, the cash-flow model in this paper also differs from the two-beta model of Campbell and Vuolteenaho (2004) in which their two betas come from the market portfolio. The role of the long-run profitability growth in asset pricing is different from that of the cash-flow duration in which the former is based on the counter-cyclical nature of the profitability growth due to learning through the business cycle, while the latter is built on the term-structure of cash-flow. This study might partly reconcile the “cash-flow risk puzzle” by Santos and Veronesi (2010) since we find that the value premium is largely explained by the long-run growth beta rather than by the cash-flow beta.

The remainder of the paper is organized as follows. The next section presents the dynamic cash-flow model and discusses its empirical implications. Section 3 describes the data and presents the estimation results for the cash-flow model. Section 4 focuses on the construction of the mimicking factors. In Section 5, an empirical two-factor cash-flow model is constructed and the empirical results for the model in explaining the existing factors are reported. Section 6 describes the testing portfolios in the cross-section for the empirical two-factor cash-flow model. Section 7 performs a series of asset pricing tests for the empirical two-factor cash-flow model in dissecting cross-sectional anomalies. Finally, Section 8 contains the concluding remarks.

## **2. The Asset Pricing Model with Learning about Long-run Profitability Growth**

### **2.1. The model**

Consider a firm in the economy whose earnings,  $Y_t$ , evolves according to the following process:

$$\frac{dY_t}{Y_t} = X_t dt + \sigma_{S,0} dW_{0,t} + \sigma_{S,S} dW_{S,t}, \quad (1)$$

where  $X_t$  is the long-run profitability growth;  $dW_{0,t}$  and  $dW_{S,t}$  are uncorrelated Wiener processes for profitability capturing systematic ( $dW_{0,t}$ ) and firm-specific ( $dW_{S,t}$ ) randomness, respectively. The long-run profitability growth,  $X_t$ , is unobservable and follows a mean-reverting process:

$$dX_t = \phi(\mu_X - X_t)dt + \sigma_{L,L} dW_{L,t}, \quad (2)$$

where  $\mu_X$  is the steady-state profitability growth;  $\phi$  is the speed of mean reversion;  $dW_{L,t}$  is another independent Wiener process capturing firm-specific ( $dW_{L,t}$ ) randomness for long-run profitability growth. Assume that the firm has a constant dividend payout ratio,  $\alpha$ , to its earnings, and therefore the dividend payout is  $D_t = \alpha Y_t$ .

The aggregate consumption,  $C_t$ , in this pure exchange economy follows the process:

$$\frac{dC_t}{C_t} = (b_0 + b_1 X_t)dt + \sigma_{C,0} dW_{0,t} \quad (3)$$

where the consumption growth is assumed to contain information about the long-run profitability growth ( $X_t$ ). The expression follows Pástor and Veronesi (2005) because such a link might be plausible ex-ante. Assume that investors are endowed with the preference of the power utility, so that the stochastic discount factor (SDF) is  $\Lambda_t = U_C = e^{-\eta t} C_t^{-\gamma}$ , where  $\eta$  is the time discount parameter and  $\gamma$  is the coefficient of risk aversion. Thus, the process for the SDF can be expressed as:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \gamma \sigma_C dW_{0,t} \quad (4)$$

where  $r_t$  is the risk-free rate and Ito's Lemma implies that  $r_t = \eta + \gamma(b_0 + b_1 X_t) - \frac{\gamma(1+\gamma)}{2} \sigma_C^2$ .

## 2.2. Bayesian learning and asset prices

Since the long-run profitability growth cannot be directly observed, investors learn about the value of  $X_t$  through the information from the current profitability  $Y_t$  and the aggregate consumption  $C_t$ . According to Liptser and Shirayayev (1977), the posterior long-run profitability growth,  $\hat{X}_t = \mathbb{E}[X_t | \mathcal{F}_t]$ , evolves as:

$$d\hat{X}_t = \phi(\mu_X - \hat{X}_t)dt + \sigma_{\hat{X},0}d\tilde{W}_{0,t} + \sigma_{\hat{X},S}d\tilde{W}_{S,t}, \quad (5)$$

where  $\sigma_{\hat{X},0} = b_1 o_t / \sigma_C$ ,  $\sigma_{\hat{X},S} = (\sigma_C - b_1 \sigma_{S,0}) o_t / (\sigma_C \sigma_{S,S})$ , and  $o_t$  is the prediction error, which is defined as  $o_t = \mathbb{E}[(X_t - \hat{X}_t)^2 | \mathcal{F}_t]$ ; the processes for  $o_t$ ,  $d\tilde{W}_{0,t}$  and  $d\tilde{W}_{S,t}$  are:

$$\begin{aligned} \frac{dY_t}{Y_t} &= \hat{X}_t dt + \sigma_{S,0} d\tilde{W}_{0,t} + \sigma_{S,S} d\tilde{W}_{S,t}, \\ \frac{dC_t}{C_t} &= (b_0 + b_1 \hat{X}_t) dt + \sigma_C d\tilde{W}_{0,t}, \end{aligned} \quad (6)$$

where

$$\frac{do_t}{dt} = \sigma_{L,L}^2 - 2\phi o_t - o_t^2 \left( \frac{\sigma_C^2 - 2b_1 \sigma_{S,0} \sigma_C + b_1^2 (\sigma_{S,S}^2 + \sigma_{S,0}^2)}{\sigma_C^2 \sigma_{S,S}^2} \right).$$

Due to Bayesian learning, the posterior long-run profitability growth  $\hat{X}_t$  contains both the systematic ( $d\tilde{W}_{0,t}$ ) and the firm-specific ( $d\tilde{W}_{S,t}$ ) randomness. Denote that  $\sigma_Y d\tilde{W}_{Y,t} = \sigma_{S,0} d\tilde{W}_{0,t} + \sigma_{S,S} d\tilde{W}_{S,t}$  and  $\sigma_{\hat{X}} d\tilde{W}_{\hat{X},t} = \sigma_{\hat{X},0} d\tilde{W}_{0,t} + \sigma_{\hat{X},S} d\tilde{W}_{S,t}$ . As a result, the rational learning mechanism not only increases the correlation between the current profitability  $Y_t$  and the long-run profitability growth (i.e.,  $\sigma_{\hat{X},Y} \equiv \frac{1}{dt} \text{Cov} \left[ d\hat{X}_t, \frac{dY_t}{Y_t} \right] = \sigma_{\hat{X},0} \sigma_{S,0} + \sigma_{\hat{X},S} \sigma_{S,S} = o_t > 0$ ) which increases from  $\sigma_{X,Y} \equiv \frac{1}{dt} \text{Cov} \left[ dX_t, \frac{dY_t}{Y_t} \right] = 0$ ) but also induces a non-trivial covariance between the long-run profitability growth and the aggregate consumption (i.e.,  $\sigma_{\hat{X},C} \equiv$

$$\frac{1}{dt} \text{Cov} \left[ d\hat{X}_t, \frac{dc}{c_t} \right] = \sigma_{\hat{x},0} \sigma_C = b_1 o_t \neq 0 \text{ compared with } \frac{1}{dt} \text{Cov} \left[ dX_t, \frac{dc}{c_t} \right] = 0.$$

It is now ready to solve the model for the equilibrium market price and the expected stock return. The market price is expressed in terms of the P/E ratio in the following proposition.

**Proposition 1.** The market value for the firm,  $M_t$ , is given by the sum of the discounted value of all future cash flows:

$$M_t = \mathbb{E} \left[ \int_t^\infty \frac{A_s}{A_t} D_s \, ds \right] = Y_t G(\hat{X}_t) \quad (7)$$

$$G(\hat{X}_t) = \frac{M_t}{Y_t} = \left( \alpha \int_t^\infty Z(\hat{X}_t, s) \, ds \right), \quad (8)$$

where

$$Z(\hat{X}_t, s) = \mathbb{E}_t \left[ \frac{A_s Y_s}{A_t Y_t} \right] = \exp(\zeta(s) + \zeta_X(s) \hat{X}_t)$$

The proof is shown in the Appendix. Note that the function  $Z(\hat{X}_t, s)$  is the expected discounted profitability growth, which is exponentially linear only in  $\hat{X}_t$ . The function  $G(\hat{X}_t)$  is exactly equal to the P/E ratio, which is a monotonic transformation of the long-run profitability growth  $\hat{X}_t$ . This property suggests that the posterior long-run profitability growth ( $\hat{X}_t$ ) can be nicely proxied by the P/E ratio in our empirical analysis.

Applying Ito's Lemma, the process for the market price can be expressed as:

$$\begin{aligned} \frac{dM_t}{M_t} &= \frac{dY_t}{Y_t} + \frac{dG}{G} + \left( \frac{dY_t}{Y_t} \right) \left( \frac{dG}{G} \right) \\ &= \mu_M dt + \sigma_Y d\tilde{W}_{Y,t} + \Delta_{\hat{X}} \sigma_{\hat{X}} d\tilde{W}_{\hat{X},t}, \end{aligned} \quad (9)$$

where

$$\Delta_{\hat{X}} = \frac{1}{G} \frac{\partial G}{\partial \hat{X}_t} = \frac{\int_t^\infty \zeta_{\hat{X}}(s) Z(\hat{X}_t, s) \, ds}{\int_t^\infty Z(\hat{X}_t, s) \, ds}$$

Then, the following proposition summarizes the results for the expected stock return and the return volatility.

**Proposition 2.** The process for the excess stock return is  $dR_t = (dM_t + D_t)/M_t - r_t dt$  and the expected excess stock return,  $\mu_R = \frac{1}{dt} \mathbb{E}_t[dR_t]$ , is given by:

$$\mu_R = -\frac{1}{dt} \text{Cov} \left[ \frac{dM_t}{M_t}, \frac{d\Lambda_t}{\Lambda_t} \right] = \gamma(\sigma_{Y,C} + \Delta_{\hat{X}} \sigma_{\hat{X},C}) \quad (10)$$

where  $\sigma_{Y,C} = \sigma_{S,0} \sigma_C$  and  $\sigma_{\hat{X},C} = \sigma_{\hat{X},0} \sigma_C = b_1 \sigma_t$ .

Define the cash-flow beta as  $\beta_Y = \sigma_{Y,C} / \sigma_C^2$  and the growth beta as  $\beta_X = \sigma_{\hat{X},C} / \sigma_C^2$ . Then, in terms of beta-pricing model, the expected stock return becomes  $\mu_R = \gamma \sigma_C^2 (\beta_Y + \Delta_{\hat{X}} \beta_X)$ . In other words, the expected stock return is determined by two sources of systematic risks: one of which is the cash-flow beta  $\beta_Y$  from  $\sigma_{Y,C}$  and the other is the growth beta  $\beta_X$  from  $\sigma_{\hat{X},C}$ .

Further, the return variance can be expressed by:

$$\begin{aligned} \sigma_R^2 &= \sigma_Y^2 + \Delta_{\hat{X}}^2 \sigma_{\hat{X}}^2 + 2\Delta_{\hat{X}} \sigma_{\hat{X},Y} \\ &= \underbrace{\sigma_C^2 (\beta_Y^2 + \Delta_{\hat{X}}^2 \beta_X^2)}_{\text{systematic}} + \underbrace{(\sigma_{S,S}^2 + \Delta_{\hat{X}}^2 \sigma_{\hat{X},S}^2) + 2\Delta_{\hat{X}} \sigma_{\hat{X},Y}}_{\text{firm-specific}} \end{aligned} \quad (11)$$

The total return variance depends on systematic variance as well as firm-specific variance. The systematic component of which comes from the squared cash-flow beta,  $\beta_Y^2$ , and the squared growth beta,  $\beta_X^2$ .

### 2.3. Empirical implications

The role of cash-flow beta is equivalent to the dividend-consumption beta in the standard CCAPM, since the dividend payout in this model is a constant fraction of the earnings. Thus, typically  $\beta_Y > 0$ , which contributes to a positive cash-flow risk premium consistent with the literature.

More importantly,  $b_1$  is the key parameter in this paper that not only drives the non-trivial systematic risk in the growth beta  $\beta_X$ , but also determines the sign of  $\Delta_{\hat{X}}$ , which affects the

pricing effect of  $\beta_X$ . In particular, as described in the Appendix, the sign of  $\Delta_{\hat{X}}$  depends on the sign of  $(1 - \gamma b_1)$ . To discuss the role of  $b_1$  in different regions,  $b_1$  is required such that  $b_1 < 1/\gamma$  to ensure the common notion that high long-run profitability growth has a positive effect on the market value. First of all, if  $b_1 = 0$ , then  $\beta_X = 0$ ,  $\Delta_{\hat{X}} > 0$ , and  $\Delta_{\hat{X}}\beta_X = 0$ , and the model is degenerated to the single-factor cash-flow model for the expected stock return. Second, if  $0 < b_1 < 1/\gamma$ , then  $\beta_X > 0$ ,  $\Delta_{\hat{X}} > 0$ , and  $\Delta_{\hat{X}}\beta_X > 0$ , and the pricing effect of the growth beta in this case is in the same direction as that of the cash-flow beta. Third, if  $b_1 < 0$ , then  $\beta_X < 0$ ,  $\Delta_{\hat{X}} > 0$ , and  $\Delta_{\hat{X}}\beta_X < 0$ , and the growth beta in this case has an opposite pricing effect as does the cash-flow beta.

The role of  $b_1$  in the third case is especially desirable because the long-run profitability growth increases the market price as the current profitability does, but the growth beta provides the other side of the risk premium in contrast to the cash-flow beta. The following corollary explores this property, which can potentially match many empirical features documented in the literature.

**Corollary 1.** Assume that  $b_1 < 0$ ,  $\partial\sigma_{Y,C}/\partial Y_t > 0$ , and  $\partial\sigma_{\hat{X},C}/\partial\hat{X}_t < 0$ . Then,

- (a) an increase in  $Y_t$  is associated with an increase in expected stock returns and an increase in stock price;
- (b) an increase in  $\hat{X}_t$  is associated with a decrease in expected stock returns and an increase in stock price.

The result in Corollary 1 (a) that short-run profitability growth is positively associated expected returns and also market price (i.e., the M/B ratio) suggests a negative value premium but a positive price momentum. In addition, since short-run profitability growth is positively associated expected returns and also the cash-flow beta, it follows that there is a positive volatility premium. The result in Corollary 1 (b) that long-run profitability growth is negatively

associated expected returns but positively with market price (i.e., the M/B ratio) suggests a positive value premium but a negative price momentum (i.e., reversal). In addition, since long-run profitability growth is negatively associated expected returns but positively with the absolute value of the growth beta (which is negative), it follows that there is a negative volatility premium.

In summary, if there is a positive risk premium for the cash-flow beta associated with the current profitability  $Y_t$ , then the positive cash-flow beta should be able to generate a positive price momentum effect. Moreover, if there is a positive risk premium for the growth beta associated with the long-run profitability growth  $\hat{X}_t$ , then the negative growth beta should be able to generate a positive value premium and a negative volatility premium. This corollary constitutes the main testing hypothesis in this paper.

