Corporate Financing under Ambiguity: A Utility-Free Multiple-Priors Approach

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Abstract

I build a novel utility-free ambiguity model using the misspecification of probabilities within good-deal price bounds. In the model managers and outside investors both are ambiguous about the compensation for invisible idiosyncratic risk on firm's partially-tradable assets, and infer the magnitude of ambiguity from arbitragers' ambition. This ambiguity model is applied to a contingent claim-based capital structure framework. I find ambiguity aversion makes lenders hold a most pessimistic belief about the firm's operating performance as well as their default decision. This key feature helps address low-leverage puzzle and credit spread puzzle about corporate debts, and highlights the relevance of ambiguity preferences in measuring hedging demand and agency conflict beyond debt services. Using a large cross section of S&P 500 firms, the magnitude of ambiguity aversion effects in explaining the patterns observed in the capital structure data is assessed. The comparative statics offer an ambiguity-based explanation for the relation between systematic risk exposures and corporate default/financing policies.

Keywords: ambiguity aversion; Sharpe ratio; optimal leverage; credit spread; agency conflicts; systematic risk.

JEL classification: G32, D81, G13, G33

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1. Introduction

Since Modigliani and Miller (1958), how firms make optimal financing decisions has received considerable attention from financiers and economists.[1] Corporate financing accompanies a variety of important issues; e.g., default decision, capital structure, risk management, debt valuation, agency conflicts, etc. Unless a very little literature on the bond pricing subject to information uncertainty (Duffie and Lando, 2001; Davis, 2008; and Boyarchenko, 2012), most existing theories are consistently based on the rational expectation equilibrium with complete information structure. In this way there leaves no role for *ambiguity* (or *uncertainty*) to play within the analytical frameworks.

The relevance of aversion to ambiguity in the contexts of decision-making, however, has been widely documented.[2] Knight (1921) firstly defines ambiguity as a case where informational-constrained agents are uncertain about probability measures used for decision-making. The Ellsberg Paradox (Ellsberg, 1961) and related experimental evidences document that the distinctions between risk and ambiguity are behaviorally meaningful. Several recent studies (e.g., Ju and Miao, 2011) conclude that the rational expectation hypothesis faces serious difficulties in confronting with asset market data. For these reasons, this paper aims at providing a first step towards understanding and assessing the impact of ambiguity aversion on the features of corporate financing.

I propose a novel utility-free multiple-priors approach to model ambiguity using the misspecification of probabilities within "good-deal" price bounds of Cochrane and Saa-Requejo (2000). My model departs from the traditional max-min model of Gilboa

[1] See, for example, Jensen and Meckling (1976), Miller (1977), Myers (1984), Leland (1994), Leland and Toft (1996), Leland (1998), Morellec (2001), Ju et al. (2005), Hackbarth et al. (2006), Bhamra et al. (2010a, 2010b), Carlson and Lazrak (2010), Chen (2010), and He (2011).

[2] The impacts of ambiguity aversion on decision-making have motivated a vast literature on financial economics. Part of this literature devotes to explaining the puzzles of interest rates and equity premium in the unified framework with a plausible risk aversion parameter (e.g., Epstein and Wang, 1994; Chen and Epstein, 2002; Maenhout, 2004; Leippold et al., 2008; and Ju and Miao, 2011). Another part of this literature puts emphases on the implications for option markets and on the distinct portfolio behavior of ambiguity-averse investors; such as Epstein and Miao (2003), Liu et al. (2005), etc.. Gagliardini et al. (2009) study the implication of ambiguity aversion for the term structure of interest rate.

and Schmeidler (1989) and smooth model of Klibanoff et al. (2005) in two aspects: (i) it uses the quasi arbitrage-free condition to exogenously determine the magnitude of ambiguity; and (ii) it permits the separation between information constraint, measured as the proportion of systematic risk to aggregate risk, and ambiguity.[3] Due to the free of estimating those abstract parameters in the utility functions (such as inter-temporal substitution or risk aversion), as well as concerning for heterogeneities among various types of preference, my model is more tractable for empirical implementation. Further, my model allows for analyzing the comparative statics with respect to informational constraint. It thus can be applied to explaining an endogenous link between systematic risk exposures and the price structure of derivative markets (Duan and Wei 2009), or the cross section of corporate decisions (Acharya et al., 2012; and Chen et al., 2012).

I embed this new ambiguity model inside the Leland's (1994) contingent claim-based capital structure framework. I explore the empirical implication of my modified capital structure model in two ways. First, I use the basic calibration to quantitatively study how lenders' aversion to ambiguity over the assets' return affects the features of corporate financing. I find that ambiguity aversion (i) makes managers choose a lower bankruptcy-triggering threshold such that the under-biased distortion in the expected asset price is less likely to fully offset the default loss suffered by lenders; (ii) makes managers use less debts and pay more interests for debt services simultaneously; (iii) discourages managerial risk-shifting incentives, causing a weaker hedge intention and asset-substitution effect; (iv) benefits lenders by saving their hedging costs as well as

[3] My model links the determination of the magnitude of ambiguity to the signals on economy situation, including market-index Sharpe ratio and the upper bound of observed Sharpe ratios. Because investors with insufficient cash display a more conservative intention to chase the trading with high Sharpe ratios (arbitrage), the upper limit of Sharpe ratios in markets inversely measure the condition of cash holding. On the other hand, the market-index Sharpe ratio represents fair market price for risk. Thus the rises in both are associated with recessions (Brennan et al., 2001; Kato, 2006; and Chen, 2010). Such a model feature can capture the empirical phenomenon of Korteweg and Polson (2010) and Boyarchenko (2012) that the amount of model misspecification as well as uncertainty about asset valuation increases during the 2007- 2008 credit crises.

mitigating agency conflict; and (v) has an increasing first-order impact but decreasing marginal effect on the decision rules, given a rising degree of information constraint.

Second, I use a large cross section of S&P 500 firms to assess the magnitude of ambiguity aversion impacts within the present model. The structural estimations show that on average, ambiguity aversion explains 22.8% of low uses of financial leverage, produces 185 bps yield spread without raising leverage, modifies 8.9% (4.5%) of over-prediction on the equity (debt) value-risk elasticity, cuts one third net tax benefit, and lowers 21% default boundary. The importance of ambiguity aversion in improving the goodness-of-fit of the model with respect to leverage, yield spread, and value-to-risk elasticity is documented using prediction-error tests and moment comparisons.

Next introduce the basic structure of my ambiguity model. I build the model on a key assumption that the representative firm's assets are partially-tradable such that the return cannot be perfectly replicated from well-diversified market portfolio.[4] Because continuous observation on the fundamental value of a thinly or non-traded asset is un-achievable, public market information is insufficient for firm's managers and outside investors to exactly measure the asset return. Information constraint makes them have heterogeneous beliefs on uncertainty over the compensation for assets' unobservable idiosyncratic risk. Such heterogeneous beliefs are described as a set of approximating asset-return models under subjective risk-adjusted measures.

Both managers and outside investors use these distorted asset models to trade all the claims on firm within the good-deal bounds of Cochrane and Saa-Requejo (2000). The value bounds reflect the upper limit of Sharpe ratios in the open markets, which corresponds to an upper constraint on the volatilities of stochastic discount factor (see

[4] Typical intangible assets are often thinly or non-traded, including human capital, trademark copyright, and product-design patents. Korteweg and Polson (2010) document that credit spreads on firms having large intangible assets are the most affected by model uncertainty. See Strebulaev (2003) and Khandani and Lo (2011), for more evidences of asset illiquidity.

Hansen and Jagannathan, 1991). In this way pure arbitrage opportunities that deliver too high Sharpe ratios are precluded.[5] The actual trading value relies on the investor's subjective belief about model uncertainty as well as his ambiguity attitude. Only when the 100% systematic risk proportion is given to eliminate informational constraint, the preference to ambiguity cannot interfere in the decision analyses such that the model is degenerated as a standard preference-free type. In the model information constraint serves as a channel to deliver the ambiguity-related impact on decision-making; while its degree acts as a *control valve* for this channel.

The straightforward implication of my ambiguity model is: due to the constraints on learning information about the asset value, agents are uncertain about the expected asset performance in making the decision. This idea echoes with traditional ambiguity theories (such as the max-min expected utility theory and smoothness theory), which interpret ambiguity as a case where the agents are uncertain about probability measure used for decision-making due to cognitive or informational constraint. My idea is also related to the robustness theory developed by Hansen and Sargent (2001) and Hansen et al. (2006). Specifically, agents in my model fear the misspecification of probability assigned to the status of asset performance. Thus they behave pessimistically to avoid model misspecification when seeking a robust decision-making.

The present paper relates to different strands of literature. First, it closely relates to a growing literature on the bond pricing under uncertainty (Duffie and Lando, 2001; David, 2008; and Boyarchenko, 2012). So far this line of studies mainly concentrates on how uncertainty explains the term structure of credit spread, but pays less attention on how much bond yield spread is attributed to uncertainty, as well as the endogenous interaction among bond pricing, hedge intention, and the joint decision on default and

[5] There is a long tradition in finance that regards the trading with high Sharpe ratios as a pure arbitrage opportunity (see Ross, 1976; Shanken, 1992; Ledoit, 1995; and Cochrane and Saa-Requejo, 2000).

financial leverage. Second, my paper relates to the tradeoff models of Leland (1994), Morellec (2001), Ju et al. (2005), Hackbarth et al. (2006), Chen (2010), or He (2011). In these papers, uncertainty over the firm's growth or performances has been largely ignored. Third, my paper relates to the vast literature examining the rules of decision making subject to ambiguity preference. Different with this literature that often works on asset pricing (e.g., Chen and Epstein, 2002; or Ju and Miao 2011), dynamic asset allocation (Liu et al., 2005), and the term structure of interest rates (Gagliardini et al., 2009), the present paper represents the earliest example to apply ambiguity theory to the issues on corporate finance. Another important difference is that the literature considers systematic-type uncertainty over monetary policy, inflation, or macroeconomic condition; while this paper discusses non-systematic-type uncertainty attributed to the idiosyncratic shocks on the individual firm's operation.