### 3. Empirical Results for the Dynamic Cash-flow Model

#### 3.1. Estimation methodology

In the model, the long-run profitability growth is unobservable and should be learned from the available information through the current profitability and the aggregate consumption. The estimation strategy is to reformulate the underlying model in a standard state space form in discrete time. Define  $y_t = \log(Y_t)$  and  $c_t = \log(C_t)$ , then  $dY_t/Y_t \approx \Delta y_t$  and  $dC_t/C_t \approx \Delta c_t$ . Thus, the measurement equation derived from Equation (1) and (3) is identified as

$$\begin{bmatrix} \Delta y_{t+1} \\ \Delta c_{t+1} \end{bmatrix} = \begin{bmatrix} 0 \\ b_0 \end{bmatrix} + \begin{bmatrix} 1 \\ b_1 \end{bmatrix} x_{t+1} + \begin{bmatrix} \varepsilon_{y,t+1} \\ \varepsilon_{c,t+1} \end{bmatrix}, \quad (12)$$

and the state equation derived from Equation (2) is defined as

$$x_{t+1} = \phi_0 + \phi_X x_t + \varepsilon_{x,t+1}, \quad (13)$$

where  $\varepsilon_{y,t+1}$ ,  $\varepsilon_{c,t+1}$ , and  $\varepsilon_{x,t+1}$  are three independent Gaussian noises with variances  $\sigma_y$ ,  $\sigma_c$ , and  $\sigma_x$ . Then, applying the Kalman filtering procedure, the maximum likelihood estimation provides the estimates for the parameters,  $\Theta = \{b_0, b_1, \phi_0, \phi_X, \sigma_y, \sigma_c, \sigma_x\}$ . With the filtered  $\hat{x}_{t+1}$ , the cash-flow beta ( $\beta_Y$ ) and the growth beta ( $\beta_X$ ) can be estimated by

$$\begin{aligned}\beta_Y &= \frac{\text{Cov}[\Delta y_{t+1} - \hat{x}_{t+1}, \Delta c_{t+1}]}{\text{Var}[\Delta c_{t+1}]}, \\ \beta_X &= \frac{\text{Cov}[\hat{x}_{t+1} - (\hat{\phi}_0 + \hat{\phi}_X x_t), \Delta c_{t+1}]}{\text{Var}[\Delta c_{t+1}]}.\end{aligned}\tag{14}$$

Since Proposition 2 implies that  $\mu_R = \gamma \sigma_c^2 (\beta_Y + \Delta \bar{x} \beta_X)$ , the main empirical tests of the cash-flow model rely on the cross-sectional regressions using the estimates of  $\beta_Y$  and  $\beta_X$ .

### 3.2. Data

The sample comprises NYSE/AMEX/NASDAQ ordinary common stocks. Annual and quarterly financial statement data are collected from COMPUSTAT. Daily and monthly stock return data (with share codes = 10 and 11) are retrieved from the Center for Research in Security Prices (CRSP). Stock returns are adjusted for stock delisting to avoid survivorship bias, following Shumway (1997). Due to the availability of the quarterly data from COMPUSTAT, the sample is from January 1972 to December 2012. Stocks with share prices less than \$1 at the end of the previous month are excluded in the construction of the testing portfolios. Financial firms are identified with one-digit standard industrial classification codes of 6.

Following Lettau and Ludvigson (2001), consumption (C) is measured as either total personal consumption expenditures or expenditures on nondurables and services, excluding shoes and clothing. The quarterly data are seasonally adjusted at annual rates, in billions of chain weighted 1996 dollars. The adjusted consumption data are downloaded from Martin Lettau's



website, in which the source is from the U.S. Department of Commerce, Bureau of Economic Analysis.

Following Ball, Gerakos, Linnainmaa, and Nikolaev (2014), operating profitability ( $OP$ ) is defined as annual revenue minus cost of goods sold and selling, general & administrative expenses, but not expenditures on research & development ( $REVT-COGS-XSGA+XRD$ ). A quarterly version of the operating profitability ( $OPQ$ ) is similarly defined using the corresponding quarterly items ( $REVTQ-COGSQ-XSGAQ+XRDQ$ ). To construct portfolios, quarterly accounting variables are used in the months immediately after the most recent public quarterly earnings announcement dates ( $RDQ$ ). After the portfolio formation, quarterly accounting variables are used in the months corresponding to the fiscal periods ( $DATADATE$ ).

To test whether my model can explain the factors in those popular factor models proposed in the literature, we also construct these factors, including the four factors of Fama-French (1993) and Carhart (1997) ( $FFC4$ ;  $MKT$ ,  $SMB$ ,  $HML$ , and  $UMD$ ), the five factors of Fama and French (2014) ( $FF5$ ;  $MKT$ ,  $SMB$ ,  $HML$ ,  $CMA$ , and  $RMW$ ), and the four factors of Hou, Xue, and Zhang (2014a) ( $Q4$ ;  $MKT$ ,  $rME$ ,  $rI/A$ , and  $rROE$ ). The four factors of Fama-French (1993) and Carhart (1997) are obtained from the online data library of Kenneth French.<sup>3</sup>

### 3.3. Test portfolios for the cash-flow model

The main test portfolios are 10 operating profitability growth ( $\% \Delta OPQ$ ) portfolios and 10 earnings-to-price ratio ( $OPQ/ME$ ) portfolios.  $\% \Delta OPQ$  is a proxy for the short-run profitability growth  $dY_t/Y_t$  in the model and the rationale for the portfolios formed on  $OPQ/ME$  follows Proposition 1 that the earnings-to-price ratio should be inversely and monotonically associated

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<sup>3</sup> <http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>

with the long-run profitability growth  $\hat{X}_t$ .

The  $\% \Delta OPQ$  portfolios: a quarterly measure of operating profitability growth ( $\% \Delta OPQ$ ) is defined as the growth rate of  $OPQ$  relative to  $avgOPQ$ , where  $avgOPQ$  is computed as the lagged average value of  $OPQ$ s over the prior three quarters ( $(OPQ - avgOPQ) / |avgOPQ|$ ).<sup>4</sup> At the beginning of each month, stocks are sorted into 10 portfolios by their recent  $\% \Delta OPQ$ s using NYSE breakpoints.

The  $OPQ/ME$  portfolios: the earnings-to-price ratio ( $OPQ/ME$ ) for each quarter is defined as the quarterly operating profitability ( $OPQ$ ) divided by the market value at the end of the fiscal quarter ( $OPQ / (PRCCQ \times CSHOQ)$ ). At the beginning of each month, stocks are sorted into 10 portfolios by their recent  $OPQ/ME$ s using NYSE breakpoints.

Following Liu, Whited, and Zhang (2009), portfolio returns are equal-weighted because equal-weighted returns are harder for asset pricing models to capture than value-weighted returns (e.g., Fama 1998). The results are similar if the value-weight portfolio returns are used. Following Fama and French (1995), firm-level earnings are first aggregated to portfolio-level earnings and the portfolio-level current profitability growth rates across periods are then computed. To match the quarterly frequency of the consumption growth and to construct a non-overlapping series of profitability growth rates for each monthly rebalanced portfolio, unlike the case for post-formation portfolio returns which are computed each month, post-formation current profitability growth rates are measured only at the end of each calendar quarter (e.g., March, June, September, and December).

For each portfolio formed at the end of the calendar quarter  $t-1$ , post-formation operating profitability growth is measured by the difference between the log of the portfolio  $OPQ$  at

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<sup>4</sup> The results are essentially the same but weaker when  $avgOPQ$  is replaced by the previous year's same quarter  $OPQ$ .

quarter  $t$  and the log of the portfolio  $avgOPQ$  computed from quarter  $t-3$  to  $t-1$ ; averaged aggregate consumption growth rates over quarter  $t-1$  and  $t$  are used. For each quarter, cross-sectional  $OPQs$  are winsorized at the bottom 1% and at the top 1%. To prevent the circumstance in which a negative value is undefined for the log function, a positive constant is pre-added to all of the portfolio earnings before the log function is taken such that these portfolio growth rates are well-defined. The positive constant is arbitrary and does not affect the empirical results in this paper.<sup>5</sup>

### 3.4. Empirical results

#### A. Parameter estimates

To understand the structure of the learning-based cash-flow model, the aggregate earnings for the market is first used to calibrate the model and the parameter estimates from the Kalman filtering procedure in reported in Table 1. The latent long-run profitability growth,  $x_t$ , for the market is quite persistent, as the mean-reversion coefficient  $\phi_X$  is estimated as 0.77 with a standard error of 0.06. More importantly, the information about  $x_t$  learned from aggregate consumption,  $b_1$ , is  $-3.83$  with a standard error of 2.19, suggesting that  $x_t$  tends to be negatively correlated with the average aggregate consumption growth.

Figure 1 shows the short-run profitability growth rate for the market ( $dlogY$ ) estimated from  $\Delta y_t$ , the filtered long-run profitability growth for the market ( $dX$ ) estimated from  $x_t$ , and aggregate consumption growth ( $dlogC$ ) estimated from  $\Delta c_t$ . The short-run market profitability tends to co-move positively with aggregate consumption, as the profitability declines considerably in the recession periods. In contrast, the filtered long-run profitability growth is

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<sup>5</sup> For each set of testing portfolios, the positive constant is computed by 3 times the full sample averaged value of portfolio earnings.

typically high during the recessions. In other words, while the market short-run profitability growth is pro-cyclical, the long-run profitability growth is counter-cyclical.

Table 1 also provides the parameter estimates for the main test portfolios. For portfolios formed on  $\% \Delta OPQ$ , the mean-reversion parameters  $\phi_X$  are roughly of the same level with that of the aggregate market and have a slightly increasing pattern ranging from 0.77 for the bottom portfolio *Low* to 0.87 for the top portfolio *High*. The learning parameters,  $b_1$ , consistent with the finding for the aggregate market, are all negative across portfolios. Moreover, the bottom portfolio *Low* has the highest steady-state long-run mean of profitability growth, which is estimated by  $\phi_0/(1 - \phi_X)$ , while the top portfolio *High* has a negative value of that. Thus, firms with low current operating profitability growth rates tend to have higher long-run profitability growth in the steady state than those with high current operating profitability growth rates. In another testing portfolios formed on  $OPQ/ME$ , similar patterns are found, as the estimates of  $\phi_X$  are persistent, all of the estimates of  $b_1$  are negative, and the bottom portfolio *Low* has the highest steady-state long-run mean.

#### B. Cross-sectional regressions

Table 2 provides the estimates of the cash-flow beta ( $\beta_Y$ ) and the growth beta ( $\beta_X$ ) for the 20 test portfolios. For each portfolio, the cash-flow beta and the growth beta are estimated from Equation (14) using the parameters estimated from the state-space model of Kalman filtering reported in Table 1. The first set of test portfolios formed on  $\% \Delta OPQ$  exhibits an increasing pattern both in  $\beta_Y$  and  $\beta_X$ . More specifically, the portfolio *High* has the highest  $\beta_Y$  at 3.66 with a significant  $t$ -statistic of 2.78, while the portfolio *Low* has the most negative  $\beta_X$  at  $-0.88$  with a significant  $t$ -statistic of  $-9.75$ . Consistent with the model prediction that the P/E ratio is

solely determined by the long-run growth profitability, the second set of portfolios formed on  $OPQ/ME$  exhibits an increasing pattern only in  $\beta_X$  but not in  $\beta_Y$ . The portfolio *Low* has the most negative  $\beta_X$  at  $-1.12$  with a significant  $t$ -statistic of  $-9.81$ .

Post-formation excess returns and pre-formation firm characteristics are also reported in Table 2. For each portfolio,  $SZ(\$m)$  is the average of the firm-level market capitalization (in million dollars);  $B/M$  is computed from the sum of the firm-level book value of equity dividend by the sum of the firm-level market value of equity;  $R\_2\_12$  is value-weighted average of the firm-level prior 11-month returns before the last month. These test portfolios meet the empirical patterns implied by the model. In the first set of the test portfolios formed on  $\% \Delta OPQ$ , firms with high operating profitability growth rates tend to have high post-formation stock returns, lower pre-formation book-to-market ratios, and high pre-formation stock returns. These return patterns indicate a negative value premium and a positive price momentum. In the second set of the test portfolios formed on  $OPQ/ME$ , firms with high earnings-to-price ratios tend to have high post-formation stock returns, higher pre-formation book-to-market ratios, and lower pre-formation stock returns. These return patterns show a positive value premium and a negative price momentum (i.e., reversal).

Table 3 provides the estimates of the risk price for the cash-flow beta ( $\lambda_Y$ ) and the risk price for the growth beta ( $\lambda_X$ ) using the 20 test portfolios. Portfolio returns ( $R_{p,t}$ ) are regressed on the cross-sectional cash-flow betas ( $\beta_Y$ ) and growth betas ( $\beta_X$ ) as follows:

$$R_{p,t} - R_{f,t} = \lambda_0 + \lambda_Y \beta_Y + \lambda_X \beta_X + \varepsilon_{p,t}, \quad (15)$$

where the  $\lambda_Y$  and  $\lambda_X$  are estimated price of risks for  $\beta_Y$  and  $\beta_X$ , respectively. Robust Newey and West (1987)  $t$ -statistics that account for autocorrelations are reported. As reported in Panel A of Table 3, the estimate of  $\lambda_Y$  is 0.26 with a significant  $t$ -statistic of 5.46 and the estimate of  $\lambda_X$

is 1.22 with a significant  $t$ -statistic of 7.46. The average realized returns and the predicted returns estimated from the model for the two sets of test portfolios are plotted in Figure 2 and Figure 3. As can be seen in the figures, the overall variations in the portfolio returns are well captured by the model. Further, the adjusted  $R^2$  for the cross-sectional regression is 0.81. Thus, the cross-sectional dispersion in the measured cash flow beta and growth beta can explain more than 80% of the cross-sectional variation in the risk premiums.

In the  $\% \Delta OPQ$ -sorted portfolios, the average spread of excess returns for *High–Low* is 1.09% per month with a significant  $t$ -statistic of 11.40. As can be seen in Figure 2, there are increasing patterns in the portfolio returns,  $\beta_Y$ , and  $\beta_X$ . From the return spread between *High* and *Low*, the difference in  $\beta_Y$  generates a risk premium of 0.73%, while the difference in  $\beta_X$  generates an additional risk premium of 0.50%. Thus, the cash-flow beta captures 58% of the risk premium explained by the model and the growth beta captures the remaining 42% of that.

In the  $OPQ/ME$ -sorted portfolios, the average spread of excess returns for *High–Low* is 1.80% per month with a significant  $t$ -statistic of 7.49. As can be seen in Figure 3, the increasing portfolio returns are accompanied by the increasing  $\beta_X$  while the pattern in  $\beta_Y$  is relatively flat. From the return spread between *High* and *Low*, the difference in  $\beta_X$  generates the risk premium of 1.25%, while the difference in  $\beta_Y$  only generates the marginal risk premium of 0.12%. In contrast to the  $\% \Delta OPQ$ -sorted portfolios, the growth beta captures more than 90% of the risk premium for the return spreads explained by the model in  $OPQ/ME$ -sorted portfolios.

In summary, consistent with Proposition 2, both the cash-flow beta and the growth beta are significantly priced in the cross-sectional expected stock returns. Further, consistent with the Proposition 1 that the P/E ratio is solely determined by the long-run growth profitability, we find that the return premium associated with the P/E ratio is mainly explained by the growth beta.

Consistent with the Corollary 1 (b), we find a positive value premium and a negative price momentum (i.e., reversal) in the  $OPQ/ME$ -sorted portfolios. Moreover, both the cash-flow beta and the growth beta are important in explaining the return effect of  $\% \Delta OPQ$ , suggesting that sorting on the operating profitability growth provides not only the information about the short-run component but also the long-run component. Nevertheless, the risk premium explained by the cash-flow beta in the  $\% \Delta OPQ$ -sorted portfolios is certainly higher than that by the growth beta. Hence, the condition in Corollary 1 (a) is satisfied, and we find the consistent evidence for a negative value premium and a positive price momentum implied by the model based on the  $\% \Delta OPQ$ -sorted portfolios.