My paper also relates to the literature on corporate risk management and agency theory (e.g., Haugen and Senbet, 1981; Smith and Stulz, 1985; Campbell and Kracaw, 1990; Leland, 1998; and Aretz and Bartram, 2010). I advance this literature by adding the effects of lenders' aversion to ambiguity to the analysis on the positive interaction between managerial risk-taking incentives and hedge benefit. Such an extension helps understand whether the ignorance of ambiguity aversion explains the theoretical overstatements on corporate hedging intention (Guay and Kothari, 2003), as well as asset-substitution agency problems (Graham and Harvey, 2001). Finally, my paper relates to a recent work by Chen et al. (2012), who studies the implications of the exposures to systematic risk for corporate credit spreads and default/financing policies. My results on the comparative statics with respect to information constraint indirectly provide an ambiguity-based explanation for the relation between systematic risk exposure and the cross section of corporate capital structure.

2. A Utility-Free Multiple-Priors Model

Consider a continuous-trading economy endowed with a complete probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ where Ω denotes the finite state space, $\mathbb{P}$ is reference belief, and continuous filtration $(\mathcal{F}_t)_{t\geq 0}$ supports a two-dimension standard Brownian motion $[B(t), W(t)]'$ that generates uncertainty over this economy. Time is continuous and varied over $[0, \infty)$. To fix the term structure of interest rates, assume that default-free bonds are allowed for trading and pay interests at a constant rate $r > 0$.

2.1. Brief Review to "Good-Deal" Asset Price Bound Theory: Reference Model

In this subsection, I firstly make a brief review to the theory of "good-deal" asset price bounds. Extending the incomplete-market setting in Cochrane and Saa-Requejo (2000) and Chen et al. (2010), I consider a representative firm that is totally equity-financed at initial time. The firm's capital assets are partially-tradable and have a total value unaffected by capital structure, denoted by V following a diffusion process as:

$$dV(t)/V(t) = \mu_V\, dt + \sigma_{VB}\, dB^{\mathbb{P}}(t) + \sigma_{VW}\, dW^{\mathbb{P}}(t) \tag{1}$$

with current value $V(0) \equiv \boldsymbol{V}$, nonnegative constant appreciation rate μ_V, and non-negative constant volatility σ_{VB} (systematic shock) and σ_{VW} (idiosyncratic shock).

Also consider a perfect-liquid asset, such as S&P 500 index or a well-diversified market portfolio, S used for twin security of V which has the following evolution:

$$dS(t)/S(t) = \mu_S\, dt + \sigma_S\, dB^{\mathbb{P}}(t) \tag{2}$$

where $S(0) \equiv \boldsymbol{S}$ is the current value, μ_S is nonnegative constant appreciation rate, and σ_S is nonnegative constant volatility. Notably, the firm's asset correlation with market, denoted by $\rho \equiv \operatorname{corr}(dS/S, dV/V) = \sigma_{VB} \big/ \sqrt{\sigma_{VB}^2 + \sigma_{VW}^2}$, never equals 1 unless $\sigma_{VW} = 0$. The imperfection in the correlation implies that a non-traded or illiquid asset cannot be perfectly replicated from its twin security. In other words, information from open markets is insufficient for both firm's managers and outside investors to exactly learn the dynamics of asset value. As a result, it is impossible to perfect hedge in such an incomplete market, suggesting that $B^{\mathbb{P}}$ and $W^{\mathbb{P}}$ must be uncorrelated. Once the replication is less than perfect, the single price law based on the standard no-arbitrage pricing argument fails. In this case exactly determining the value of claims on a firm's assets is unachievable for whole market participants due to information constraint.

According to Proposition 3 of Cochrane and Saa-Requejo (2000), the stochastic discount factor of any economic agent Λ is governed by a dynamic process:

$$d\Lambda(t)/\Lambda(t) = -r\,dt - h_S\,dB^{\mathbb{P}}(t) + \gamma\sqrt{\mathcal{A}^2 - h_S^2}\,dW^{\mathbb{P}}(t) \tag{3}$$

In (3), $h_S \equiv (\mu_S - r)/\sigma_S$ denotes the Sharpe ratio of market index; $\mathcal{A}^2$ denotes the upper limit for volatility; and parameter $\gamma \in [-1,1]$ controls the range of its variation to ensure the fulfillment of volatility constraint $\mathrm{E}_t^{\mathbb{P}}[(d\Lambda(t)/\Lambda(t))^2] \le \mathcal{A}^2$. The diffusion term $-\gamma\sqrt{\mathcal{A}^2 - h_S^2}$ and h_S measures the required compensation for idiosyncratic risk and systematic risk respectively. Thus the lower good-deal bound solves

$$\underline{C}(0) = \min_{\Lambda} \mathrm{E}_0^{\mathbb{P}}[\Lambda(0)^{-1}\int_0^{\infty}\Lambda(s)\,x^c(V(s))\,ds] \tag{4}$$

subject to $\mathbf{P}_0 = \Lambda(0)^{-1}\mathrm{E}_0^{\mathbb{P}}[\Lambda(t)\mathbf{X}_t]$; $\Lambda(t) \ge 0$; $\mathrm{E}_t^{\mathbb{P}}[(d\Lambda(t)/\Lambda(t))^2] \le \mathcal{A}^2$; for $0 \le t \le \infty$

where $\underline{C}(0)$ is the lower good-deal bound; $x^c(\cdot)$ is the focus payoff associated with V to be valued; $\mathbf{P}_0$ and $\mathbf{X}_t$ are the price and payoffs of basis assets (vectors); and the upper good-deal bound solves the corresponding maximum.

The volatility constraint on the discount factor is at the heart of good-deal pricing bound theory. Hansen and Jagannathan (1991) interpret the restriction on the discount factor volatility as an upper limit on Sharpe ratio of mean excess return to standard deviation. Cochrane and Saa-Requejo (2000) choose $\mathcal{A}^2 = 2h_S$ by assuming that an investor will chase any opportunity or asset that delivers a Sharpe ratio twice that of market index. They argue that investors are attracted by good deals—large Sharpe ratios—as well as pure arbitrage opportunities. The size of $\mathcal{A}^2$, therefore, depends on the ceiling of Sharpe ratios least required by potential arbitragers. If $\mathcal{A}^2$ is chosen at a higher level, the range of good-deal bounds will be wider, suggesting that investors display a more conservative intention to arbitrage, and will be more careful in judging the arbitrage opportunities. In brief, values outside good-deal price bounds signal pure arbitrage opportunities. If the value above upper bound appears, investors consistently short sell the corresponded asset, and vice versa. So long as the offer prices are within good-deal bounds, however, the trading strategies adopted by investors are no longer consistent. This is because, in such a situation, the implied Sharpe ratios are likely not high enough for a portion of potential arbitragers, but still are regarded as good-deals by some investors with lower discount factor volatilities.

2.2. Model Misspecification within "Good-Deal" Asset Price Bounds

Using the change from physical measure $\mathbb{P}$ to risk-adjusted measure $\mathbb{Q}$ with discount factor $\tilde{\Lambda}(t) \equiv e^{-rt}$, I follow Cochrane and Saa-Requejo (2000) to rewrite the reference model (1) as

$$dV(t)/V(t) = (\mu_V - \sigma_{VB} h_S - \sigma_{VW}\, \hat{h})\, dt + \sigma_{VB}\, dB^{\mathbb{Q}}(t) + \sigma_{VW}\, dW^{\mathbb{Q}}(t)\,. \tag{5}$$

Notice that, only when the agent guesses the true compensation for unobservable idiosyncratic shock $\hat{h}$, the drift of (5) can be reduced as risk-free interest rate *r*. Because continuous observation on the fundamental value of firm's partially-tradable assets is unachievable, however, public market information is insufficient for agents to exactly measure the asset return as well as the fair risk compensation. Due to this information constraint, the agent must subjectively choose the compensation for idiosyncratic risk $h=-\gamma\sqrt{\mathcal{A}^2-h_S^2}\equiv\hat{h}-h'$ from a set of distorted risk-adjusted drifts $\{h\in\mathbb{R}:h^2\le\mathcal{A}^2-h_S^2\}$ when modeling the data-generating process of asset return.

The agent in fact uses an approximating asset-return model

$$dV(t)/V(t)=[\mu_V-\sigma_{VB}h_S-\sigma_{VW}(\hat{h}-h')]dt+\sigma_{VB}\,dB^{\mathbb{Q}h}(t)+\sigma_{VW}\,dW^{\mathbb{Q}h}(t) \tag{6}$$

in determining the price of the contingent claims on firm within good-deal bounds

$$C(0)=\mathrm{E}_0^{\mathbb{P}}[\Lambda(0)^{-1}\int_0^\infty\Lambda(s)x^c(V(s))\,ds]=\mathrm{E}_0^h[\tilde{\Lambda}(0)^{-1}\int_0^\infty\tilde{\Lambda}(s)x^c(V(s))\,ds] \tag{7}$$

where $\mathbb{Q}h$ represents absolutely continuous contaminations with respect to reference risk-neutral belief $\mathbb{Q}$; $\mathrm{E}_0^h(\cdot)$ is the expectation operator at time-0 under measure $\mathbb{Q}h$; $B^{\mathbb{Q}h}=B^{\mathbb{Q}}$ is a standard Brownian motion under $\mathbb{Q}h$; $W^{\mathbb{Q}h}$ follows a form of measure change similar to the transformation of probability scenario mentioned by Epstein and Miao (2003), Liu et al. (2005), or Gagliardini et al. (2009): $W^{\mathbb{Q}h}(t)=W^{\mathbb{Q}}(t)-\int_0^t h'ds$; and h' is the contaminating drift that satisfies *good-deals (arbitrage)-free* condition:

$$(\hat{h}-h')^2\le\mathcal{A}^2-h_S^2\equiv(\eta-h_S)h_S\,. \tag{8}$$

Hence ambiguity takes the form of the changes of expected appreciation in the asset value under a set of distorted risk-adjusted measures.