#### **4. Construction of the Empirical Two-factor Model**

Inspired by the pricing structure of the dynamic cash-flow model, we construct an empirical asset pricing model consisted of a short-run cash-flow factor and a long-run profitability growth factor using portfolios formed on operating profitability growth and the P/E ratio. Two mimicking cash-flow factors are constructed as follows. To construct the short-run cash-flow factor ( $F_S$ ), stocks are sorted into three portfolios (*Low*, *Med*, *High*) based on  $\% \Delta OPQ$  using the 30th and 70th percentiles for NYSE stocks as breakpoints. The three portfolios are then intersected with the firms below the NYSE median market capitalization.  $F_S$  is the difference between the returns on *High* and the returns on *Low*. Similarly, to construct the long-run profitability growth factor ( $F_L$ ), stocks are sorted into three portfolios (*Low*, *Med*, *High*) based on  $OPQ/ME$  using the 30th and 70th percentiles for NYSE stocks as breakpoints. The three portfolios are then intersected with the firms below the NYSE median market capitalization.  $F_L$  is the difference between the returns on *High* and the returns on *Low*. Portfolio returns for the

construction of the mimicking factors are equal-weighted and all NYSE/AMEX/NASDAQ common stocks (including financial firms) are used.

The methodology and the sample coverage for the construction of the mimicking factors largely follow the procedure suggested by Fama and French (1993) with only two exceptions. First, only stock returns for small firms are used for the mimicking factors because as reported in Table 2, small firms have more dispersed cash flow beta and the growth beta. Second, the portfolio returns are equal-weighted rather than value-weighted, because the value-weighted returns tilt toward the effect from large firms which typically have less dispersed cash flow beta and growth beta and therefore might underestimate the risk premium associated with the underlying systematic risks. Hence, the procedure used in this paper follows the logic that small firms are more sensitive to the business cycle.

Table 4 reports the performance of the mimicking factors. In Panel A,  $F_S$  is significantly positive at 0.97% per month ( $t$ -stat = 11.95) and  $F_L$  is also significantly positive at 1.31% ( $t$ -stat = 7.34). The risk-adjusted returns for both  $F_S$  and  $F_L$  remain significant. For example,  $F_S$  has a significant alpha of 0.84% ( $t$ -stat = 10.37) in the  $FFC4$  model, a significant alpha of 0.89% ( $t$ -stat = 11.30) in the  $FF5$  model, and a significant alpha of 0.73% ( $t$ -stat = 8.37) in the  $Q4$  model. Further,  $F_L$  also has a significant alpha of 1.08% ( $t$ -stat = 7.61) in the  $FFC4$  model, a significant alpha of 0.81% ( $t$ -stat = 7.16) in the  $FF5$  model, and a significant alpha of 0.82% ( $t$ -stat = 3.78) in the  $Q4$  model. Therefore, the two mimicking factors cannot be fully explained by the existing factor models in the literature.

Panel B of Table 4 presents the Spearman correlations. The correlation between the two mimicking factors is 0.01, suggesting that they separately capture different pricing effects. In the first column,  $F_S$  has positive correlations of 0.25 with the market factor  $MKT$ , 0.13 with the size



factor  $SMB$ , 0.22 with the momentum factor  $UMD$ , and 0.27 with the profitability factor  $rROE$ . In the second column,  $F_L$  has positive correlations of 0.54 with the value factor  $HML$ , 0.33 with the investment factor  $CMA$ , 0.27 with the profitability factor  $RMW$ , 0.34 with the investment factor  $rI/A$ , and 0.13 with the profitability factor  $rROE$ .

The plots in Figure 4 show the time-series evolution for the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ). Shaded areas denote NBER recessions. As can be seen,  $F_S$  is pro-cyclical to the business cycle while  $F_L$  is counter-cyclical. Figure 5 plot the difference between the two factors,  $F_S - F_L$ , together with the  $HML$  factor. In the figure, it seems that they are negatively correlated. Thus, consistent with previous findings,  $F_L$  predicts a positive value premium, while  $F_S$  predicts a negative value premium.

## 5. Empirical Performance of the Empirical Two-factor Cash-flow Model

### 5.1. An empirical two-factor cash-flow model

We consider an empirical two-factor cash-flow model ( $L2$ ), consisted of the newly constructed short-run cash-flow factor ( $F_S$ ) and long-run profitability growth factor ( $F_L$ ). The expected excess return of asset  $p$  can be described as follows:

$$\mathbb{E}[R_p] - R_f = \lambda_S \beta_{p,S} + \lambda_L \beta_{p,L} \quad (16)$$

where  $\lambda_S$  is risk premium associated with  $F_S$ ,  $\lambda_L$  is risk premium associated with  $F_L$ ,  $\beta_{p,S}$  is the empirical cash-flow beta with respect to the short-run cash-flow factor and  $\beta_{p,L}$  is the empirical growth beta with respect to the long-run growth profitability growth factor. The two empirical cash-flow betas,  $\beta_{p,S}$  and  $\beta_{p,L}$ , are the factor loadings estimated from the following time-series regression:

$$R_{p,t} - R_{f,t} = \alpha_p + \beta_{p,S}F_{S,t} + \beta_{p,L}F_{L,t} + \varepsilon_{p,t}. \quad (17)$$

Since the mimicking factors are tradable assets, the empirical two-factor model implies that

$$\mathbb{E}[R_p] - R_f = \beta_{p,S}\mathbb{E}[F_S] + \beta_{p,L}\mathbb{E}[F_L]. \quad (18)$$

Thus, the standard asset pricing tests can be conducted either through Equation (16) with cross-sectional regressions or through Equation (17) with time-series regressions.

## 5.2. Dissecting factors

Table 5 reports the performance of the empirical two-factor cash-flow model ( $L2$ ) in explaining the five factors of Fama and French (2014) ( $FF5$ ;  $MKT$ ,  $SMB$ ,  $HML$ ,  $CMA$ , and  $RMW$ ), the momentum factor of Carhart (1997) ( $UMD$ ), and the four factors of Hou, Xue, and Zhang (2014a) ( $Q4$ ;  $MKT$ ,  $rME$ ,  $rIA$ , and  $rROE$ ) from time-series regressions. In Panel A of Table 5, all nine factors have significant average returns, but, as reported in Panel B of Table 5, none of them is significant in the  $L2$ -adjusted returns. The market factor  $MKT$  and the momentum factor  $UMD$  are mainly captured by  $F_S$ , despite negatively correlated with  $F_L$ . Moreover, the value factor  $HML$ , the investment factor  $CMA$ , the profitability factor  $RMW$ , and the investment factor  $rIA$  are largely explained by  $F_L$ . Another profitability factor  $rROE$  are well explained by the both factors. Although the magnitude of the average returns for the two size factors,  $SMB$  and  $rME$ , is not reduced by the  $L2$  model, both of them are insignificant. In Panel C, the  $L2$  model is augmented with the market factor, and the results are very similar.

In summary, we find that our two-factor cash-flow model can characterize all the 9 pricing factors and explain their risk premiums proposed by Fama and French (1993, 2014), Carhart (1997), and Hou, Xue, and Zhang (2014a). Thus, the dimensionality of the existing factors could be well spanned by the two cash-flow factors.

## 6. Test Portfolios for the Empirical Two-factor Cash-flow Model

Besides the previous 20 test portfolios motivated by the model, additional 80 test portfolios are formed based on four categories of firm characteristics: profitability, growth, momentum, and volatility. Specifically, the additional portfolios are consisted of 10 portfolios each formed on the quarterly return on equity (*ROEQ*), annual operating profitability (*OP/BE*), annual book-to-market ratios (*B/M*), annual asset growth rates (*% $\Delta$ AT*), the prior 11-month returns (*R\_2\_12*), standardized unexpected earnings (*SUE*), total return volatility (*TVOL*), and the distress risk (*O-score*). Understanding the pricing effects for these variables is important since many of them have been factorized into the *FFC4*, *FF5*, and *Q4* models. Moreover, the rationale for testing the pricing effects of these variables are well motivated by the Corollary 1 (a) and (b). The portfolios are constructed as follows and the portfolio returns are value-weighted.

The *B/M* portfolios: the annual book-to-market ratio (*B/M*), following Davis, Fama, and French (2000), is defined as the book value of equity (*BE*) at the fiscal-year end dividend by the market value of equity at the calendar year-end.<sup>6</sup> At the end of June of year *t*, following Fama and French (1993), stocks are sorted into 10 portfolios by their *B/M* ratios at year *t-1* using NYSE breakpoints. Firms with negative book equity are excluded.

The *% $\Delta$ AT* portfolios: the annual asset growth rate at year *t* (*% $\Delta$ AT*), following Cooper, Gulen, and Schill (2008), is defined as the year-on-year percentage change in total assets ( $AT(t)/AT(t-1)-1$ ). At the end of June of year *t*, stocks are sorted into 10 portfolios by their *% $\Delta$ ATs* at year *t-1* using NYSE breakpoints. Financial firms are excluded.

The *ROEQ* portfolios: the quarterly return on equity (*ROEQ*), following Hou, Xue, and

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<sup>6</sup> As in Davis, Fama, and French (2000), *BE* is the stockholders' book equity (*SEQ* if available, or *CEQ+PSTK* otherwise) plus balance sheet deferred taxes and investment tax credit (*TXDITC* if available), minus the book value of preferred stock (*PSTKRV* if available, or *PSTKL* if available, or *PSTK* otherwise).

Zhang (2014a), is defined as the income before extraordinary items (IBQ) divided by 1-quarter-lagged book equity (lagged  $BEQ$ ), where  $BEQ$  is the quarterly version of  $BE$  as in Davis, Fama, and French (2000) using the corresponding quarterly items. At the beginning of each month, stocks are sorted into 10 portfolios by their recent  $ROEQ$ s using NYSE breakpoints. Financial firms and firms with negative book equity are excluded.

The  $OP/BE$  portfolios: the annual operating profitability ( $OP/BE$ ), following Ball, Gerakos, Linnainmaa, and Nikolaev (2014), is defined as operating profitability ( $OP$ ) divided by the book equity ( $BE$ ). At the end of June of year  $t$ , stocks are sorted into 10 portfolios by their  $OP/BE$ s at year  $t-1$  using NYSE breakpoints. Financial firms and firms with negative book equity are excluded.

The  $R_{2\_12}$  portfolios: the prior 11-month return at month  $t$  ( $R_{2\_12}$ ), following Jegadeesh, and Titman (1993), is defined as the cumulative stock returns from month  $t-12$  to  $t-2$ . At the beginning of each month  $t$ , stocks are sorted into 10 portfolios by their  $R_{2\_12}$ s using NYSE breakpoints.

The  $SUE$  portfolios: the measure of standardized unexpected earnings ( $SUE$ ), following Foster, Olsen, and Shevlin (1984) and Chan, Jegadeesh, and Lakonishok (1996), is defined by the change in the most recently announced quarterly earnings per share from its value 4 quarters ago, divided by the standard deviation of this change in quarterly earnings over the prior 8 quarters (6 quarters minimum). Following Livnat and Mendendall (2006), the quarterly earnings per share ( $EPSPXQ$ ) is adjusted for any stock splits and stock dividends using the cumulative factor ( $AJEXQ$ ). At the beginning of each month, stocks are sorted into 10 portfolios by their recent  $SUE$ s using NYSE breakpoints.

The  $TVOL$  portfolios: the total return volatility ( $TVOL$ ), following Ang, Hodrick, Xing,

and Zhang (2006), is defined as the standard deviation of daily stock returns in the past one month, where a minimum of 17 daily returns are required. At the beginning of each month  $t$ , stocks are sorted into 10 portfolios by their *TVOLs* using NYSE/AMEX/NADAQ breakpoints.

The *O-score* portfolios: following Dichev (1998), the model of bankruptcy risk proposed by Ohlson (1980) is used to measure of distress risk.<sup>7</sup> At the end of June of year  $t$ , stocks are sorted into 10 portfolios by their *O-scores* at year  $t-1$  using NYSE/AMEX/NADAQ breakpoints. Financial firms are excluded. The sample is from January 1981 to December 2012.

## 7. Dissecting Cross-sectional Anomalies

Table 6 reports the overall performance of various factor models in explaining the 10 *High-Low* portfolio returns. As shown in Panel A, each of the 10 *High-Low* portfolio returns is significant. Panel A also reports the risk-adjusted returns. First, the Fama-French (1993) and Carhart (1997) four-factor model (*FFC4*) performs well only in explaining the *High-Low* deciles for the book-market ratio (*B/M*) and the asset growth (*%ΔAT*). Further, similar to the findings in Novy-Marx (2013), the *FFC4* model cannot capture the price momentum as the corresponding risk-adjusted *High-Low* return associated with *R\_2\_12* is 0.32% with a significant *t*-statistic of 2.40. The Fama and French (2014) five-factor model (*FF5*) explains one more variable better than does the *FFC4* model, in which the *FF5* risk-adjusted *High-Low* return associated annual operating profitability (*OP/BE*) becomes insignificant.

The Hou, Xue, and Zhang (2014a) q-theory four-factor model (*Q4*) perform relatively well in explaining seven out of the ten *High-Low* returns associated with *%ΔOPQ*, *ROEQ*, *OP/BE*,

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<sup>7</sup> The variable *O-score* for the fiscal year  $t$  is computed using the following Compustat annual items:

$$\begin{aligned} O - score = & -1.32 - 0.407(\log[AT_t]) + 6.03 (DLC_t + DLTT_t)/AT_t - 1.43 (ACT_t - LCT_t)/AT_t \\ & + 0.076 (LCT_t)/ACT_t - 1.72 (1 \text{ if } LT_t > AT_t, \text{ else } 0) - 2.37 (NI_t)/AT_t - 1.83 (PI_t)/LT_t \\ & + 0.285(1 \text{ if } NI_t < 0 \text{ or } NI_{t-1} < 0, \text{ else } 0) - 0.521 (NI_t - NI_{t-1})/(|NI_t| + |NI_{t-1}|). \end{aligned}$$

*B/M*, *%ΔAT*, *R\_2\_12*, and *SUE*. Consistent with the findings in Hou, Xue, and Zhang (2014a, 2014b), the *Q4* model is better than *FFC4* and *FF5* models in capturing the price momentum and earnings momentum anomalies as well as the profitability premium. Thus, the *Q4* model explains the pricing effects for a majority of the variables used for the test portfolios except for the earnings-to-price ratio, *OPQ/ME*, and the two volatility variables, *TVOL* and *O-score*.<sup>8</sup>

In Panel B of Table 6, all the 10 *High-Low* portfolios become insignificant in the *L2*-adjusted returns. Thus, the *L2* model captures the premiums not only for the underlying two variables, but also for the other eight variables in four categories. Panel B of Table 6 also reports the estimated factor loadings with respect to the *L2* model. Consistent with the previous findings in dissecting factors as well as the implication from the Corollary 1, the cash-flow factor *F<sub>S</sub>* captures the momentum premium while the long-run growth factor *F<sub>L</sub>* explains the value premium as well as the puzzling association between high volatility and low returns.

Table 7 reports the performance of the *L2* model for each portfolio of the 100 total test portfolios. As reported in Panel A, none of them is significant in the *L2*-adjusted returns. Thus, for each portfolio, the null hypothesis that the *L2*-adjusted return is zero cannot be rejected. In Panel B, the Gibbons, Ross, and Shanken (1989, GRS) statistics are used to test the null hypothesis for a given model that the risk-adjusted returns are jointly zero across portfolios. The *L2* model is rejected by the GRS test in only two sets of deciles, which are *R\_2\_12* portfolios and *TVOL* portfolios. While all the competing models are rejected in *R\_2\_12* portfolios, the *L2* model provides the smallest pricing error. In contrast, *FFC4* is rejected in nine, *FF5* is rejected in eight, and *Q4* is rejected in five sets of the decile portfolios. Overall, the *L2* model well characterizes the portfolio returns in the broad cross-section and provides reasonable small pricing errors

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<sup>8</sup> The two volatility variables have stronger pricing effects in this paper than in Hou, Xue, and Zhang (2014a) because they construct portfolios for the two based on NYSE breakpoints, while the results in this paper, following Ang, Hodrick, Xing, and Zhang (2006) and Dichev (1998) are based on NYSE/AMEX/NASDAQ breakpoints.

among them. In other words, the dimensionality of the cross-sectional stock returns could be well spanned by the two cash-flow factors.

Table 8 reports the performance of the empirical cash-flow beta and the empirical growth beta in the  $L2$  model in explaining the test portfolios in the cross-section. For each set of the test portfolios, parameters are jointly estimated from a one-stage generalized method of moments (GMM), stacking the orthogonal conditions in the time-series and in the cross-section. As reported in Panel A of Table 8, the risk price  $\lambda_S$  for the factor loadings with respect to the short-run cash-flow factor is significantly priced in all sets of the testing portfolios. Consistent with the previous findings, the risk price  $\lambda_L$  for the factor loadings with respect to the long-run profitability growth factor is also significantly priced in many sets of the testing portfolios, except for  $\% \Delta OPQ$  portfolios,  $ROEQ$  portfolios, and the two sets of portfolios,  $R\_2\_12$  and  $SUE$ , in the momentum category. Figure 6 shows the average realized returns and the predicted returns estimated from the  $L2$ -model for the eight sets of test portfolios. As can be seen in the figures, the variations of portfolio returns in each set of test portfolios are well captured by the model.

The empirical cash-flow beta  $\beta_{p,S}$  generates the major portion of the risk premium for the High-Low returns in  $\% \Delta OPQ$  portfolios,  $ROEQ$  portfolios,  $R\_2\_12$  portfolios and  $SUE$  portfolios, while the empirical growth beta  $\beta_{p,L}$  generates the major portion of the risk premium for the High-Low returns in  $OPQ/ME$  portfolios,  $OP/BE$  portfolios,  $B/M$  portfolios,  $\% \Delta AT$  portfolios,  $TVOL$  portfolios, and  $O$ -score portfolios. In summary, consistent with the previous findings, the empirical cash-flow beta captures the momentum premium while the empirical growth beta explains the value premium and accounts for the low average return association with the high volatility.

In Panel B of Table 8, the Gibbons, Ross, and Shanken (1989, GRS) statistics are used to

test the null hypothesis for a given model that the pricing errors based on the factor loadings are jointly zero across portfolios. The *L2* model is rejected by the GRS test in only three sets of deciles, which are *%ΔAT* portfolios, *TVOL* portfolios, and *O-score* portfolios. In contrast, *FFC4* is rejected in nine, *FF5* is rejected in eight, and *Q4* are rejected in seven sets of deciles.

Overall, similar to the results for time-series regressions, the empirical cash-flow beta and the empirical growth beta well characterize the portfolio returns in the broad cross-section with small pricing errors. Moreover, the results confirm the factor structure of the *L2* model in explaining the cross-sectional stock returns. Thus, the results imply that the cash-flow beta and the growth beta are two common elements in asset pricing.

## **8. Conclusions**

In our proposed dynamic cash-flow model, the stock price is determined by the current profitability and the long-run profitability growth. The learning mechanism further suggests that that expected stock returns are associated with two systematic risks: cash-flow beta and growth beta. The implications derived from the model characterize a broad set of empirical patterns.

Using the optimal filtering technique to estimate the dynamic cash-flow model, we find the evidence that the cash-flow beta and the growth beta are significantly priced. Moreover, in the current profitability growth sorted portfolios in which a high short-run profitability is associated with a high cash-flow beta, we find a negative value premium (i.e., higher B/M is associated with lower expected returns) and a positive price momentum. In contrast, in the earnings-to-price ratio sorted portfolios in which a high long-run profitability is associated with a more negative growth beta, we find a positive value premium and a negative price momentum (i.e., reversal).



The empirical two-factor model consisted of a short-run cash-flow factor and a long-run profitability growth factor can characterize all the nine pricing factors and explain their risk premiums proposed in the literature and the cross section of average stock returns formed on profitability, growth, momentum, and volatility. The findings imply that the cash-flow beta and the growth beta are two common elements in asset pricing. Thus, the issue of multidimensional challenge for the cross-sectional anomalies might be partly resolved with the refined dynamic fundamental valuation model proposed in this paper.

## Appendix

Proof of Proposition 1:

$$M_t = \mathbb{E} \left[ \int_t^\infty \frac{\Lambda_s}{\Lambda_t} D_s \, ds \right] = \alpha \int_t^\infty Y_t \mathbb{E} \left[ \frac{\Lambda_s Y_s}{\Lambda_t Y_t} \right] \, ds$$

Define  $p_t \equiv \log(\Lambda_t Y_t)$ , then  $Z(\hat{X}_t, s) \equiv \mathbb{E} \left[ \frac{\Lambda_s Y_s}{\Lambda_t Y_t} \right] = \exp \left( \mathbb{E}[p_s] - p_t + \frac{1}{2} \text{Var}[p_s] \right)$ . The process

for  $p_t$  evolves as  $dp_t = \left( -\eta - \gamma b_0 + (1 - \gamma b_1) \hat{X}_t + \frac{\gamma}{2} \sigma_C^2 - \frac{1}{2} \sigma_Y^2 \right) dt - \gamma \sigma_C d\tilde{W}_{0,t} + \sigma_Y d\tilde{W}_{Y,t}$ .

Solving the stochastic differential equation for  $dp_t$  yields the result:  $Z(\hat{X}_t, s) = \exp(\zeta(s) + \zeta_X(s) \hat{X}_t)$

$$\zeta_X(s) = \frac{(1 - \gamma b_1)}{\phi} (1 - e^{-s\phi})$$

$$\begin{aligned} \zeta(s) = & -s \left( \eta + \gamma b_0 - \frac{1}{2} \gamma (1 + \gamma) \sigma_C^2 \right) + \frac{(1 - \gamma b_1)}{\phi} \left( s\phi - (1 - e^{-s\phi}) \right) \mu_X \\ & + \frac{(1 - \gamma b_1)^2}{4\phi^3} \left[ (1 - e^{-2s\phi}) - 4(1 - e^{-s\phi}) + 2s\phi \right] \sigma_X^2 \\ & - \frac{s}{2} \gamma \sigma_{S,0} \sigma_C - \frac{\gamma(1 - \gamma b_1)}{\phi^2} \left( s\phi - (1 - e^{-s\phi}) \right) \gamma \sigma_{\hat{X},0} \sigma_C \end{aligned}$$

Q.E.D.

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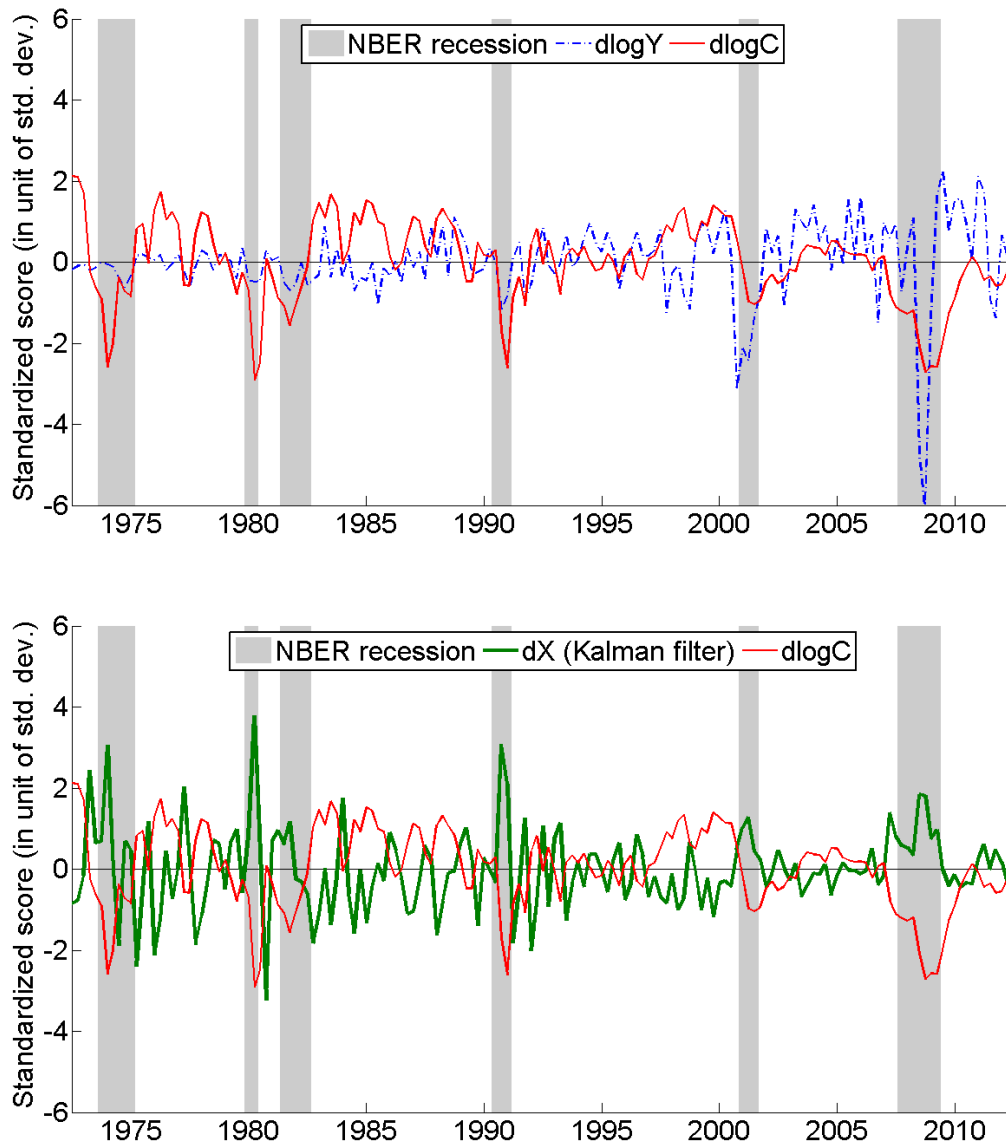
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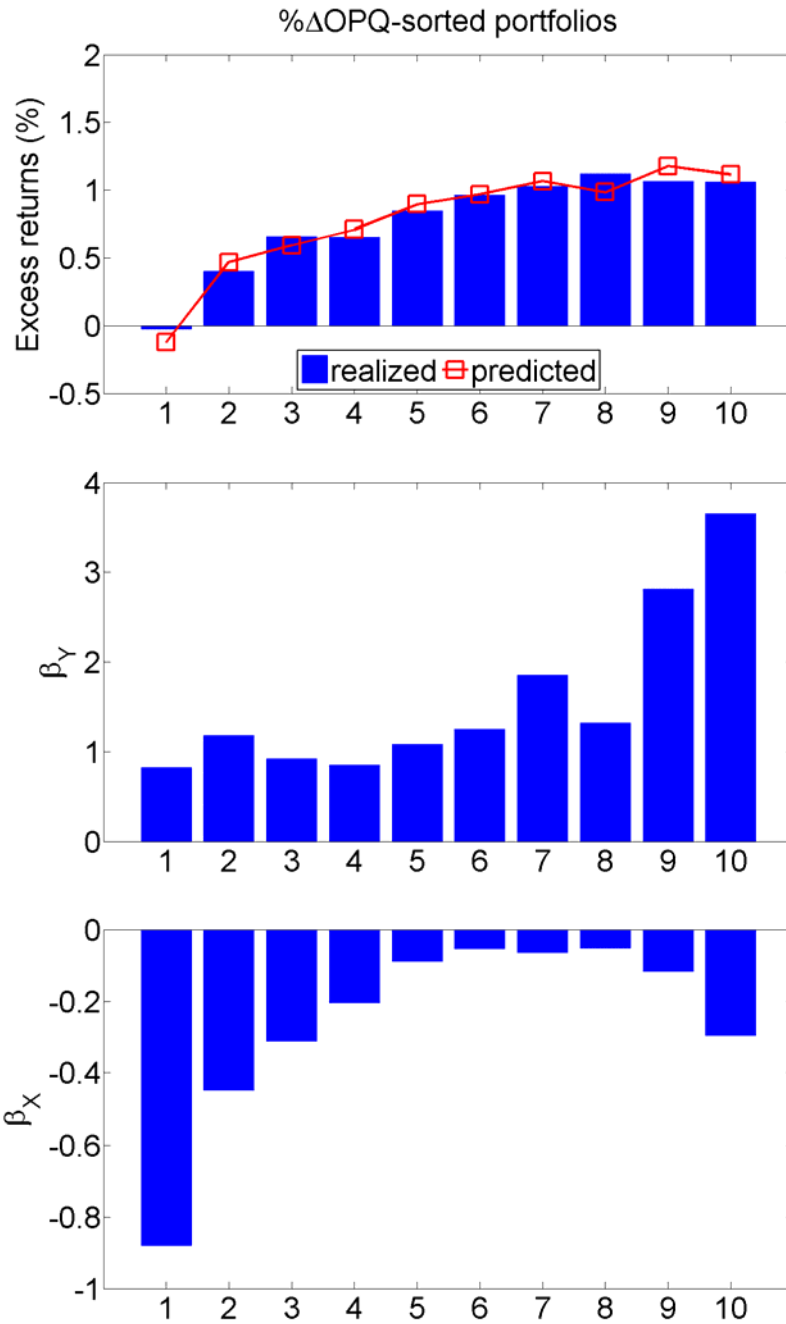
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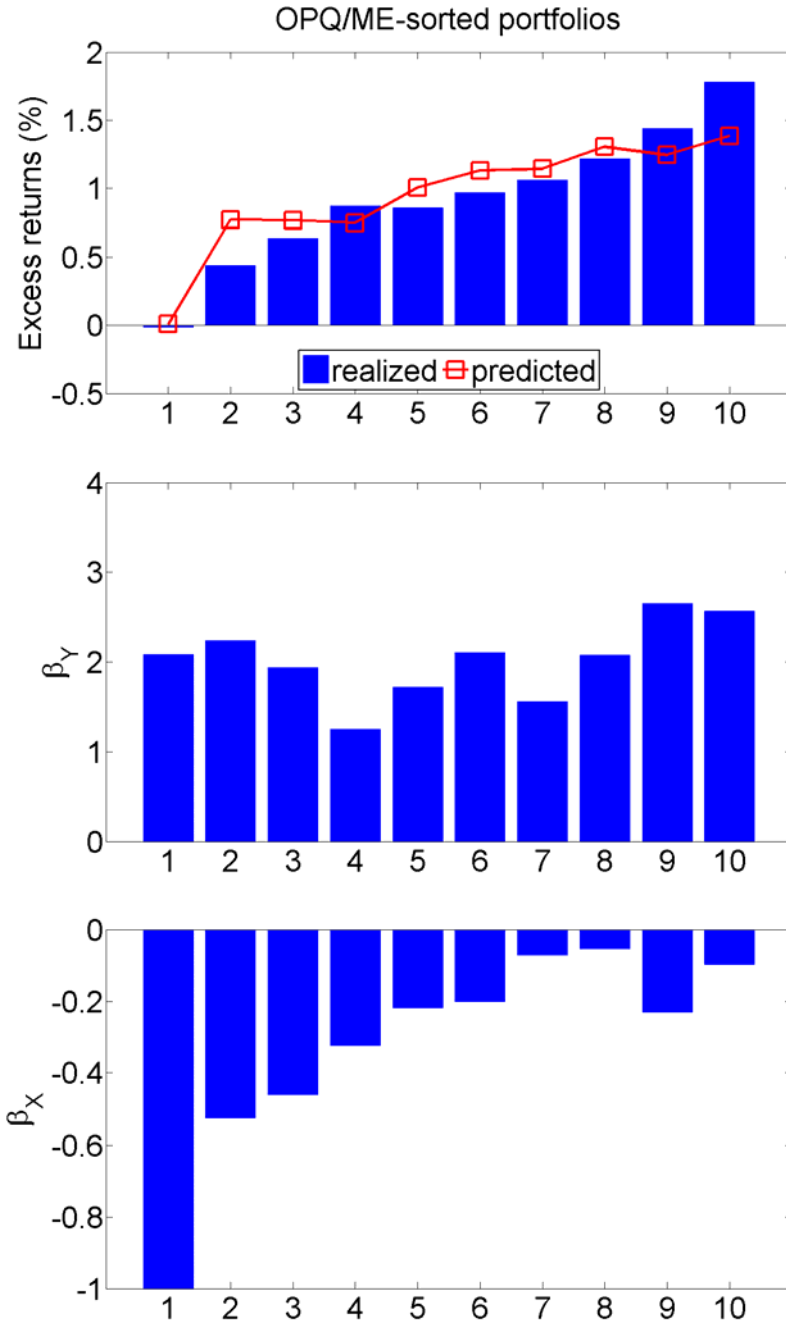