2.3. Measuring the Model Misspecification: Discounted Relative Entropy

The bound (8) is useful for measuring the range of discrepancy between the true model (1) and distorted model (6). According to the robustness theory of Hansen and Sargent (2001) and Hansen et al. (2006), the discounted relative entropy is defined as $\mathcal{R}(\mathbb{Q}h)\equiv\int_0^\infty-\mathrm{E}_0^h(\log m(t))\,d\tilde{\Lambda}(t)$ where $m(\cdot)$ is the Radon-Nikodym derivative of $\mathbb{P}$ with respect to $\mathbb{Q}h$. Given $W^{\mathbb{Q}h}(t)=W^{\mathbb{Q}}(t)-\int_0^t h'ds$, $W^{\mathbb{Q}}(t)=W^{\mathbb{P}}(t)+\int_0^t\hat{h}\,ds$, and

$m(t) = e^{\int_0^t -h\,dW^{\mathbb{Q}h}(s) + \int_0^t h^2/2\,ds}$, we thus can derive the model-misspecification constraint as

$$\mathscr{R}(\mathbb{Q}h) \leq \phi \equiv \int_0^{\infty} -0.5(\eta - h_S)\,h_S\; t\,d\tilde{\Lambda}(t) = 0.5(\eta - h_S)\,h_S\,r^{-1} \tag{9}$$

In the specification-error constraint (9), ϕ describes the degree of ambiguity aversion displayed by an agent. It is jointly determined by the Sharpe ratio of market index h_S, the upper bound for Sharpe ratios in the market $\mathcal{A}^2 = \eta h_S$, and the risk-free interest rate r. An extended implication behind constraint (9) is that the preference of agents to ambiguity is related to economic scenarios. Brennan et al. (2001) document that recessions are often associated with a growing market Sharpe ratio. Perez-Quiros and Timmermann (2000) and Whitelaw (1997) both find similar cyclical patterns in the Sharpe ratios. On the other hand, the upper limit on the observable Sharpe ratios reflects investors' ambition to arbitrage as well as the condition of cash holding. If the cash holding is insufficient, investors would display a more conservative intention to arbitrage such that more high-Sharpe-ratio assets are left in the market. Kato (2006) finds pro-cyclical corporate demands for liquid assets. Chen (2010) argues that firms hold more cash in good times because of lower marginal cash cost. Briefly speaking, within the model agents learn the magnitude of ambiguity from the signal of economy state. As the observed economy state deteriorates, the magnitude of model uncertainty increases so that the ambiguity-based impact on agents' decision-making gets stronger.

2.4. Max-Min Infinite Value Program under Ambiguity Aversion

Next consider the decision program of managers of the representative firm under ambiguity. In making the financing policies, the subjective belief managers hold about uncertainty over the asset's return depends on the attitude of external bond investors toward ambiguity. In order to restrict our attention to the central issues, assume that potential lenders faced by managers are ambiguity-averse. For simplicity, also assume their knowledge about Sharpe ratios in the market is symmetric.[6] Thus the preference of ambiguity aversion forces managers to hold a most pessimistic belief on the future asset return. In the later section I will explain why lenders prefer ambiguity aversion to ambiguity loving from the arguments concerning bond hedge and agency conflict.

One important thing we must notice when defining managers' objective function is that the changes in the fundamental value of firm's total assets cannot be observed continuously. Managers only conjecture the asset value at $V_h(t) \equiv \mathrm{E}_t^h(V(t))$ based on the public information during the period of debt service. This conjectural asset price offers managers an alternative criterion to consider the decision on default. It is jointly determined by the realized market index return and managers' subjective belief about model uncertainty: with $\boldsymbol{V}_h \equiv V_h(0) = \boldsymbol{V}$,

$$V_h(t) = \boldsymbol{V} \exp\{\int_0^t [r + (h_V - h\sqrt{1-\rho^2})\sigma_V - 0.5\sigma_{VB}^2 - \sigma_{VB}\,\sigma_S^{-1}\mu_S)]du + \int_0^t \sigma_{VB}\,\sigma_S^{-1} dS(u)/S(u)\}.$$

As suggested by the Ellsberg Paradox, an ambiguity-averse agent behaves as if he maximizes his expected utility under a most pessimistic belief chosen from the set of conditional probabilities (Epstein and Schneider, 2008). Following this notion, the objective of managers is thus to solve a max-min infinite value program

$$\max_{x \in \mathbb{R}_+} \min_{\mathbb{Q}h \in \mathcal{H}(\phi)} \mathrm{E}_0^h [\int_0^\infty \tilde{\Lambda}(t) L(h, x; V_h(t))\, dt] \tag{10}$$

$$\textit{subject to}\;\; dV_h(t)/V_h(t) = [r + (h_V - h_S\rho - h\sqrt{1-\rho^2})\sigma_V]dt + \rho\sigma_V\, dB^{\mathbb{Q}h}(t)\,. \tag{11}$$

The program (10) is equivalent to the so-called "constrained" robust control problem in Hansen and Sargent (2001, 2008) and Hansen et al. (2006), and fits the max-min expected utility theory of Gilboa and Schmeidler (1989). It implicates that ambiguity-averse managers optimally make the financing strategy *x* (e.g., debt issuing amount or coupon level) that maximizes their net leverage value $L(\cdot)$ under a worst-case belief

[6] The assumption of symmetric-information structure is unnecessary, but makes models more tractable. In appendix I will extend the model to include informational asymmetry between managers and lenders to show that my basic results do not rely on the assumption of information structure.

about asset return chosen from the set of multiple priors $\mathcal{H}(\phi)=\{\mathbb{Q}h\in\mathcal{H}:\mathfrak{R}(\mathbb{Q}h)\le\phi\}$.[7] The role of leverage value here is similar to a utility function in traditional ambiguity theories. Actually, my model is much more tractable for empirical analyses due to the free of estimating those abstract parameters in the utility function (e.g., risk aversion), as well as concerning for heterogeneities among various types of preferences.

Notably, the time subscript is redundant for the control of the contaminating drift h. The reason is that, as in Cochrane and Saa-Requejo (2000), both the market-index Sharpe ratio and the upper limit of observed Sharpe ratios here are treated as constants calibrated from historical data such that the magnitude of ambiguity is time-irrelevant. This assumption directly restricts the model's application to static decision problems.[8] The following lemma shows the Bellman-Isaacs condition implied by program (10).

Lemma. *Let* $F(V_{h^*}(t))\equiv\max_{x\in\mathbb{R}_+}\min_{\mathbb{Q}h\in\mathcal{H}(\phi)}\mathrm{E}_t^h[\int_t^\infty\tilde{\Lambda}(s-t)L(h,x;V_h(s))ds]$ *denote the indirect value of a perpetual claim on the firm's leverage benefits. Given the dynamics of asset return as equation (11), the value function F satisfies:*

$$\max_x\min_h 0.5\rho^2\sigma_V^2\boldsymbol{V}_h^2F_{V_hV_h}(\boldsymbol{V}_h)+[r+(h_V-h_S\rho-h\sqrt{1-\rho^2})\sigma_V]\boldsymbol{V}_hF_{V_h}(\boldsymbol{V}_h)$$
$$-rF(\boldsymbol{V}_h)+L(h,x;\boldsymbol{V}_h)=0.$$

The Bellman-Isaacs condition above defines an inhomogeneous ordinary differential equation that has a general solution (see also Leland, 1994; Hansen and Sargent, 2001; and Gagliardini et al., 2009). Since the explicit pricing formula for a perpetual

[7] See equation (24) for the explicit form of value function of a firm's net leverage benefits.
[8] To relax this assumption, there are at least two ways to capture the time-varying properties of Sharpe ratio. Wachter (2002) models the market price of risk as a mean-reverting process (Ornstein-Uhlenbeck process). Some macroeconomic literature, such as Chen (2010), links the risk price to macroeconomic state, modeled as a Markov-switching jump process. Taking the dynamics of Sharpe ratios into account, however, is redundant for static decision analyses. Because this paper mainly puts the focus on a static capital structure problem, I leave such an interesting extension for future research topics. See appendix for an extended model including dynamic upward capital restructuring.

claim on the firm's leverage benefits is available, I could solve the max-min program (10) explicitly. In this way the optimal controls x^* and h^* can be easily solved using the first-order and second-order differentiation conditions.

3. Revisiting Leland (1994): Optimal Capital Structure

This section embeds the utility-free multiple-priors model developed in Section 2 inside Leland's (1994) contingent claim-based capital structure framework.