**Figure 1. Market short-run profitability, filtered market long-run profitability growth, and aggregate consumption.** These plots show the short-run profitability growth rate for the market ( $d\log Y$ ), the filtered long-run profitability growth for the market ( $dX$ ), and aggregate consumption growth ( $d\log C$ ). The sample period is first quarter of 1972 to fourth quarter of 2012. Shaded areas denote NBER recessions.

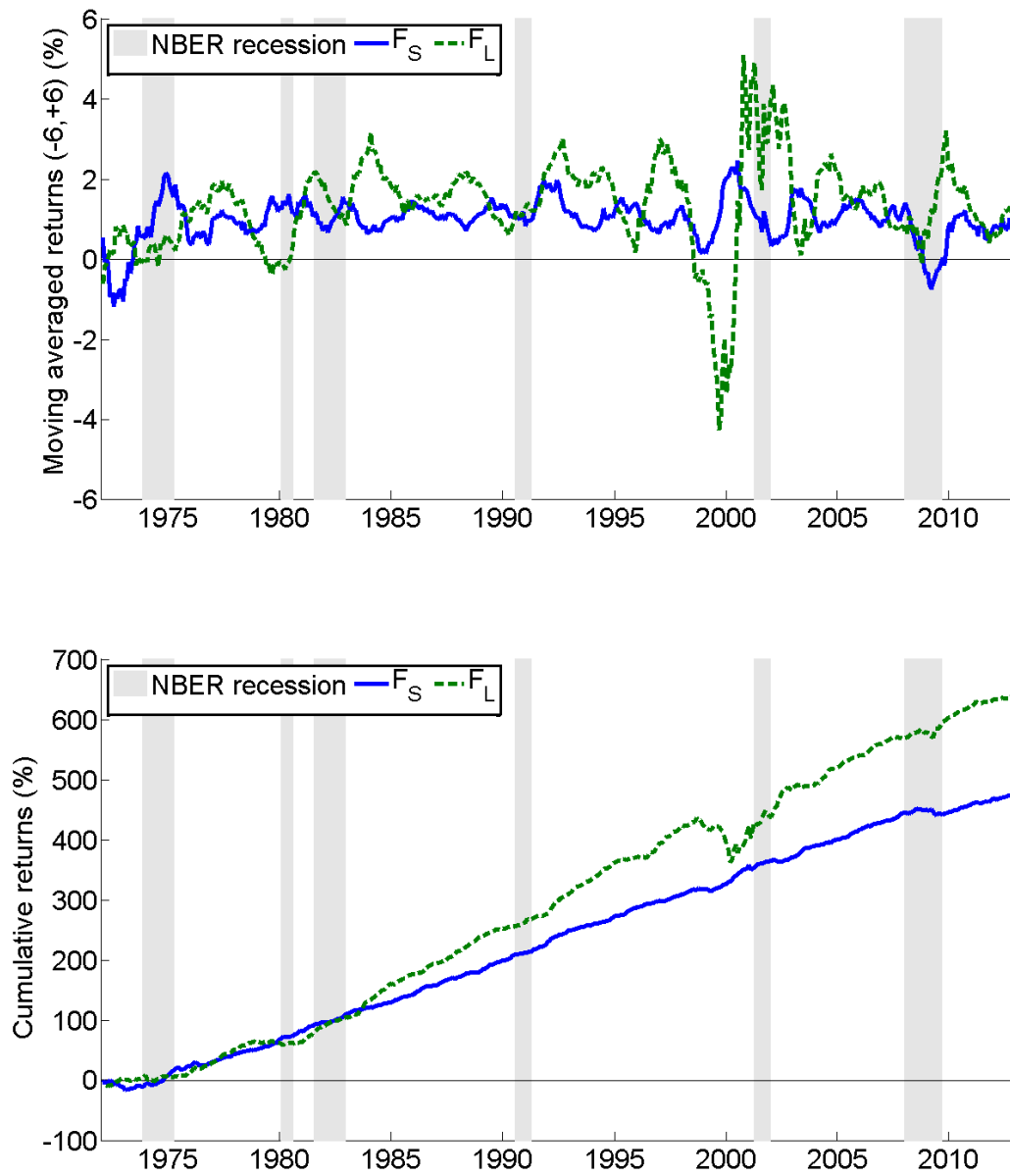


**Figure 2.  $\% \Delta OPQ$ -sorted portfolios: realized returns and predicted returns.** The first plot shows the average realized returns and the predicted returns estimated from the dynamic cash-flow model for the portfolios formed on  $\% \Delta OPQ$ . The estimates of the cash-flow beta and the estimates of the growth beta are shown in the second plot and the third plot, respectively. The sample period is first quarter of 1972 to fourth quarter of 2012.

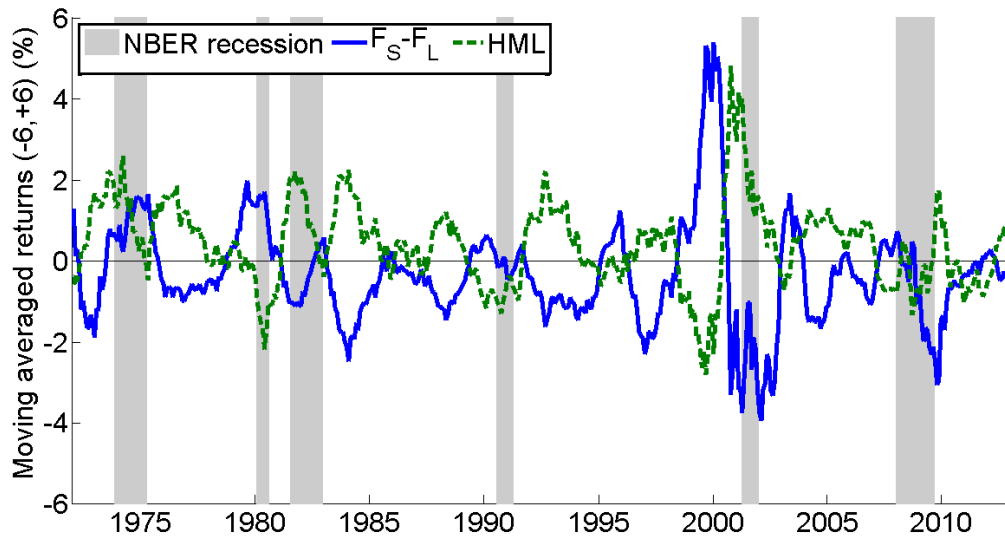




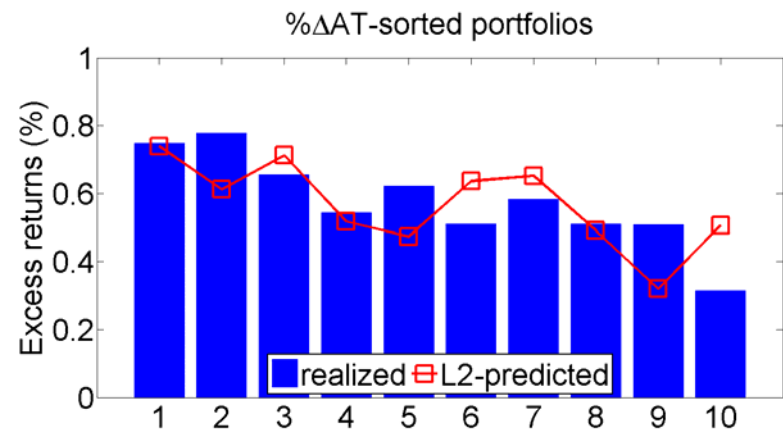
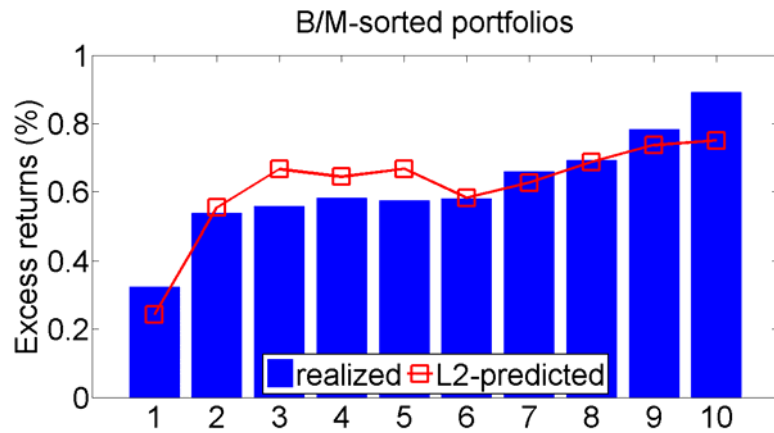
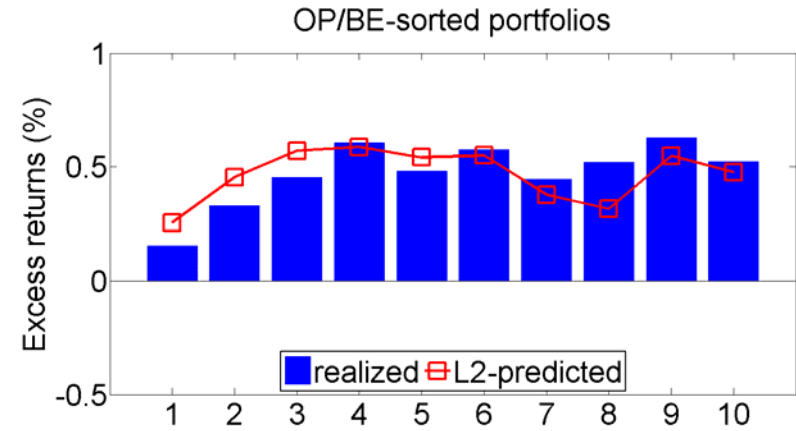
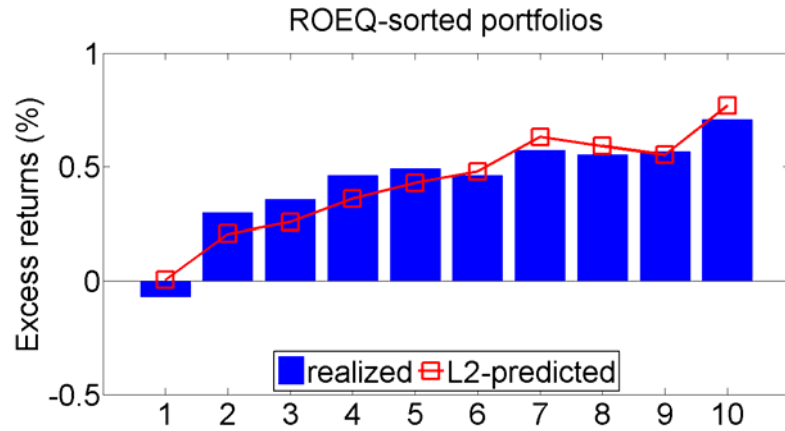
**Figure 3. *OPQ/ME*-sorted portfolios: realized returns and predicted returns.** The first plot shows the average realized returns and the predicted returns estimated from the dynamic cash-flow model for the portfolios formed on *OPQ/ME*. The estimates of the cash-flow beta and the estimates of the growth beta are shown in the second plot and the third plot, respectively. The sample period is first quarter of 1972 to fourth quarter of 2012.



**Figure 4. The mimicking factors.** These plots show the time-series evolution for the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ). The figure in the top shows their moving-averaged returns from month  $t-6$  to  $t+6$  in each month  $t$ . The figure in the bottom shows the cumulative returns for these two factors. The sample period is from 1972 to 2012. Shaded areas denote NBER recessions.



**Figure 5. HML factor and the difference in the mimicking factors.** These plots show the time-series evolution for the HML factor and the difference between the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ). The figure in the top shows their moving-averaged returns from month  $t-6$  to  $t+6$  in each month  $t$ . The figure in the bottom shows the cumulative returns for these two factors. The sample period is from 1972 to 2012. Shaded areas denote NBER recessions.



**Figure 6. Realized returns and predicted returns in the  $L2$ -model.** These plots show the average realized returns and the predicted returns estimated from the  $L2$ -model for the portfolios formed on  $ROEQ$ ,  $OP/BE$ ,  $B/M$ ,  $\% \Delta AT$ ,  $R_{2\_12}$ ,  $SUE$ ,  $TVOL$ , and  $O$ -score. The sample period is from 1972 to 2012.