3.1. Time-Independent Security Model under Ambiguous Beliefs

Following Leland (1994), consider a circumstance where the representative un-levered firm intends to sell a perpetual debt at the value equal its par. Let $F(\boldsymbol{V}_h)$ be the price of a claim on the firm's assets that pays a continuous coupon (rate) C so long as the firm is solvent. Based on lemma, $F(\boldsymbol{V}_h)$ satisfies

$$(1/2)\,\rho^2\sigma_V^2\boldsymbol{V}_h^2F_{V_hV_h}(\boldsymbol{V}_h)+[r+(h_V-h_S\,\rho-h\sqrt{1-\rho^2})\,\sigma_V\,]\boldsymbol{V}_h\,F_{V_h}(\boldsymbol{V}_h)-r\,F(\boldsymbol{V}_h)+C=0\,.$$

The general solution to this ODE can be easily derived as:

$$F\,(\boldsymbol{V}_h)=C\,r^{-1}+A_1\boldsymbol{V}_h^{\,-[a(h,\rho)+b(h,\rho)]}+A_2\boldsymbol{V}_h^{\,-[a(h,\rho)-b(h,\rho)]}\equiv F\,(\boldsymbol{V}_h\,;h,C,\rho) \qquad (12)$$

where

$$a(h,\rho)=[\,r+(h_V-h_S\,\rho-h\sqrt{1-\rho^2}\,)\,\sigma_V-0.5\,\rho^2\sigma_V^2\,]\,\rho^{-2}\sigma_V^{-2}\,;$$

$$b(h,\rho)=[\,(a(h,\rho)\,\rho^2\sigma_V^2)^2+2\,r\rho^2\sigma_V^2\,]^{0.5}\,\rho^{-2}\sigma_V^{-2}\,;$$

and the constants (A_1,A_2) are determined by boundary conditions. We then specify the associated conditions to obtain the values of debt, bankruptcy cost, and tax shield.

3.1.1. Perpetual Debt

The debt sold by firm carries a perpetual coupon payment *c* whose level remains constant until the bankruptcy is declared. For convenience I express the debt value as $D(\boldsymbol{V}_h; h, c, \rho)$ with suppressing other arguments. Let V_B represents an endogenously-determined level of asset value at which the firm goes to bankruptcy. Once the bank-ruptcy occurs, only a portion 1-α of the remaining asset value can be redeemed by debt holders. That means,

$$\lim_{\boldsymbol{V}_h \downarrow V_B} D(\boldsymbol{V}_h; h, c, \rho) = (1-\alpha)\, V_B \tag{13i}$$

$$\lim_{\boldsymbol{V}_h \uparrow \infty} D(\boldsymbol{V}_h; h, c, \rho) = c\int_0^{\infty} \tilde{\Lambda}(t)\, dt \tag{13ii}$$

Given equation (12) with letting $C = c$, condition (13ii) yields $A_2 = 0$ and condition (13i) implies $A_1 = [(1-\alpha)V_B - c\,r^{-1}]V_B^{a(h,\rho)+b(h,\rho)}$. Thus

$$D(\boldsymbol{V}_h; h, c, \rho) = c\,r^{-1} + [(1-\alpha)V_B - c\,r^{-1}]\, (\boldsymbol{V}_h / V_B)^{-a(h,\rho)-b(h,\rho)}. \tag{14}$$

3.1.2. Tax Benefits and Bankruptcy Costs

The well-known tradeoff theory has described the significance of tax effects and bankruptcy loss in determining the optimal capital structure. Tax benefits enjoyed by a perpetual debt $TB(\boldsymbol{V}_h; h, c, \rho)$ resemble a security that pays a constant coupon equal to the tax-sheltering value of interest payment (with setting $C = \tau c$ in (12)) so long as the firm is solvent and pays nothing in the bankruptcy. This security also satisfies the form as (12) with boundary conditions:

$$\lim_{\boldsymbol{V}_h \downarrow V_B} TB(\boldsymbol{V}_h; h, c, \rho) = 0 \tag{15i}$$

$$\lim_{\boldsymbol{V}_h \uparrow \infty} TB(\boldsymbol{V}_h; h, c, \rho) = \tau c\int_0^{\infty} \tilde{\Lambda}(t)\, dt \tag{15ii}$$

Condition (15i) reflects the loss of tax benefits once the firm enters into bankruptcy. Condition (15ii) implicates that, as the bankruptcy becomes increasingly unlikely in the future, the value of tax benefits approaches the capitalized value of the tax benefit flow. Given the above two conditions, using (12) gives

$$TB(\boldsymbol{V}_h;h,c,\rho)=\tau c r^{-1}[1-(\boldsymbol{V}_h/V_B)^{-a(h,\rho)-b(h,\rho)}] \tag{16}$$

Bankruptcy costs are the present value of expected loss in bankruptcy, denoted by $BC(\boldsymbol{V}_h;h,c,\rho)$. This pays no coupon (with setting C=0 in (12)), but has the value equals to αV_B at $\boldsymbol{V}_h=V_B$, suggesting that the boundary conditions are

$$\lim_{\boldsymbol{V}_h\downarrow V_B} BC(\boldsymbol{V}_h;h,c,\rho)=\alpha V_B \tag{17i}$$

$$\lim_{\boldsymbol{V}_h\uparrow\infty} BC(\boldsymbol{V}_h;h,c,\rho)=0 \tag{17ii}$$

Applying conditions (17i) and (17ii) to general solution (12) yields

$$BC(\boldsymbol{V}_h;h,c,\rho)=\alpha V_B(\boldsymbol{V}_h/V_B)^{-a(h,\rho)-b(h,\rho)} \tag{18}$$

3.1.3. Total Levered Value and Equity

The firm's initial total levered value $TV_L(\boldsymbol{V}_h;h,c,\rho)$ consists of three parts: the asset value, plus the tax deduction of coupon payment, less the bankruptcy costs; i.e.,

$$TV_L(\boldsymbol{V}_h;h,c,\rho)=\boldsymbol{V}_h+\tau c r^{-1}[1-(\boldsymbol{V}_h/V_B)^{-a(h,\rho)-b(h,\rho)}]-\alpha V_B(\boldsymbol{V}_h/V_B)^{-a(h,\rho)-b(h,\rho)} \tag{19}$$

The value of equity equals the firm's total value minus the value of debt:

$$\begin{aligned} E(\boldsymbol{V}_h;h,c,\rho)&=TV_L(\boldsymbol{V}_h;h,c,\rho)-D(\boldsymbol{V}_h;h,c,\rho)\\ &=\boldsymbol{V}_h-[V_B-(1-\tau)c r^{-1}](\boldsymbol{V}_h/V_B)^{-a(h,\rho)-b(h,\rho)}-(1-\tau)c r^{-1}\end{aligned} \tag{20}$$

3.2. Endogenous Bankruptcy: Smooth-Pasting Condition

Throughout this research I define the random default time on the firm's assets as $\xi:=\inf(t>0:\mathrm{E}_t^h(V(t))\le V_B(c,h,\rho))$.[9] For simplicity, only the endogenous bankruptcy case is considered. By using smooth-pasting condition (see e.g., Leland, 1994; Leland

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[9] Note that in the definition I use expected price rather than actual price due to the fact that managers cannot continuously learn the changes in the fundamental value of firm's partially-tradable assets. They can only guess the asset's value under a distorted belief based on market information during the period of debt services. Thus the occurrence of default time means the moment at which this conjectural asset value firstly fall below the predetermined threshold $V_B(c,h,\rho)$.

and Toft, 1996; or Hackbarth et al., 2006), the equilibrium bankruptcy-triggered threshold is chosen to maximize the equity's value, meaning that V_B^* solves the equation

$$\partial E(\boldsymbol{V}_h; h, c, \rho) / \partial \boldsymbol{V}_h \big|_{\boldsymbol{V}_h = V_B} = 0 . \tag{21}$$

The solution to equation (21) can be derived differentiating (20) with respect to $\boldsymbol{V}_h$, setting this expression equal to zero with $\boldsymbol{V}_h = V_B$, and solving for $V_B \equiv V_B^*$, which has

$$V_B^* = (1-\tau) c r^{-1} [(a(h,\rho) + b(h,\rho)) / (1 + a(h,\rho) + b(h,\rho))] \equiv V_B^*(h, c, \rho) \tag{22}$$

In addition to coupon choice c, managers' ambiguous belief about uncertainty over the firm's growth h enters the formula of V_B^*. In contrast, the existing models (e.g., Leland, 1994; Hackbarth et al., 2006; Chen, 2010) typically imply that V_B^* is based on the rational expectation equilibrium. This feature leads to important difference in the effects of ambiguity preference on the debt value between their models and mine.

The bankruptcy conditions can be further analyzed by computing the expected appreciation of equity around the bankruptcy trigger. Due to $E_{V_h} = 0$ when $\boldsymbol{V}_h = V_B$, expressing the equity value as a function of $\boldsymbol{V}_h$ and using Ito's lemma can simplify the appreciation of equity as

$$\begin{aligned} dE(\boldsymbol{V}_h; h, c, \rho) \big|_{\boldsymbol{V}_h = V_B} &= (1/2) \rho^2 \sigma_V^2 \boldsymbol{V}_h^2 E_{V_h V_h} \big|_{\boldsymbol{V}_h = V_B} dt \geq 0 \\ &= (1-\tau) c [(a(h,\rho) + b(h,\rho)) / (a(h,1) + b(h,1))] dt \end{aligned} . \tag{23}$$

The left-hand-side of equation (23) implies the expected change in the equity value at $\boldsymbol{V}_h = V_B$. The right-hand-side consists of two parts: (i) the ambiguity-based adjustment factor $(a(h,\rho) + b(h,\rho)) / (a(h,1) + b(h,1))$, which reflects managers' subjective belief about uncertainty; and (ii) the after-tax coupon expense $(1-\tau)c$, which represents the additional cash flow that must be provided by equity holders to keep the firm solvent at bankruptcy point. The expected appreciation of equity equals to the net cost of debt services only in the rational expectation equilibrium (when letting $\rho = 1$ and $h_V = h_S$).

Once the preference to ambiguity is involved in the decision on default, the expected appreciation of equity at bankruptcy point no longer matches the contribution required from shareholders to keep the firm solvent. This naturally echoes with the breakdown of rational expectation hypothesis.

3.3. Financial Variables under Optimal Financing Strategies

This subsection intends to derive a firm's financial variables at optimal leverage in the presence of ambiguity aversion, including optimal debt ratio, debt capacity, and yield spread on debt. According to capital structure tradeoff theory, I explicitly define the contingent payoff of claims on the firm's net levered benefits as

$$L(h,c;V_h(t)) \equiv \tau c 1_{(\xi > t)} - \alpha V_B(c,h,\rho) 1_{(\xi = t)}, \tag{24}$$

which equals the value of tax benefits minus bankruptcy costs. The model solution is summarized in the following theorem (see Appendix A for a detailed proof).

Theorem. *Given the max-min objective as program* (10)*, the dynamics of asset return as equation* (11)*, and the functional form of leverage value as expression* (24)*, ambiguity-averse managers choose the coupon and bankruptcy-triggering threshold at:*

$$c_A^* = \boldsymbol{V}_h [(1 + a(h^*,\rho) + b(h^*,\rho)) l(h^*,\rho)]^{-1/(a(h^*,\rho)+b(h^*,\rho))},$$

$$V_B^*(h^*, c_A^*, \rho) = (1-\tau) c_A^* r^{-1} [(a(h^*,\rho) + b(h^*,\rho)) / (1 + a(h^*,\rho) + b(h^*,\rho))],$$

under the most pessimistic belief about asset return $h^* = (\eta h_S - h_S^2)^{0.5}$.

The value of whole firm, equity, and debt at optimal leverage are as follows:

$$\begin{aligned} TV_L^A \equiv TV_L(\boldsymbol{V}_h; h^*, c_A^*, \rho) = \boldsymbol{V}_h \{ & 1 + \tau r^{-1} [(1 + a(h^*,\rho) + b(h^*,\rho)) l(h^*,\rho)]^{-1/(a(h^*,\rho)+b(h^*,\rho))} \\ & \times [1 - (1 + a(h^*,\rho) + b(h^*,\rho))^{-1}] \} \end{aligned},$$

$$E^A \equiv E(\boldsymbol{V}_h; h^*, c_A^*, \rho) = TV_L(\boldsymbol{V}_h; h^*, c_A^*, \rho) - D(\boldsymbol{V}_h; h^*, c_A^*, \rho),$$

$$D^A \equiv D(\boldsymbol{V}_h;h^*,c_A^*,\rho) = \boldsymbol{V}_h r^{-1}[(1+a(h^*,\rho)+b(h^*,\rho))l(h^*,\rho)]^{-1/(a(h^*,\rho)+b(h^*,\rho))} \\ \times[1-k(h^*,\rho)l(h^*,\rho)^{-1}(1+a(h^*,\rho)+b(h^*,\rho))^{-1}];$$

where

$$j(h,\rho) = [(1-\tau)r^{-1}(1-(1+a(h,\rho)+b(h,\rho))^{-1})]^{a(h,\rho)+b(h,\rho)}[1+a(h,\rho)+b(h,\rho)]^{-1}$$

$$l(h,\rho) = [1+\alpha(1-\tau)(a(h,\rho)+b(h,\rho))\tau^{-1}+a(h,\rho)+b(h,\rho)]\,j(h,\rho)$$

$$k(h,\rho) = [1-(1-\alpha)(1-\tau)(a(h,\rho)+b(h,\rho))+a(h,\rho)+b(h,\rho)]\,j(h,\rho)\,.$$

Optimal debt ratio represents the most ideal leverage choice for a firm, equaling the ratio of debt value to total capital; namely, $L^A = D^A / TV_L^A$. An important issue that arises here is whether optimal leverage chosen by managers is lower in the presence of ambiguity aversion. If the answer is positive, the preference to ambiguity would be justified in explaining the so-called "under-leverage puzzle".

The coupon rate can be straightforwardly computed from dividing the coupon by debt value: $c_A^* / D(\boldsymbol{V}_h;h^*,c_A^*,\rho) = r[1-k(h^*,\rho)l(h^*,\rho)^{-1}(1+a(h^*,\rho)+b(h^*,\rho))^{-1}]^{-1}$. The distance between the rate of coupon and of risk-free interest measures the yield spread on debt. The implication of ambiguity aversion effect on the credit spread may help explain another puzzle-"credit spread puzzle", if the positive ambiguity premium does exist given a small leverage.

Debt capacity demonstrates the maximal value of debt issued by firm. The target coupon that maximizes the debt value is derived differentiating $D(\boldsymbol{V}_h;h^*,c,\rho)$ with respect to *c*, setting the resulting equation to be zero, and solving for $c = c_A^{max}$. Thus we have $c_A^{max} = \boldsymbol{V}_h[(1+a(h^*,\rho)+b(h^*,\rho))k(h^*,\rho)]^{-1/(a(h^*,\rho)+b(h^*,\rho))}$ and debt capacity

$$D(\boldsymbol{V}_h;h^*,c_A^{max},\rho) = Vr^{-1}[k(h^*,\rho)^{-1/(a(h^*,\rho)+b(h^*,\rho))}(1+a(h^*,\rho)+b(h^*,\rho))^{-(1+1/(a(h^*,\rho)+b(h^*,\rho)))} \\ \times(a(h^*,\rho)+b(h^*,\rho))].$$

4. Quantitative Results

I conduct the numerical analyses to quantitatively study how ambiguity aversion affects the features of corporate financing, including leverage choice, default decision, credit spread, debt capacity, agency conflicts, and hedging demand. For comparison, I use the endogenous case of Leland (1994) as my benchmark model ($\rho = 1$).

4.1. A Basic Calibration

I choose the baseline parameters at the values that roughly reflect a typical U.S. corporation. I normalize the initial unlevered value of firm's assets ***V*** to \$100. While this value is arbitrary, I show below that neither credit spread nor debt ratio at optimal leverage depends on this parameter. Because there is no difference in the variance of return between the firm's assets and equity, I choose the asset's aggregate risk σ_V at 20% that is close to the empirical estimate of equity volatility in Cecchetti et al. (2000) and Chen (2010). I choose the risk-free interest rate *r* to be 1.47%, which is consistent with the consumption-based calibrated results of Chen (2010). According to Carlson and Lazrak (2010), the U.S. firms have the average effective tax rate of 32%. Thus I use τ =32% throughout the section. Following Leland (1994), Leland and Toft (1996), and He and Xiong (2011), I use bankruptcy cost α =50% that is close to the empirical estimates of Ju et al. (2005) and Morellec et al. (2011), around 48.52% to 49.1%.

I choose the parameter governing the upper bound for the volatility of stochastic discount factor $\mathcal{A}^2$ to be $2h_S$. Such a choice implies $\eta = 2$, which suggests that no portfolio traded in the market has more than twice the market index Sharpe ratio (see also Ross, 1976; Shanken, 1992; Cochrane and Saa-Requejo, 2000; and Hung and Liu, 2005). I borrow the estimates of Carlson and Lazrak (2010), Chen (2010), and Ju and Miao (2011) to choose market-portfolio Sharpe ratio h_S at 0.33. I set the correlation coefficient between the return on the firm's assets and market index ρ at 0.9, which is also used by Cochrane and Saa-Requejo (2000). Using the standard CAPM theory, I derive the asset's Sharpe ratio under market-based expectation as $\mathrm{E}^{\mathbb{P}}(h_V|\mathcal{F}_B) = h_S\rho$.[10]

4.2. The Decision to Default

I firstly study how managers make responses to ambiguity aversion displayed by potential lenders in considering the decision to default. In the model managers choose an optimal bankruptcy-triggering threshold based on the smooth-pasting condition to make their default policies. Thus I plot the equilibrium bankruptcy-triggered threshold as well as the expected recovery rate as a function of debt value in Figure 1.

[Insert Figure 1 here]

During the period of debt service, managers just know the subjectively-expected value of firm's partially-tradable assets from the public market information. Because ambiguity aversion forces managers distortedly hold a worst-case belief on uncertainty over the firm's performance, assets are always under-valued in terms of expectation. Managers learn the actual value of assets in place only through a bankrupt liquidation. Thus a troublesome problem for managers that arises here is how to determine an ideal bankruptcy point where the liquidated value of remaining assets left for debt holders is not too much. Choosing a lower bankruptcy-triggering threshold intuitively could be a realistic counterplot to such a problem. In this way the under-bias in the expected asset value is less likely to fully offset the expected loss on the debt's principal. Indeed, this

[10] Because of informational constraints, the true expected asset return μ_V is unknown in the economy. Agents can only conjecture the return rate based on public market information. Following such an idea, I therefore apply the standard CAPM theory to the derivation for the market-based expected return rate for assets: $\mathrm{E}^{\mathbb{P}}(dV/V|\mathcal{F}_B) = rdt + (\mathrm{E}^{\mathbb{P}}(dS/S - rdt|\mathcal{F}_B)\mathrm{cov}(dS/S, dV/V))\mathrm{var}(dS/S)^{-1}$.

idea is confirmed by the pattern of Panel A in Figure 1. As we see, the interference of ambiguity aversion in the default decision lowers the equilibrium bankruptcy-triggered threshold, given an arbitrary debt level. Moreover, this negative effect gets stronger as the rises in the debt's issuing amount. Therefore, the expected debt recovery rate under ambiguity aversion is smaller than the standard case without ambiguity (Panel B).

[Insert Figure 2 here]

Within the present model, ambiguity aversion affects the subjective probability of default on the firm's debt in two ways. The first is that, as shown by Figure 1, it delays default by lowering the bankruptcy-triggered threshold. On the other hand, it advances default by making managers' subjective belief about the asset return more pessimistic. From Figure 2, we can find that the latter dominates the former. More specifically, the preference to ambiguity delivers a significant amplifying effect on the term structure of cumulative default probability. For example, when the coupon level is calibrated to match the debt value at $50, the total probabilities of default within 50 and 100 years under ambiguity aversion approach 95% and 100% respectively, whereas those under the standard case sharply falls to 30% and 50%. This suggests that ambiguity aversion makes lenders pessimistically expect that managers are likely to default earlier.