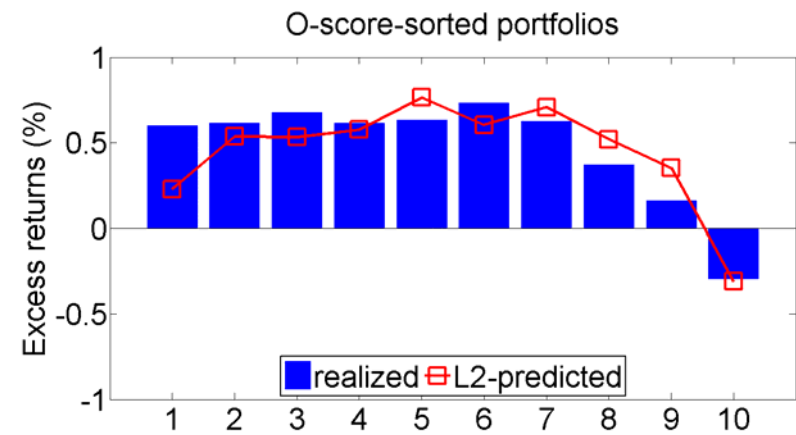
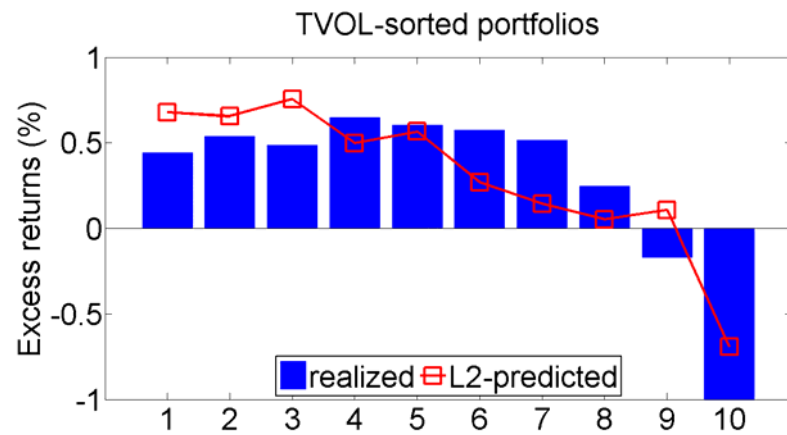
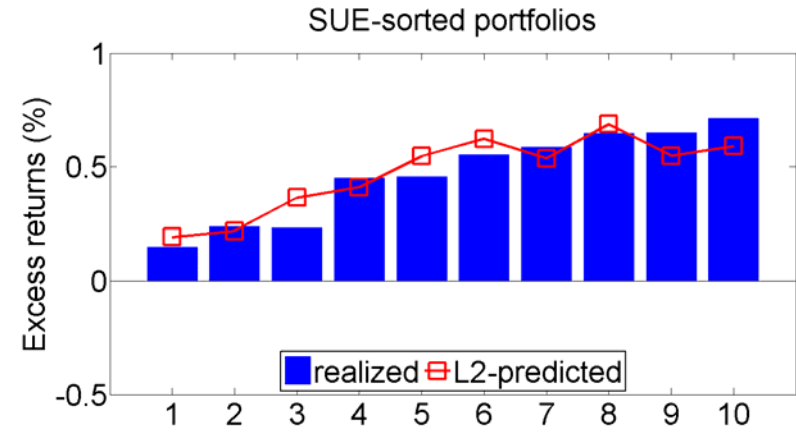
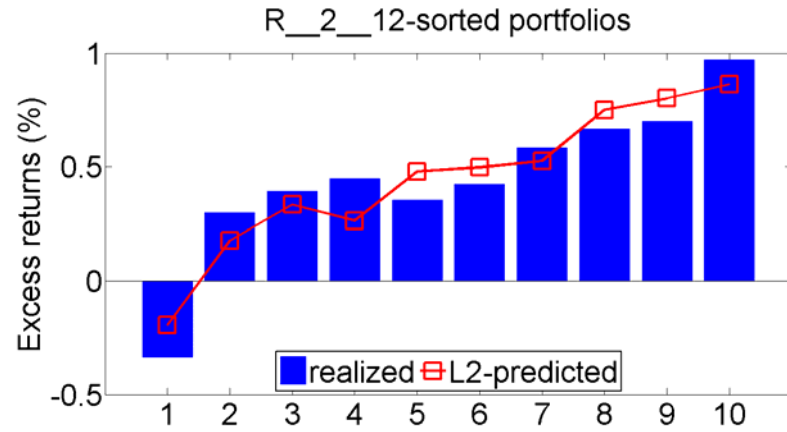


Figure 6. (Continued.)

**Table 1. Parameter estimates**

This table provides the parameter estimates from the Kalman filtering procedure. Results for the aggregate market, for 10 portfolios formed on operating profitability growth rates ( $\% \Delta OPQ$ ), and for 10 portfolios formed on earnings-to-price ratios ( $OPQ/ME$ ) are presented. For each portfolio, the measurement equation in the state-space form is identified as  $[\Delta y_{t+1}, \Delta c_{t+1}]' = [0, b_0]' + [1, b_1]'x_{t+1} + [\varepsilon_{y,t+1}, \varepsilon_{c,t+1}]'$  and the state equation is defined as  $x_{t+1} = \phi_0 + \phi_x x_t + \varepsilon_{x,t+1}$ , where  $\Delta y_{t+1}$  is the short-run profitability growth rate,  $\Delta c_{t+1}$  is the aggregate consumption growth rate, and  $x_{t+1}$  is the latent long-run profitability growth. Standard errors are reported in parentheses. The sample period is from January 1972 to December 2012.

	$\phi_x$	$\phi_0$	$\phi_0/(1 - \phi_x)$	$b_1$	$b_0$	<i>loglike</i>
<i>Market</i>	0.77 (0.06)	8.8E-04 (0.00)	0.00	-3.83 (2.19)	0.02 (0.01)	1173.62
Portfolios formed on $\% \Delta OPQ$						
<i>Low</i>	0.77 (0.06)	8.0E-03 (0.00)	0.03	-0.44 (0.09)	0.02 (0.00)	1049.39
<i>2</i>	0.76 (0.06)	4.4E-03 (0.00)	0.02	-0.87 (0.21)	0.02 (0.00)	1100.19
<i>3</i>	0.77 (0.05)	3.0E-03 (0.00)	0.01	-1.26 (0.31)	0.02 (0.00)	1141.13
<i>4</i>	0.77 (0.06)	1.7E-03 (0.00)	0.01	-1.90 (0.82)	0.02 (0.01)	1137.76
<i>5</i>	0.77 (0.06)	6.8E-04 (0.00)	0.00	-4.32 (3.98)	0.02 (0.01)	1136.65
<i>6</i>	0.76 (0.06)	4.4E-04 (0.00)	0.00	-7.04 (8.90)	0.02 (0.01)	1147.45
<i>7</i>	0.78 (0.06)	4.2E-04 (0.00)	0.00	-5.84 (14.38)	0.02 (0.03)	1140.38
<i>8</i>	0.78 (0.06)	3.0E-04 (0.00)	0.00	-7.18 (19.09)	0.01 (0.03)	1124.84
<i>9</i>	0.80 (0.06)	-1.7E-04 (0.00)	-0.00	-3.07 (7.88)	0.00 (0.00)	1086.08
<i>High</i>	0.87 (0.05)	-1.5E-03 (0.00)	-0.01	-1.05 (0.40)	-0.01 (0.00)	1067.52
Portfolios formed on $OPQ/ME$						
<i>Low</i>	0.76 (0.05)	1.1E-02 (0.00)	0.05	-0.35 (0.07)	0.02 (0.00)	993.70
<i>2</i>	0.76 (0.05)	5.5E-03 (0.00)	0.02	-0.75 (0.18)	0.02 (0.00)	1053.02
<i>3</i>	0.76 (0.06)	4.4E-03 (0.00)	0.02	-0.87 (0.28)	0.02 (0.00)	1038.82
<i>4</i>	0.77 (0.05)	2.9E-03 (0.00)	0.01	-1.20 (0.54)	0.02 (0.01)	1083.66
<i>5</i>	0.77 (0.05)	1.8E-03 (0.00)	0.01	-1.78 (1.05)	0.02 (0.01)	1067.54
<i>6</i>	0.77 (0.05)	1.7E-03 (0.00)	0.01	-1.94 (1.10)	0.02 (0.01)	1065.80
<i>7</i>	0.76 (0.06)	5.0E-04 (0.00)	0.00	-5.46 (12.93)	0.02 (0.03)	1119.38
<i>8</i>	0.77 (0.06)	2.3E-04 (0.00)	0.00	-6.94 (17.63)	0.01 (0.01)	1118.85
<i>9</i>	0.83 (0.05)	-9.0E-04 (0.00)	-0.01	-1.48 (0.83)	0.00 (0.00)	1089.69
<i>High</i>	1.00 (0.05)	-7.3E-06 (0.00)	-0.00	-2.10 (2.44)	-0.03 (0.04)	1087.36

**Table 2. Cash-flow beta and growth beta**

This table provides the estimates of the cash-flow beta ( $\beta_Y$ ) and the growth beta ( $\beta_X$ ) for the 20 testing portfolios: 10 portfolios formed on operating profitability rates ( $\% \Delta OPQ$ ) and 10 portfolios formed on earnings-to-price ratios ( $OPQ/ME$ ). After the portfolio formation, monthly equal-weighted portfolio returns are calculated and the excess returns are reported. For each portfolio, the cash-flow beta and the growth beta are estimated from

$$\beta_Y = \frac{\text{Cov}[\Delta y_{t+1} - \hat{x}_{t+1}, \Delta c_{t+1}]}{\text{Var}[\Delta c_{t+1}]} \quad \text{and} \quad \beta_X = \frac{\text{Cov}[\hat{x}_{t+1} - (\hat{\phi}_0 + \hat{\phi}_X x_t), \Delta c_{t+1}]}{\text{Var}[\Delta c_{t+1}]},$$

where  $\Delta y_{t+1}$  is the short-run profitability growth rate,  $\Delta c_{t+1}$  is the aggregate consumption growth rate,  $\hat{x}_{t+1}$  is the filtered long-run profitability growth, and  $\hat{\phi}_0$  and  $\hat{\phi}_X$  are the parameters estimated from the state-space model of Kalman filtering. Pre-formation firm characteristics are reported; for each portfolio,  $SZ(\$m)$  is the average of the firm-level market capitalization (in million dollars);  $B/M$  is computed from the sum of the firm-level book value of equity dividend by the sum of the firm-level market value of equity;  $R\_2\_12$  is value-weighted average of the firm-level prior 11-month returns before the last month. Robust Newey and West (1987)  $t$ -statistics that account for autocorrelations are reported in parentheses. The sample period is from January 1972 to December 2012.

	<i>Excess returns</i>	$\beta_Y$	$\beta_X$	<i>SZ(\$m)</i>	<i>B/M</i>	<i>R_2_12</i>
Portfolios formed on $\% \Delta OPQ$						
<i>Low</i>	-0.03 (-0.09)	0.82 (0.53)	-0.88 (-9.75)	457	0.73	10.84
2	0.40 (1.43)	1.18 (1.49)	-0.45 (-9.79)	1167	0.69	11.09
3	0.66 (2.52)	0.92 (1.82)	-0.31 (-9.77)	1884	0.64	12.57
4	0.66 (2.63)	0.85 (1.55)	-0.21 (-9.72)	2532	0.60	13.88
5	0.84 (3.37)	1.09 (1.68)	-0.09 (-9.68)	2783	0.58	16.13
6	0.96 (3.79)	1.25 (1.92)	-0.06 (-9.81)	2738	0.57	18.42
7	1.03 (3.82)	1.85 (2.45)	-0.07 (-9.36)	2176	0.57	20.62
8	1.12 (3.93)	1.32 (1.59)	-0.05 (-9.37)	1800	0.56	23.75
9	1.06 (3.41)	2.81 (2.28)	-0.12 (-8.59)	1195	0.62	24.83
<i>High</i>	1.06 (3.08)	3.66 (2.78)	-0.30 (-6.84)	652	0.64	23.59
Portfolios formed on $OPQ/ME$						
<i>Low</i>	-0.02 (-0.04)	2.08 (1.00)	-1.12 (-9.81)	639	0.55	21.92
2	0.44 (1.54)	2.23 (1.98)	-0.53 (-9.81)	2291	0.39	20.15
3	0.64 (2.49)	1.93 (1.73)	-0.46 (-10.01)	2370	0.41	17.13
4	0.87 (3.43)	1.25 (1.48)	-0.33 (-9.74)	2156	0.48	16.37
5	0.86 (3.34)	1.73 (1.66)	-0.22 (-9.71)	2128	0.57	16.07
6	0.97 (3.85)	2.10 (2.01)	-0.20 (-9.72)	2035	0.65	15.74
7	1.06 (4.09)	1.56 (2.12)	-0.07 (-9.84)	1937	0.74	14.42
8	1.22 (4.59)	2.08 (2.15)	-0.06 (-9.65)	1598	0.85	13.41
9	1.44 (4.98)	2.65 (2.42)	-0.23 (-7.91)	1290	0.96	13.23
<i>High</i>	1.78 (5.02)	2.56 (2.23)	-0.10 (-3.97)	638	1.15	13.22

**Table 3. Cross-sectional regressions**

This table provides the estimates of the risk price for the cash-flow beta ( $\lambda_Y$ ) and the risk price for the growth beta ( $\lambda_X$ ) using the 20 testing portfolios: 10 portfolios formed on operating profitability rates ( $\% \Delta OPQ$ ) and 10 portfolios formed on earnings-to-price ratios ( $OPQ/ME$ ). After the portfolio formation, monthly equal-weighted portfolio returns are calculated. For each set of the testing portfolios, portfolios returns ( $R_{p,t}$ ) are regressed on the cross-sectional cash-flow betas ( $\beta_Y$ ) and growth betas ( $\beta_X$ )

$$R_{p,t} - R_{f,t} = \lambda_0 + \lambda_Y \beta_Y + \lambda_X \beta_X + \varepsilon_{p,t},$$

and the estimates of  $\lambda_Y$  and  $\lambda_X$  are reported in Panel A; the excess returns for High-Low and the predicted premium are shown in Panel B;  $\lambda_Y^{[3]}$  and  $\lambda_X^{[3]}$  are the estimates from the 20 testing portfolios in column [3]. Robust Newey and West (1987)  $t$ -statistics that account for autocorrelations are reported in parentheses. The sample period is from January 1972 to December 2012.

	[1]	[2]	[3]
	10 $\% \Delta OPQ$ portfolios	10 $OPQ/ME$ portfolios	20 testing portfolios
<i>Panel A: Cross-sectional regressions</i>			
<i>Intercept</i>	<b>0.84</b>	<b>0.52</b>	<b>0.80</b>
	<b>(3.75)</b>	<b>(2.29)</b>	<b>(3.61)</b>
$\lambda_Y$	<b>0.22</b>	<b>0.41</b>	<b>0.26</b>
	<b>(5.46)</b>	<b>(4.44)</b>	<b>(7.30)</b>
$\lambda_X$	<b>1.17</b>	<b>1.37</b>	<b>1.22</b>
	<b>(7.09)</b>	<b>(6.25)</b>	<b>(7.61)</b>
<i>Adj. R<sup>2</sup></i>	0.95	0.81	0.81
<i>Panel B: Excess returns for High-Low and the predicted premium</i>			
<i>High-Low</i>	<b>1.09</b>	<b>1.80</b>	
	<b>(11.40)</b>	<b>(7.49)</b>	
$\lambda_Y(\beta_Y^{High} - \beta_Y^{Low})$	0.62	0.20	
$\lambda_X(\beta_X^{High} - \beta_X^{Low})$	0.48	1.40	
$\lambda_Y^{[3]}(\beta_Y^{High} - \beta_Y^{Low})$	0.73	0.12	
$\lambda_X^{[3]}(\beta_X^{High} - \beta_X^{Low})$	0.50	1.25	



**Table 4. Properties of the mimicking factors**

Two mimicking cash-flow factors are constructed as follows. To construct the short-run cash-flow factor ( $F_S$ ), stocks are sorted into three portfolios (*Low*, *Med*, *High*) based on  $\% \Delta OPQ$  using the 30th and 70th percentiles for NYSE stocks as breakpoints and the three portfolios are then intersected with the firms below the NYSE median market capitalization;  $F_S$  is the difference between the returns on *High* and the returns on *Low*. Similarly, to construct the long-run profitability growth factor ( $F_L$ ), stocks are sorted into three portfolios (*Low*, *Med*, *High*) based on  $OPQ/ME$  using the 30th and 70th percentiles for NYSE stocks as breakpoints and the three portfolios are then intersected with the firms below the NYSE median market capitalization;  $F_L$  is the difference between the returns on *High* and the returns on *Low*. Portfolio returns for the construction of the mimicking factors are equal-weighted. For each portfolio, Panel A reports the excess return and the risk-adjusted returns (alpha or intercept) with respect to the four factors of Fama-French (1993) and Carhart (1997) ( $FFC4$ ;  $MKT$ ,  $SMB$ ,  $HML$ , and  $UMD$ ), the five factors of Fama and French (2014) ( $FF5$ ;  $MKT$ ,  $SMB$ ,  $HML$ ,  $CMA$ , and  $RMW$ ), and the four factors of Hou, Xue, and Zhang (2014a) ( $Q4$ ;  $MKT$ ,  $rME$ ,  $rI/A$ , and  $rROE$ ) from time-series regressions. Panel B presents the Spearman correlations. Robust Newey and West (1987)  $t$ -statistics that account for autocorrelations are reported in parentheses. The sample period is from January 1972 to December 2012.

*Panel A: Performance of the mimicking factors*

	Ranking on $\% \Delta OPQ$			$F_S$ <i>High- Low</i>	Ranking on $OPQ/ME$			$F_L$ <i>High- Low</i>
	<i>Low</i>	<i>Med</i>	<i>High</i>		<i>Low</i>	<i>Med</i>	<i>High</i>	
Excess Returns	0.36 (1.11)	1.06 (3.65)	1.33 (3.88)	<b>0.97</b> <b>(11.95)</b>	0.37 (1.02)	1.11 (4.02)	1.68 (4.99)	<b>1.31</b> <b>(7.34)</b>
$\alpha$ - $FFC4$	-0.15 (-1.05)	0.43 (4.64)	0.69 (5.32)	<b>0.84</b> <b>(10.37)</b>	-0.13 (-0.78)	0.45 (6.10)	0.95 (7.51)	<b>1.08</b> <b>(7.61)</b>
$\alpha$ - $FF5$	-0.24 (-1.41)	0.33 (2.88)	0.65 (4.82)	<b>0.89</b> <b>(11.30)</b>	-0.09 (-0.56)	0.30 (3.38)	0.72 (4.80)	<b>0.81</b> <b>(7.16)</b>
$\alpha$ - $Q4$	0.06 (0.33)	0.43 (3.30)	0.79 (5.07)	<b>0.73</b> <b>(8.12)</b>	0.13 (0.64)	0.41 (4.13)	0.95 (4.87)	<b>0.82</b> <b>(3.78)</b>

*Panel B: Spearman correlations*

	$F_S$	$F_L$	$MKT$	$SMB$	$HML$	$UMD$	$CMA$	$RMW$	$rME$	$rI/A$	$rROE$
$F_S$	1.00										
$F_L$	0.01	1.00									
$MKT$	0.25	-0.21	1.00								
$SMB$	0.13	-0.13	0.22	1.00							
$HML$	-0.18	0.54	-0.34	-0.06	1.00						
$UMD$	0.22	-0.03	-0.10	-0.02	-0.09	1.00					
$CMA$	-0.12	0.33	-0.34	-0.04	0.66	-0.01	1.00				
$RMW$	-0.01	0.27	-0.22	-0.26	-0.14	0.15	-0.22	1.00			
$rME$	0.15	-0.10	0.16	0.98	-0.03	0.04	-0.02	-0.24	1.00		
$rI/A$	-0.05	0.34	-0.35	-0.09	0.59	0.11	0.92	-0.09	-0.05	1.00	
$rROE$	0.27	0.13	-0.12	-0.23	-0.27	0.44	-0.25	0.65	-0.14	-0.04	1.00

**Table 5. Dissecting factors**

This table reports the performance of the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ) in explaining the five factors of Fama and French (2014) ( $FF5$ ;  $MKT$ ,  $SMB$ ,  $HML$ ,  $CMA$ , and  $RMW$ ), the momentum factor of Carhart (1997) ( $UMD$ ), and the four factors of How, Xue, and Zhang (2014a) ( $Q4$ ;  $MKT$ ,  $rME$ ,  $rI/A$ , and  $rROE$ ) from time-series regressions. The learning-based two-factor model ( $L2$ ), consisting of the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ), is

$$R_{p,t} - R_{f,t} = \alpha_p + \beta_{p,S}F_{S,t} + \beta_{p,L}F_{L,t} + \varepsilon_{p,t},$$

where  $R_{p,t}$  is the portfolio return and  $\alpha_p$  is the risk-adjusted return. For each factor as the dependent variable, Panel A presents the average raw return and Panel B reports the risk-adjusted return and the corresponding factor loadings with respect to the learning-based two cash-flow factors ( $L2$ ;  $F_S$  and  $F_L$ ); Panel C reports the results using the L3 model, which consists of the market factor and the two cash-flow factors ( $L3$ ;  $MKT$ ,  $F_S$  and  $F_L$ ). Robust Newey and West (1987)  $t$ -statistics that account for autocorrelations are reported in parentheses. The sample period is from January 1972 to December 2012.

	<i>MKT</i>	<i>SMB</i>	<i>HML</i>	<i>UMD</i>	<i>CMA</i>	<i>RMW</i>	<i>rME</i>	<i>rI/A</i>	<i>rROE</i>
<i>Panel A: Raw returns</i>									
<i>Mean</i>	<b>0.48</b>	<b>0.24</b>	<b>0.40</b>	<b>0.71</b>	<b>0.34</b>	<b>0.31</b>	<b>0.33</b>	<b>0.45</b>	<b>0.59</b>
	<b>(2.16)</b>	<b>(1.73)</b>	<b>(2.48)</b>	<b>(3.39)</b>	<b>(3.64)</b>	<b>(2.41)</b>	<b>(2.42)</b>	<b>(5.04)</b>	<b>(4.99)</b>
<i>Panel B: L2 adjusted returns and factor loadings</i>									
$\alpha_{L2}$	0.18	0.35	-0.08	-0.02	0.14	-0.19	0.35	0.15	-0.18
	(0.53)	(1.56)	(-0.52)	(-0.05)	(1.13)	(-1.13)	(1.56)	(1.56)	(-1.08)
$\beta_S$	<b>0.76</b>	<b>0.25</b>	<b>-0.34</b>	<b>0.94</b>	-0.10	-0.05	<b>0.28</b>	-0.05	<b>0.46</b>
	<b>(2.91)</b>	<b>(1.90)</b>	<b>(-4.21)</b>	<b>(2.65)</b>	(-1.52)	(-0.53)	<b>(2.24)</b>	(-0.76)	<b>(3.11)</b>
$\beta_L$	<b>-0.34</b>	<b>-0.26</b>	<b>0.62</b>	-0.13	<b>0.23</b>	<b>0.42</b>	<b>-0.22</b>	<b>0.26</b>	<b>0.24</b>
	<b>(-2.59)</b>	<b>(-2.35)</b>	<b>(14.89)</b>	(-0.50)	<b>(4.30)</b>	<b>(4.86)</b>	<b>(-1.87)</b>	<b>(5.93)</b>	<b>(2.71)</b>
<i>Panel C: L3 adjusted returns and factor loadings</i>									
$\alpha_{L3}$	N.A.	0.33	-0.06	0.02	0.16	-0.18	0.34	<b>0.18</b>	-0.15
	N.A.	(1.43)	(-0.41)	(0.04)	(1.52)	(-1.05)	(1.47)	<b>(2.10)</b>	(-0.98)
$\beta_{MKT}$	N.A.	<b>0.11</b>	<b>-0.11</b>	<b>-0.25</b>	<b>-0.13</b>	-0.07	0.07	<b>-0.13</b>	<b>-0.13</b>
	N.A.	<b>(2.23)</b>	<b>(-3.00)</b>	<b>(-2.73)</b>	<b>(-4.51)</b>	(-1.56)	(1.39)	<b>(-5.13)</b>	<b>(-2.07)</b>
$\beta_S$	N.A.	0.17	<b>-0.26</b>	<b>1.13</b>	-0.01	0.00	<b>0.23</b>	0.05	<b>0.56</b>
	N.A.	(1.28)	<b>(-3.44)</b>	<b>(3.26)</b>	(-0.09)	(-0.03)	<b>(1.84)</b>	(0.86)	<b>(3.61)</b>
$\beta_L$	N.A.	<b>-0.23</b>	<b>0.58</b>	-0.21	<b>0.19</b>	<b>0.40</b>	-0.20	<b>0.22</b>	<b>0.19</b>
	N.A.	<b>(-1.89)</b>	<b>(14.03)</b>	(-0.90)	<b>(4.10)</b>	<b>(4.37)</b>	(-1.58)	<b>(6.39)</b>	<b>(2.17)</b>

**Table 6. Dissecting anomalies: explaining ‘High-Low’ portfolio returns**

This table reports the performance of the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ) in explaining the portfolios formed on operating profitability rates ( $\% \Delta OPQ$ ), earnings-to-price ratios ( $OPQ/ME$ ), quarterly return-on-equity ( $ROEQ$ ), annual operating profitability ( $OP/BE$ ), book-to-market ratios ( $B/M$ ), asset growth rates ( $\% \Delta AT$ ), prior 11-month returns ( $R_{2\_12}$ ), standardized unexpected earnings ( $SUE$ ), total volatility ( $TVOL$ ), and Ohlson’s O-score ( $O\text{-score}$ ). The learning-based two-factor model ( $L2$ ), consisting of the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ), is

$$R_{p,t} - R_{f,t} = \alpha_p + \beta_{p,S}F_{S,t} + \beta_{p,L}F_{L,t} + \varepsilon_{p,t}$$

where  $R_{p,t}$  is the portfolio return and  $\alpha_p$  is the risk-adjusted return. Portfolios formed on operating profitability rates ( $\% \Delta OPQ$ ), earnings-to-price ratios ( $OPQ/ME$ ) are motivated by the  $L2$  model, and other testing portfolios are classified into the categories of profitability, growth, momentum, and volatility. For each set of the testing portfolios in deciles, portfolio returns are value-weighted and the ‘High-Low’ denotes the portfolio that longs the top decile and shorts the bottom decile. For each ‘High-Low’ portfolio return as the dependent variable, Panel A presents the excess return and the risk-adjusted returns with respect to the four factors of Fama-French (1993) and Carhart (1997) ( $FFC4$ ;  $MKT$ ,  $SMB$ ,  $HML$ , and  $UMD$ ), the five factors of Fama and French (2014) ( $FF5$ ;  $MKT$ ,  $SMB$ ,  $HML$ ,  $CMA$ , and  $RMW$ ), and the four factors of Hou, Xue, and Zhang (2014a) ( $Q4$ ;  $MKT$ ,  $rME$ ,  $r/A$ , and  $rROE$ ); Panel B reports the risk-adjusted return and the corresponding factor loadings with respect to the learning-based two cash-flow factors ( $L2$ ;  $F_S$  and  $F_L$ ). Robust Newey and West (1987)  $t$ -statistics that account for autocorrelations are reported in parentheses. The sample period is from January 1972 to December 2012.

	<i>L2-model</i>		<i>Profitability</i>		<i>Growth</i>		<i>Momentum</i>		<i>Volatility</i>	
	<i>%ΔOPQ</i>	<i>OPQ/ME</i>	<i>ROEQ</i>	<i>OP/BE</i>	<i>B/M</i>	<i>%ΔAT</i>	<i>R_2_12</i>	<i>SUE</i>	<i>TVOL</i>	<i>O-score</i>
<i>Panel A: Excess returns and FFC4, FF5, and Q4 adjusted returns</i>										
<i>Ex.Ret.</i>	<b>0.42</b>	<b>1.01</b>	<b>0.78</b>	<b>0.37</b>	<b>0.57</b>	<b>-0.43</b>	<b>1.31</b>	<b>0.56</b>	<b>-1.46</b>	<b>-0.90</b>
	(2.75)	(4.40)	(3.01)	(1.83)	(2.37)	(-2.38)	(4.43)	(3.67)	(-3.22)	(-2.34)
<i>α-FFC4</i>	<b>0.26</b>	<b>0.75</b>	<b>0.85</b>	<b>0.47</b>	-0.10	-0.12	<b>0.32</b>	<b>0.52</b>	<b>-1.52</b>	<b>-1.11</b>
	(1.82)	(4.73)	(3.96)	(2.81)	(-0.95)	(-0.73)	(2.40)	(3.86)	(-4.98)	(-3.99)
<i>α-FF5</i>	<b>0.46</b>	<b>0.37</b>	<b>0.64</b>	0.08	-0.08	0.12	<b>1.43</b>	<b>0.62</b>	<b>-1.08</b>	<b>-0.92</b>
	(2.80)	(2.27)	(4.08)	(0.69)	(-0.83)	(0.81)	(3.43)	(3.86)	(-4.20)	(-3.49)
<i>α-Q4</i>	0.16	<b>0.53</b>	0.06	0.05	0.10	0.10	0.32	0.01	<b>-0.80</b>	<b>-0.84</b>
	(0.91)	(2.16)	(0.37)	(0.33)	(0.51)	(0.63)	(0.73)	(0.03)	(-2.49)	(-2.81)
<i>Panel B: L2 adjusted returns and factor loadings</i>										
<i>α-L2</i>	-0.22	0.05	-0.49	-0.18	-0.04	0.00	0.28	-0.02	0.11	0.01
	(-1.04)	(0.15)	(-1.31)	(-0.83)	(-0.14)	(-0.02)	(0.36)	(-0.06)	(0.17)	(0.01)
<i>β<sub>S</sub></i>	<b>0.65</b>	-0.19	<b>0.42</b>	-0.17	-0.13	0.12	<b>1.39</b>	<b>0.49</b>	0.45	0.55
	(4.88)	(-0.98)	(1.74)	(-0.97)	(-0.77)	(0.84)	(2.83)	(2.36)	(0.69)	(1.04)
<i>β<sub>L</sub></i>	0.01	<b>0.87</b>	<b>0.66</b>	<b>0.54</b>	<b>0.56</b>	<b>-0.41</b>	-0.24	0.08	<b>-1.51</b>	<b>-0.97</b>
	(0.13)	(9.65)	(3.82)	(4.91)	(7.50)	(-4.79)	(-0.73)	(0.57)	(-4.98)	(-3.29)

**Table 7. Time-series regressions and the GRS-statistics**

This table reports the performance of the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ) in explaining the portfolios formed on operating profitability rates ( $\% \Delta OPQ$ ), earnings-to-price ratios ( $OPQ/ME$ ), quarterly return-on-equity ( $ROEQ$ ), annual operating profitability ( $OP/BE$ ), book-to-market ratios ( $B/M$ ), asset growth rates ( $\% \Delta AT$ ), prior 11-month returns ( $R\_2\_12$ ), standardized unexpected earnings ( $SUE$ ), total volatility ( $TVOL$ ), and Ohlson's O-score ( $O\text{-score}$ ). Portfolio returns are value-weighted. The learning-based two-factor model ( $L2$ ), consisting of the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ), is

$$R_{p,t} - R_{f,t} = \alpha_p + \beta_{p,S} F_{S,t} + \beta_{p,L} F_{L,t} + \varepsilon_{p,t},$$

where  $R_{p,t}$  is the portfolio return and  $\alpha_p$  is the risk-adjusted return. For each portfolio return as the dependent variable, Panel A presents the  $t$ -statistics for the risk-adjusted return with respect to the  $L2$  model. For each set of the testing portfolios, Panel B reports the Gibbons, Ross, and Shanken (1989) statistics ( $GRS$ -statistics) for the pricing errors in the intercept from the time-series regressions with respect to the  $L2$ -model, four factors of Fama-French (1993) and Carhart (1997) ( $FFC4$ ;  $MKT$ ,  $SMB$ ,  $HML$ , and  $UMD$ ), the five factors of Fama and French (2014) ( $FF5$ ;  $MKT$ ,  $SMB$ ,  $HML$ ,  $CMA$ , and  $RMW$ ), and the four factors of Hou, Xue, and Zhang (2014a) ( $Q4$ ;  $MKT$ ,  $rME$ ,  $rI/A$ , and  $rROE$ ); Numbers reported in brackets are the  $p$ -values. Panel C reports the risk-adjusted return and the factor loadings with respect to the  $L2$  model for each portfolio. Robust Newey and West (1987)  $t$ -statistics that account for autocorrelations are reported in parentheses. The sample period is from January 1972 to December 2012.