The reason cumulative default probabilities computed by my model are so high is highlighted by Panel B in Figure 2. Observe that the shape of distribution of subjective default probability density is more concentrative in the presence of ambiguity aversion. Further, the convergence speed of density with respect to the extension of time horizon is clearly faster. These two features reflect an ambiguity-based clustering effect on the default density distribution, and as a result, a significant corresponded increment in the cumulative default probability appears. Due to this clustering effect, the expected time to default from my model is earlier. Such a result is supported by Boyarchenko (2012), and echoes with Jaimungal and Sigloch (2010) who argue that in a model of corporate default, ambiguity aversion plays a similar role to risk aversion but has a distinct effect.

[Insert Figure 3 here]

Figure 3 plots the term structure of cumulative default probability with different combination of model parameters. The pattern in Panel A indicates that an increase in the asset risk always leads to a higher default probability, in line with the findings of prior literature; e.g., Leland (1994) and Ju et al. (2005). Panels B and D consider the impacts of the changes in investors' arbitrage intention and market index Sharpe ratio on default likelihood respectively. The patters in these two panels help clarify how the shifts in economy state affect a firm's default policies, because a rising market Sharpe ratio and more conservative intention to arbitrage both signal recessions (Brennan et al., 2001; Kato, 2006; and Chen, 2010). As we expect, default probabilities are found to be inversely related to economy state. The key intuition is that, if the observed state of economy deteriorates, the degree of ambiguity will increase, resulting in a stronger ambiguity aversion effect on default. My results can be further linked to the empirical implications of the clustering of default (or credit contagion). For example, Giesecke (2004) and Driessen (2005) both hold that, when economy enters recession, the shifts in the aggregate shock increase the likelihood of individual default such that firms in a common market will default simultaneously. Chen (2010) has a similar simulation of higher default rates given a worse economy state.

Another interesting detection in Figure 3 is the positive relation between default probability and assets' correlation with market (Panel C). From equation (11), observe that the changes in the asset's correlation cause a twofold impact on the subjectively-expected asset return dynamics. On the one hand, with holding a fixed aggregate asset risk, a weaker correlation implies a lower systematic risk such that the market-based conjectural asset returns are less volatile. On the other hand, the imperfection in correlation amplifies the negative ambiguity effects on the asset's expected appreciation through informational constraint. It seems that the former clearly dominate the latter.

4.3. Optimal Leverage and Debt Capacity

Now examine how lenders' attitude toward ambiguity affects managers' decision on the financial leverage. Figure 4 plots the firm's total levered value as a function of leverage. The peak of each curve here represents optimal choice of leverage ratio that maximizes the tax-bankruptcy tradeoff value on debt issuance. Observe that, relative to the standard case, both the firm's total value and optimal leverage under ambiguity aversion are lower. Such outcomes are attributed to the amplifying effect of ambiguity aversion on the subjective default probabilities. Precisely speaking, the preference to ambiguity marginally increases bankruptcy costs but reduces tax benefits on the debt issuance via default decision. Thus the total levered value enjoyed by a firm becomes fewer, leading a weaker intention to use leverage. The pattern of Figure 4 also echoes with the numbers in Table 1, which report that optimal leverage ratio obtained from the benchmark model and mine equals 57.55% and 48.36% respectively.

[Insert Figure 4 and Table 1 here]

We have been aware from Figure 4 and Table 1 that ambiguity-averse managers will undertake a more conservative debt-issuing strategy when facing uncertainty over the firm's performance. I then discuss whether the implications of ambiguity aversion for leverage usage explain the so-called under-leverage puzzle. This puzzle is initially highlighted by Miller (1977), who finds that the value of bankruptcy costs alone is too small to offset the tax benefits of debt, causing the over-prediction for the net value as well as the usage of leverage.[11] Thus the key to address this capital structure puzzle is to reduce errors in the estimated leverage value by introducing other cost-side factors into the tax-bankruptcy tradeoff analysis.

Different with existing literature, my key intuition to address the under-leverage puzzle is based on the lenders' aversion to economic uncertainty. The intuition is two-stage. First, ambiguity aversion makes lenders pessimistically force managers assign higher probabilities to the lower statuses of asset performance. Then, because of the ambiguity-based amplifying effect on subjective default probabilities, the firm enjoys a smaller tax benefit but carries a greater bankruptcy cost on the financial leverage. In sum, I utilize an ambiguity-driven comovement among default likelihood, bankruptcy cost, and tax benefit to match the observed value of leverage. The well prediction on leverage value thus naturally helps mitigate the overestimation on leverage choice.

The arguments above can be verified by the numbers in Table 1. It shows that net tax benefit relative to initial firm value based on my model equals 8.49%, which is far lower than that of Leland's model, equaling 13.41%, and much closer to the empirical estimates, around 6%-9%, of Graham (2000), Korteweg (2010), Van Binsbergen et al. (2010), and Morellec et al. (2011). My value, however, is still large compared to the data, because other factors of interest rate risk or macroeconomic shock are ignored in the modeling. Besides, we note that, comparing with market data in Huang and Huang (2003) which reports that firms using 53.53% leverage have a 10-year yield spread of 320 bps and default rate 20.6% in average, these values from the benchmark model at optimal leverage (57.55%) are counterfactually low, equaling 76 bps and 5.56%. But

[11] The under-leverage puzzle has motivated a plenty of academic works on agency theory (e.g., Jensen and Meckling, 1976; Myers, 1984; and Leland, 1998), information asymmetry (e.g., Myers and Majluf, 1984), macroeconomic risk (Chen, 2010), and capital structure empirical test (Ju et al., 2005).

this puzzling phenomenon disappears after incorporating ambiguity aversion into the analysis. When leverage is optimally chosen at 48.36%, my model produces the credit spread and default rate at 279 bps and 4.9% respectively. Huang and Huang (2003) report those quantities at 194 bps and 4.39%, given leverage ratio of 43.28%.

[Insert Figure 5 here]

To understand the sensitivities of optimal leverage ratio to the model parameters, Figure 5 is provided. Panel A shows that optimal leverage is a decreasing function of asset risk, consistent with the argument of prior empirical studies that riskier firms do in fact use less leverage (see, e.g., Titman and Wessels, 1988; and Rajan and Zingales, 1995). The negative slope of lines in Panels B and D suggest that firms facing a worse economy state are more reluctant to use debts, because a higher market Sharpe ratio and more conservative intention to arbitrage both imply a larger degree of ambiguity and are usually accompanied by economic recession. Numbers in Table 1 replicate the consistency in these facts. When choosing the market's Sharpe ratio at 0.55 and 0.15, optimal leverage ratio respectively equals 47.58% and 49.98%. By varying the upper limit for Sharpe ratios η from 3 to 1, optimal leverage ratio climbs to 50.55% from 47.38%. My results are in line with Bhamra et al. (2010b) and Chen (2010), who find that capital structure is pro-cyclical at dates when firms re-lever.

The relation between optimal leverage and the assets' correlation with market is considered by Panel C. In the model the correlation serves as a channel to deliver the ambiguity-related impact on decision-making; while its degree can be conceptualized as a control valve for this channel. Thus, given a fixed level of ambiguity, ambiguity aversion effects on optimal leverage will be stronger but marginal effects weaker as the declines in the correlation. Indeed, such an argument echoes with the pattern in the figure as well as numbers in Table 1. It is found that the optimal leverage-correlation line has an increasing positive slope, and optimal leverage ratio reduces from 57.55% to 46.12%, as the correlation falls from 1 to 0.5.

[Insert Figure 6 here]

We now move our attention to the analysis of debt capacity, which represents the maximum amount of debt that can be sold against the firm's asset. As argued by much literature, the variations in debt capacity and optimal leverage are often synchronous (see, e.g., Leland, 1994; and Asvanunt et al., 2011). Take the rise in asset risk as the illustration. Debt capacity and optimal debt level for firms with riskier assets both are relatively lower. Hence, a natural issue that arises here is whether ambiguity aversion displayed by lenders has identical impact on these two measures of capital structure.

Patterns in Figure 6 help clarify the answer to this issue. It is found from Panel A that debt capacity implied by my model (equaling $79.27) is clearly smaller than that by benchmark model (equaling $85.86) due to the ambiguity-based shrinking effects on the expected time to default. Some interesting behavior of debt capacity is depicted in Panels B-D. As we see, similar to optimal leverage, choosing a higher market index Sharpe ratio and a more conservative intention to arbitrage both lead to a smaller debt capacity. These results implicate that the maximum value of firm's debt that could be sold in recession is relatively less, in line with the finding of Hackbarth et al. (2006). Debt capacity and asset-market correlation are positively-correlated, because a weaker correlation causes a stronger ambiguity aversion effect within the model (see Panel C).

4.4. Yield Spread Curves

In this subsection I attempt to analyze the credit spreads on corporate debt. After calibrating a wide range of structural models to match Moody's default data, Huang and Huang (2003) find that those models produce credit spreads well below historical averages, especially for investment grade bonds. This is so-called credit spread puzzle. Hence the main challenge to the puzzle is to explain the spreads between investment grade bonds and treasury bonds. In view of a robust inverse correlation between credit rating and leverage ratio, I assume, only when the firm uses leverage below 45%, the debt will be rated at investment grade. To study the implication of ambiguity aversion for credit spread puzzle, I plot the credit spread as a function of leverage in Figure 7.

[Insert Figure 7 here]

Observe from the figure that the preference of ambiguity aversion does generate a large premium on credit spread, no matter for investment grade or junk bonds. The premium is still significant even if the leverage is very small (i.e., highly-rated bonds). The reasons to explain the ambiguity-based premium on credit spread are twofold. On the one hand, ambiguity aversion causes an amplifying effect on the subjective default probabilities. On the other hand, ambiguity aversion motivates managers to choose a lower bankruptcy-triggered threshold such that the expected recovery rate gets smaller. An immediate implication that arises here is: the debt sold by firm is more worthless in the presence of ambiguity aversion. Hence, given a same coupon level, the implied coupon rate of my model is higher than the benchmark model. Such results echo with Duffie and Lando (2001), David (2008), Jaimungal and Sigloch (2010), Korteweg and Polson (2010), and Boyarchenko (2012), who find that the yield spreads on corporate bonds and CDS premium are larger after taking ambiguity/uncertainty into account.