	<i>L2-model</i>		<i>Profitability</i>		<i>Growth</i>		<i>Momentum</i>		<i>Volatility</i>	
	<i>%ΔOPQ</i>	<i>OPQ/ME</i>	<i>ROEQ</i>	<i>OP/BE</i>	<i>B/M</i>	<i>%ΔAT</i>	<i>R_2_12</i>	<i>SUE</i>	<i>TVOL</i>	<i>O-score</i>
<i>Panel A: T-statistics for L2 adjusted returns</i>										
<i>Low</i>	(1.45)	(1.10)	(1.06)	(0.90)	(0.52)	(0.54)	(0.18)	(0.57)	(-0.59)	(1.45)
<i>2</i>	(1.25)	(0.03)	(0.76)	(0.17)	(0.20)	(0.81)	(0.33)	(0.67)	(0.01)	(0.65)
<i>3</i>	(0.87)	(-0.27)	(0.67)	(-0.12)	(-0.14)	(0.11)	(0.18)	(0.07)	(-0.33)	(0.75)
<i>4</i>	(0.91)	(0.02)	(0.72)	(0.35)	(-0.05)	(0.45)	(0.50)	(0.53)	(0.80)	(0.46)
<i>5</i>	(0.60)	(0.11)	(0.60)	(0.20)	(-0.17)	(0.74)	(-0.40)	(0.16)	(0.59)	(0.04)
<i>6</i>	(0.81)	(0.07)	(0.24)	(0.41)	(0.10)	(-0.04)	(-0.19)	(-0.05)	(1.14)	(0.57)
<i>7</i>	(0.62)	(-0.26)	(-0.13)	(0.85)	(0.15)	(0.16)	(0.28)	(0.54)	(1.31)	(0.26)
<i>8</i>	(1.15)	(0.45)	(0.16)	(1.20)	(0.05)	(0.67)	(-0.17)	(0.24)	(1.00)	(0.37)
<i>9</i>	(-0.05)	(0.74)	(0.54)	(0.69)	(0.19)	(1.36)	(-0.09)	(0.85)	(0.20)	(0.35)
<i>High</i>	(0.81)	(1.09)	(0.14)	(0.57)	(0.41)	(0.49)	(0.90)	(0.64)	(-0.04)	(0.67)
<i>Panel B: GRS-statistics [p-value] for L2, FFC4, FF5, and Q4 adjusted returns</i>										
<i>GRS-L2</i>	1.85	1.61	1.09	1.54	0.51	1.48	<b>2.19</b>	1.17	<b>4.05</b>	1.79
	[0.05]	[0.10]	[0.37]	[0.12]	[0.88]	[0.15]	<b>[0.02]</b>	[0.31]	<b>[0.00]</b>	[0.06]
<i>GRS-FFC4</i>	<b>2.40</b>	<b>2.80</b>	<b>2.38</b>	<b>3.29</b>	0.89	<b>2.00</b>	<b>3.23</b>	<b>3.57</b>	<b>5.82</b>	<b>3.67</b>
	<b>[0.01]</b>	<b>[0.00]</b>	<b>[0.01]</b>	<b>[0.00]</b>	[0.54]	<b>[0.03]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>
<i>GRS-FF5</i>	<b>3.09</b>	1.75	<b>2.50</b>	<b>2.02</b>	0.74	<b>3.01</b>	<b>4.78</b>	<b>3.48</b>	<b>5.02</b>	<b>3.77</b>
	<b>[0.00]</b>	[0.07]	<b>[0.01]</b>	<b>[0.03]</b>	[0.69]	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>	<b>[0.00]</b>
<i>GRS-Q4</i>	1.59	<b>2.64</b>	1.20	1.37	0.70	<b>2.07</b>	<b>2.74</b>	1.01	<b>3.55</b>	<b>3.15</b>
	[0.11]	<b>[0.00]</b>	[0.29]	[0.19]	[0.72]	<b>[0.03]</b>	<b>[0.00]</b>	[0.44]	<b>[0.00]</b>	<b>[0.00]</b>

**Table 7. (Continued.)**

	<i>L2-model</i>		<i>Profitability</i>		<i>Growth</i>		<i>Momentum</i>		<i>Volatility</i>	
	<i>%ΔOPQ</i>	<i>OPQ/ME</i>	<i>ROEQ</i>	<i>OP/BE</i>	<i>B/M</i>	<i>%ΔAT</i>	<i>R_2_12</i>	<i>SUE</i>	<i>TVOL</i>	<i>O-score</i>
<i>Panel C: L2 adjusted returns and factor loadings</i>										
	$\alpha$									
<i>Low</i>	0.59	0.47	0.54	0.38	0.20	0.22	0.14	0.25	-0.14	0.65
2	0.48	0.01	0.34	0.06	0.07	0.28	0.19	0.23	0.00	0.26
3	0.30	-0.09	0.24	-0.04	-0.05	0.03	0.08	0.03	-0.11	0.31
4	0.31	0.01	0.22	0.11	-0.02	0.14	0.21	0.19	0.31	0.21
5	0.22	0.04	0.20	0.07	-0.06	0.23	-0.14	0.05	0.26	0.02
6	0.29	0.03	0.08	0.16	0.04	-0.01	-0.06	-0.02	0.56	0.31
7	0.22	-0.09	-0.04	0.29	0.05	0.05	0.09	0.18	0.66	0.15
8	0.41	0.15	0.05	0.44	0.02	0.24	-0.06	0.08	0.54	0.25
9	-0.02	0.28	0.19	0.22	0.06	0.54	-0.03	0.34	0.12	0.23
<i>High</i>	0.37	0.52	0.05	0.20	0.16	0.22	0.42	0.23	-0.03	0.66
	$\beta_s$									
<i>Low</i>	0.41	0.79	0.65	0.95	0.86	1.03	0.24	0.37	0.48	0.72
2	0.38	0.90	0.51	0.79	0.91	0.74	0.34	0.38	0.56	0.67
3	0.44	0.90	0.47	0.78	0.87	0.79	0.45	0.56	0.71	0.64
4	0.54	0.87	0.57	0.81	0.78	0.65	0.38	0.62	0.68	0.65
5	0.57	0.66	0.67	0.82	0.75	0.57	0.59	0.81	0.90	0.71
6	0.85	0.70	0.70	0.83	0.68	0.76	0.64	0.88	0.98	0.71
7	0.87	0.71	0.80	0.74	0.62	0.79	0.73	0.79	1.08	0.86
8	0.99	0.68	0.83	0.69	0.63	0.82	1.00	1.00	1.25	1.14
9	1.30	0.70	0.86	0.84	0.69	0.87	1.14	0.84	1.46	1.11
<i>High</i>	1.06	0.60	1.07	0.77	0.73	1.15	1.63	0.86	0.93	1.27
	$\beta_L$									
<i>Low</i>	-0.67	-0.88	-0.95	-0.87	-0.54	-0.36	-0.54	-0.34	0.09	-0.52
2	-0.30	-0.42	-0.41	-0.38	-0.31	-0.17	-0.17	-0.28	-0.01	-0.22
3	-0.23	-0.22	-0.26	-0.20	-0.18	-0.11	-0.09	-0.26	-0.07	-0.19
4	-0.25	-0.21	-0.24	-0.22	-0.12	-0.17	-0.09	-0.26	-0.24	-0.17
5	-0.20	-0.15	-0.27	-0.29	-0.07	-0.12	-0.06	-0.29	-0.40	-0.08
6	-0.29	-0.10	-0.22	-0.29	-0.09	-0.16	-0.10	-0.22	-0.71	-0.20
7	-0.36	0.05	-0.12	-0.43	0.01	-0.18	-0.16	-0.27	-0.90	-0.27
8	-0.51	0.01	-0.23	-0.45	0.05	-0.40	-0.19	-0.30	-1.13	-0.69
9	-0.55	0.00	-0.35	-0.31	0.04	-0.66	-0.29	-0.39	-1.29	-0.80
<i>High</i>	-0.66	-0.01	-0.29	-0.33	0.01	-0.77	-0.78	-0.27	-1.42	-1.49

**Table 8. GMM cross-sectional regressions and the GRS-statistics**

This table reports the performance of the factor loadings with respect to the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ) in explaining the portfolios formed on operating profitability rates ( $\% \Delta OPQ$ ), earnings-to-price ratios ( $OPQ/ME$ ), quarterly return-on-equity ( $ROEQ$ ), annual operating profitability ( $OP/BE$ ), book-to-market ratios ( $B/M$ ), asset growth rates ( $\% \Delta AT$ ), prior 11-month returns ( $R_{2\_12}$ ), standardized unexpected earnings ( $SUE$ ), total volatility ( $TVOL$ ), and Ohlson's O-score ( $O\text{-score}$ ). Portfolio returns are value-weighted. The learning-based two-factor model ( $L2$ ), consisting of the short-run cash-flow factor ( $F_S$ ) and the long-run profitability growth factor ( $F_L$ ) is

$$\mathbb{E}[R_{p,t}] - R_{f,t} = \lambda_S \beta_{p,S} + \lambda_L \beta_{p,L},$$

where  $R_{p,t}$  is the portfolio return,  $\beta_{p,S}$  and  $\beta_{p,L}$  are from the time-series  $L2$ -model:  $R_{p,t} - R_{f,t} = \alpha_p + \beta_{p,S} F_{S,t} + \beta_{p,L} F_{L,t} + \varepsilon_{p,t}$ . For each set of the testing portfolios, parameters are jointly estimated from a one-stage GMM, stacking the orthogonal conditions in the time series and in the cross-section; Panel A reports the GMM estimates with respect to the  $L2$ -model; Panel B reports the Gibbons, Ross, and Shanken (1989) statistics ( $GRS$ -statistics) for the pricing errors in the cross-sectional regressions using the factor loadings with respect to the  $L2$ -model, four factors of Fama-French (1993) and Carhart (1997) ( $FFC4$ ;  $MKT$ ,  $SMB$ ,  $HML$ , and  $UMD$ ), the five factors of Fama and French (2014) ( $FF5$ ;  $MKT$ ,  $SMB$ ,  $HML$ ,  $CMA$ , and  $RMW$ ), and the four factors of Hou, Xue, and Zhang (2014a) ( $Q4$ ;  $MKT$ ,  $rME$ ,  $rI/A$ , and  $rROE$ ). Numbers reported in brackets are the p-values. Robust Newey and West (1987)  $t$ -statistics that account for autocorrelations are reported in parentheses. The sample period is from January 1972 to December 2012.

	<i>L2-model</i>		<i>Profitability</i>		<i>Growth</i>		<i>Momentum</i>		<i>Volatility</i>	
	<i>%ΔOPQ</i>	<i>OPQ/ME</i>	<i>ROEQ</i>	<i>OP/BE</i>	<i>B/M</i>	<i>%ΔAT</i>	<i>R_2_12</i>	<i>SUE</i>	<i>TVOL</i>	<i>O-score</i>
<i>High-Low</i>	<b>0.42</b>	<b>1.01</b>	<b>0.78</b>	<b>0.37</b>	<b>0.57</b>	<b>-0.43</b>	<b>1.31</b>	<b>0.56</b>	<b>-1.46</b>	<b>-0.90</b>
	(2.75)	(4.40)	(3.01)	(1.83)	(2.37)	(-2.38)	(4.43)	(3.67)	(-3.22)	(-2.34)
<i>Panel A: Prices of risks in L2-model</i>										
$\lambda_S$	<b>0.77</b>	<b>1.05</b>	<b>0.88</b>	<b>0.91</b>	<b>1.00</b>	<b>1.02</b>	<b>0.89</b>	<b>0.77</b>	<b>1.18</b>	<b>1.21</b>
	(4.26)	(2.23)	(2.87)	(2.50)	(2.24)	(2.49)	(3.33)	(4.43)	(2.68)	(1.65)
$\lambda_L$	0.27	<b>1.05</b>	0.60	<b>0.70</b>	<b>1.14</b>	<b>0.86</b>	0.76	0.26	<b>1.26</b>	<b>1.24</b>
	(0.49)	(2.67)	(1.26)	(2.11)	(2.52)	(2.73)	(1.13)	(0.24)	(2.64)	(1.88)
$\lambda_S \beta_S^{High-Low}$	0.50	-0.20	0.37	-0.16	-0.13	0.12	1.24	0.38	0.53	0.67
$\lambda_L \beta_L^{High-Low}$	0.00	0.91	0.40	0.38	0.64	-0.35	-0.18	0.02	-1.90	-1.20
<i>Panel B: GRS-statistics [p-value] for L2, FFC4, FF5, and Q4 models</i>										
<i>GRS-L2</i>	1.83	1.11	1.11	1.51	0.42	<b>2.26</b>	1.51	0.99	<b>3.20</b>	<b>2.17</b>
	[0.05]	[0.35]	[0.35]	[0.13]	[0.94]	[0.01]	[0.13]	[0.45]	[0.00]	[0.02]
<i>GRS-FFC4</i>	<b>3.75</b>	<b>4.22</b>	<b>2.33</b>	<b>4.72</b>	1.15	<b>2.17</b>	<b>2.85</b>	<b>3.55</b>	<b>6.11</b>	<b>3.55</b>
	[0.00]	[0.00]	[0.01]	[0.00]	[0.32]	[0.02]	[0.00]	[0.00]	[0.00]	[0.00]
<i>GRS-FF5</i>	<b>3.99</b>	<b>2.91</b>	<b>2.80</b>	<b>2.11</b>	0.77	1.77	<b>3.12</b>	<b>4.06</b>	<b>6.89</b>	<b>3.56</b>
	[0.00]	[0.00]	[0.00]	[0.02]	[0.66]	[0.06]	[0.00]	[0.00]	[0.00]	[0.00]
<i>GRS-Q4</i>	<b>2.42</b>	<b>3.54</b>	1.79	<b>2.12</b>	0.66	<b>1.99</b>	<b>2.20</b>	1.23	<b>4.78</b>	<b>3.47</b>
	[0.01]	[0.00]	[0.06]	[0.02]	[0.76]	[0.03]	[0.02]	[0.27]	[0.00]	[0.00]