I further explore the impacts of ambiguity aversion on the estimations of credit spread, using Moody's 10-year default data reported by Huang and Huang (2003) as a benchmark. According to the market data, a debt issuance accompanied with 48.36% leverage ratio is probably rated at Baa- or Ba-rating. For consistency in the parameter choices, I merely take the issuance of Baa-rated and Ba-rated bonds as the illustration. After calibrating the coupon level to match leverage ratio at 43.28%, and 53.53%, my model and Leland's model respectively generates the credit spread at 251 bps and 48 bps for a Baa-rating bond, and 312 bps and 66 bps for a Ba-rating bond. On the other hand, market data respectively reports the average spread at 194 bps and 320 bps. In sum, incorporating ambiguity aversion with bond pricing can substantially modify the biases in the estimation of credit spread.

My estimate of credit spread is higher for highly-rated bonds but lower for junk bonds compared to market data. The reason is referred to an asymmetric time horizon effect on credit spread. This asymmetric effect demonstrates that the extension of debt maturity causes an increasing effect on credit spread when the chosen leverage is low, but a decreasing effect when the chosen leverage is high (see Leland and Toft, 1996; Hackbarth et al., 2006).[12] Hence, the difference in time horizon between market data and my model of consol debt naturally highlights discrepancies in the comparison.

[Insert Figure 8 here]

Figure 8 presents the comparative statics of credit spreads. Consider first asset's risk. Panel A indicates that credit spread increase with asset's risk, implying that debt holders must require additional compensation for default likelihood marginally raised by the rise in asset risk. Within the present model, variations in the asset's risk affect credit spread through debt value. Since higher asset risk often leads to greater default

[12] A similar point is made in Leland and Toft (1996) and Hackbarth et al. (2006). The two papers show that the term structure of credit spread on an investment grade bond is strictly increasing and concave, whereas that on a junk bond has an inverted U-shape.

likelihood, debt on riskier assets is cheaper such that the implied coupon rate is larger.

Consider next the investors' intention to arbitrage and market index Sharpe ratio. As argued by my preceding discussion, a higher Sharpe ratio and more conservative arbitrage behavior both play the signals of recession. Hence, patterns in Panels B and D, which make responses to Figure 3, suggest that, as economy state deteriorates, debt value falls and the implied coupon rate goes up due to the rises in default probability accompanied with the upward-adjustments of ambiguity preference. Such a feature of model is related to Hackbarth et al. (2006), Bhamra et al. (2010a, 2010b), and Chen (2010), who found countercyclical credit spreads.

Consider finally the asset's correlation with market. Notably, this parameter acts as a "controller" for the effects of ambiguity aversion, not a determinant or factor of ambiguity. Only when the correlation is less than perfect, the preference to ambiguity enters the context of decision-making. As a result, the decline in correlation amplifies the magnitude of ambiguity aversion effect within the model, resulting in an inverse relation between credit spread and the level of correlation (see Panel C).

4.5. Risk-Shifting Incentives: Asset Substitution Effect and Hedging Intention

Jensen and Meckling (1976) raises a tenet of financial economics that, after debt is issued, shareholders would prefer to increase the riskiness of the firm's activities. This is presumed to transfer value from debt to equity, leading to "asset substitution"-agency problem. The costs of agency conflict intuitively are related to default risk the debt is exposed to, because as firms are closer to default, shareholders will have less to lose and tend to pursue riskier investment projects. Leland and Toft (1996) argue that debt holders must request a higher coupon if shareholders benefit from increasing the riskiness of firm's activities. For this reason, the subsection intends to investigate whether ambiguity aversion affects the agency conflict between shareholders and debt holders through managerial risk-shifting incentives.

[Insert Figure 9 here]

Within the present model, asset aggregate risk consists of two parts: systematic shock σ_{VB} and idiosyncratic shock σ_{VW}. Assume the risk-shifting strategies adopted by managers are irrelevant with idiosyncratic risk, because it is unobservable and non-hedgeable. In this way Figure 9 plots the value-to-risk sensitivity for debt, equity, and whole firm as a function of aggregate asset riskiness. Observe from Panel A that the sensitivities of equity (solid and dotted lines) are positive whereas those of debt (dash-dotted and dashed lines) are negative. Such dramatic difference confirms the so-called asset substitution effect (Leland, 1994; Leland and Toft, 1996; and Ju and Ou-Yang, 2005). This substitution effect disappears as the asset risk approaches 100%, since an extremely-risky debt is valueless such that no more value can be transferred to equity.

Also observe that the curves of sensitivities under ambiguity aversion (solid and dashed lines) are clearly flatter than those under standard preference (dash-dotted and dotted lines). An important implication behind this result is that the value *stolen* from debt holders to shareholders through risk-taking strategies is fewer in the presence of ambiguity aversion. In other words, previous capital structure theories that ignore the preference to ambiguity may overstate managerial risk-taking incentives as well as the prevalence of asset substitution problem within leveraged firms. The overstatement on asset-substitution problem has been documented by Graham and Harvey (2001), who finds very weak evidences of asset substitution effect on capital structure choices.

In the literature on corporate risk management and agency theory, there has been substantial evidence that firms' hedging activities are motivated by alleviating agency conflict (see Haugen and Senbet, 1981; Smith and Stulz, 1985; Campbell and Kracaw, 1990; and Aretz and Bartram, 2010). Leland (1998) and Campello et al. (2010) find that hedging permits greater leverage and smaller credit spread. DeMarzo and Duffie (1995) hold that financial hedging improves the informativeness of corporate earnings as a signal of management ability. The arguments above leave twofold open questions: Does the preference of ambiguity aversion discourage managers' hedging intention? Does uncertainty over asset performance signal the imperfection in hedge? To explore them, we now turn the attention to Panel B of Figure 9.

[Insert Figure 10 here]

From the figure, note that the value-to-risk sensitivities of whole firm are strictly negative, meaning that undertaking a hedging strategy does increase the firm's value. Relative to the case without ambiguity, hedging effect in my base case is quite weak, and also has a faster convergence speed due to nonhedgeable idiosyncratic risk. More specifically, in the presence of ambiguity aversion, the firm's value is less sensitive to the risk-shifting strategies, causing a smaller hedging benefit.[13] The reason corporate hedging under ambiguity aversion is more inefficient lies in the information constraint on assets' value, measured by the asset's correlation with market index ($\rho = \sigma_{VB} / \sigma_V$). The patterns in Figure 10 indicate that the hedging effect on the asset's total riskiness $\partial \sigma_V / \partial \sigma_{VB}$ gets weaker as the declines in the correlation. Only when the information constraint is released $\rho = 1$, the perfect hedging efficiency appears. Therefore, given a same hedging policy or target, the total costs on the risk-shifting strategies in my base case are naturally higher than those without ambiguity. In sum, within the model there is a coexistence of ambiguity aversion effect and hedging inefficiency. Changes in the

[13] Leland (1998) defines hedging benefits as the increases in the firm's value attributed to the strategies of reducing investment riskiness.

degree of informational constraint drive an inverse comovement between the hedging benefit and ambiguity-related impact on the agency conflict. Such an idea echoes with the argument of above mentioned literature that the effort managers put in the hedging activities is positively associated with the prevalence of agency problems.

5. Empirical Results

This section assesses the main prediction of my modified capital structure model illustrated in Section 4 empirically. There are two objectives for empirical analyses. I firstly measure how big the magnitude of ambiguity aversion effects on the features of corporate financing are, and then compare the goodness-of-fit of my modified model with the benchmark model of Leland (1994).

5.1. Data, Parameter Estimations, and Target Variables

I construct a sample of S&P 500 firms covering the period of January 2, 1992 to December 31, 2011. Since estimating the proposed model requires merging data from various standard sources, I collect data on stock price and S&P 500 index from 3000 Xtra of Thomson Reuters, corporate bond price from Datastream, and annual financial statements from Compustat. As in literature, I screen the data on several fronts: (i) all regulated and financial firms are removed; (ii) observations with missing total assets, market value, long-term debt, debt in current liabilities, and total interest expenses are excluded; (iii) firms with less than five firm-year observations or 3% book debt-total assets ratio are eliminated; and (iv) firms which do not appear on the list of S&P 500 components at the end of 2011 are dropped. In addition, I restrict my sample to bonds with straight type, fixed coupon rate, and no sinking fund options. I also require bonds sold by the same firm to totally have at least five firm-year observations. As a result of these selection criteria, my base sample consists of 3980 firm-years for 274 firms

(133 firms have analyzable bond price data). Tables 2 and 3 offer detailed definition and descriptive statistics for the variables of interest.

[Insert Tables 2 and 3 here]

The estimates of the firm-specific parameters are constructed as follows. I proxy the aggregate risk on each firm's asset return using the standard deviation of monthly return on their equity, given the linear relation between equity value and total assets at optimal leverage as in Theorem: $E_i^A = \boldsymbol{V}^i \mathbb{F}_i$ where $i = 1, 2, 3, \cdots, 274$ and

$$\begin{aligned}\mathbb{F}_i \equiv 1 + r^{-1}[(1 + a(h_i^*, \rho_i) + b(h_i^*, \rho_i))\, l(h_i^*, \rho_i)]^{-1/(a(h_i^*, \rho_i) + b(h_i^*, \rho_i))} \\ \{\tau_i[1 - (1 + a(h_i^*, \rho_i) + b(h_i^*, \rho_i))^{-1}] - [1 - k(h_i^*, \rho_i)\, l(h_i^*, \rho_i)^{-1}(1 + a(h_i^*, \rho_i) + b(h_i^*, \rho_i))^{-1}]\}.\end{aligned}$$

Notably, despite such a linear relation, one still cannot learn the real asset return from stock market price exactly. The reason is that the subjective beliefs on the firm growth implied by price information unceasingly vary with investors' attitude towards risk or ambiguity such that the true belief is impossible to be captured. Heterogeneity among investors' beliefs, however, will not be involved into the calculation of equity/assets risk, ensuring the reasonableness of my estimations.

I use the Capital Asset Pricing Model in estimating the beta of each firm's equity return β_i. These betas can be used for deriving the firms' systematic risk proportions that measure their degree of information constraint; i.e., $\rho_i = \beta_i \sigma_S / \sigma_V^i$. I calculate the effective tax rate as dividing the total tax payments by net pretax income. Following Berger et al. (1996) and Morellec et al. (2011), I estimate bankruptcy costs as:

$$\alpha_i = 1 - (Tangibility_i + Cash_i) / Total\ Assets_i$$

where $Tangibility_i = 0.715 \times \mathrm{Re}ceivables_i + 0.547 \times Inventory_i + 0.535 \times Capital_i$. To acquire the proxy for the Sharpe ratio of a well-diversified market portfolio, I calculate the ratio of average monthly S&P 500-index excess return to their standard deviation, as in Duan and Wei (2009). Finally, I use one-year treasury rate as the proxy for riskless interest rate by following Morellec et al. (2011).

For comparison of the model's goodness-of-fit, I construct four target variables by individual firm as follow. *Market leverage* is computed as the ratio of book debt to the sum of market capitalization values and book debt. For comparability in the risk-shifting incentives across firms, I use value-risk elasticity as the proxy for value-risk sensitivity. *Value-risk elasticity of equity (debt)* is measured as the coefficient of OLS regression of stock (bond) monthly return rate on the percentage changes in asset risk.

Longstaff et al. (2005) offer a useful guide to compute corporate spreads. In view of a fact that less than half of sample firms have sufficient bond data (only 109 firms have at least two analytical bonds), however, I do not adopt their excellent procedure to estimate corporate spreads. Besides, it is always impossible to aggregate individual bond values to obtain the total market value of a firm's debt, because the low trading frequency of corporate bonds naturally causes the lack of market-price data. For these reasons, I proxy *Aggregate yield spread* by the modified book yield spread, calculated as $\kappa_i\, Interest\ expense\ total_i \big/ Book\ debt_i - Riskless\ interest\ rate$. Here κ_i is the adjustment factor for mitigating errors attributed to the refinancing costs, the amortization of debt discount/premium, and non-debt interest expense.[14]

Estimating this factor by intra-firm data unfortunately could be unachievable due to the undisclosed structure of corporate interest expenses. To overcome such a data-missing problem, I jointly use the merging data on bond yield spread and credit rating

[14] As suggested by Compustat's data definition, accounting interest expenses cover not only the actual debt-interest expenses, but also the amortization of debt discount/premium, non-debt interest expenses, and various types of financing charges (e.g., issuing costs). Thus the straight uses of accounting interest expense in calculating the individual firm's aggregate yield spread must suffer substantial biases.

from Huang and Huang (2003) and Chen et al. (2007), and the formulas of S&P 500 corporate spread change of Collin-Dufresne et al. (2001) in calibrating the adjustment factor (see Appendix C for details). The results on estimation are given in Table 4.

[Insert Table 4 here]

5.2. How Big Are the Impacts of Ambiguity Aversion on Corporate Financing?

The analytical results of Section 4 provide a first step towards understanding the quantitative impacts of ambiguity aversion on the features of corporate financing. Yet, these cannot convincingly justify the importance of ambiguity aversion in explaining the patterns found in the capital structure data. For this reason, I assess the magnitude of ambiguity aversion effects within the present model. The results of assessment are displayed in Table 5.

[Insert Table 5 here]

Observe that ambiguity aversion causes the first-order impacts on both the bond pricing and leverage choice. On average, ambiguity aversion explains about 22.8% of under uses of financial leverage, and generates yield spread of 185 bps without raising leverage. In particular, these ambiguity-based impacts become stronger within firms using more leverage, and are robust for firms in different leverage groups. For firms with leverage lower (higher) than 8% (32%), ambiguity aversion reduces 17% (49%) of prediction errors on the leverage choice and 49% (66%) of prediction errors on the yield spread simultaneously. Such a finding echoes with Korteweg and Polson (2010) who argue that the relation between uncertainty and credit spreads is highly non-linear, dependent with the level of default risk. Also, it confirms the relevance of ambiguity aversion in explaining the dual puzzles about corporate debts again.

Now turn the attention to equity and debt value-risk elasticity. As we see, taking ambiguity aversion into account lowers the firm's value-risk elasticity. Specifically, it cuts 0.067 (0.025) value-risk elasticity for equity (debt) on average. While this second -order effect seems weaker compared to the above two cases, it explains 8.9% (4.5%) of over-predictions on the equity (debt) value-risk elasticity at optimal leverage. The negative ambiguity aversion impact on the value-risk elasticity represents a potential explanation for theoretical overstatements on corporate hedging intention (Guay and Kothari, 2003) and asset-substitution agency problems (Graham and Harvey, 2001).

Still, it is observed that on average, ambiguity aversion raises 20.15% long-term (20-year) default probabilities, and reduces 5.5% tax benefits on debts. Jaimungal and Sigloch (2010) similarly conclude a positive relation between ambiguity aversion and the probability of corporate defaults. Using the structural estimations, Graham (2000), Korteweg (2010), Van Binsbergen et al. (2010), and Morellec et al. (2011) report that the tax benefits of debt broadly equals to 6-9% of initial firm value. The range of this value is close to my estimate (10.571%) but far lower than that by benchmark model (16.088%). Overall, ambiguity aversion helps us modify 54-77% of over-biases in the estimated tax benefits on corporate debts.

5.3. Robustness in Goodness-of-Fit

This subsection explores whether the magnitude of ambiguity aversion effects is sufficient for improving the goodness-of-fit of models with respect to capital structure data. Two natural methods I adopt to assess the model's empirical performance are: (i) to compare various model moments to their empirical analog; and (ii) to examine the sizes of prediction errors. The comparisons of goodness-of-fit with respect to leverage, yield spreads, and value-risk elasticity are reported in Table 6, 7, and 8 respectively.

[Insert Tables 6-8 here]

I first give a discussion on the moment comparison. From tables, observe that my model performs better along moments that the literature has identified to be of first-order importance. The average and median level of leverage ratio and yield spreads by my model all are much closer to empirical data, relative to Leland (1994) benchmark model. Take the case of full sample as an illustration. My model generates the mean (median) of leverage ratio and yield spread at 48.8% (49.9%) and 262 bps (226 bps) respectively; Leland's model generates these at 58.1% (58.7%) and 77 bps (44 bps) respectively; while real data respectively reports these at 17.6% (15.1%) and 262 bps (231bps). The same holds true for higher-order moments (i.e., skewness and kurtosis), except for the case of highly-leveraged firms' yield spreads. Roughly, the successes of matching the empirical moments closer are robust for different leverage groups.

Here I consider two standard measures of prediction error, including the mean of absolute errors (MAE) and the root of mean squared errors (RMSE). In addition, I use the *t*-statistic tests for verifying whether the presence of ambiguity aversion mitigates the prediction errors of the model significantly. The numbers in tables reveal that after taking ambiguity aversion into account, the model observably has smaller RMSE and MAE. Exactly speaking, RMSE on leverage ratio, yield spreads, and the equity (debt) value-risk elasticity falls from 0.433, 0.0220, and 0.814 (0.586) to 0.347, 0.0169, and 0.728 (0.545) respectively; while MAE reduces from 0.410, 0.0193, and 0.760 (0.566) to 0.322, 0.0123, and 0.693 (0.542) respectively. Consistently, the results on *t*-statistic test support the hypothesis that incorporating ambiguity aversion into capital structure models can effectively reduce the errors on predicting leverage level, bond price, and the sensitivity of equity and debt value to the firm's riskiness.

6. Why Do Lenders Tend to Be Ambiguity-Averse?

Theoretical results we have discussed are consistently based on a key assumption that potential lenders are ambiguity-averse. In this section I clarify why aversion is the preferred ambiguity attitude for lenders from the arguments concerning bond hedging and agency conflict. For comparability in the analyses, consider *ambiguity loving* as the alternative ambiguity attitude. By intuition, I define the preference of ambiguity loving as a case where the agent behaves as if he maximizes his expected value program under a most *optimistic* belief chosen from the set of multiple-priors. That is,

$$\max_{c\in\mathbb{R}_+}\max_{\mathbb{Q}h\in\mathcal{H}(\phi)} \mathrm{E}_0^h[\int_0^\infty \tilde{\Lambda}(t)(\tau c 1_{(\xi>t)} - \alpha V_B(c,h,\rho)1_{(\xi=t)})dt].$$

[Insert Figure 11 here]

We begin with bond hedging. Duration is one of common risk-sensitive measure for bonds. Macaulay (1938) measure duration as the percent change of a bond price in response to a uniform change in risk-free interest rates. Following this notion, I plot duration, $-(\partial D(\cdot)/\partial r)D(\cdot)^{-1}$, as a function of debt value in Panels A and B of Figure 11. It is observed that, given a same par value, bond duration computed by ambiguity-averse model is strictly shorter than that by ambiguity-loving model. Durations fall as the increase in debt issuing amount and in implied yield spread. In particular, duration under ambiguity aversion is concave, but that under ambiguity loving is convex. Such dramatic differences imply that lenders displaying ambiguity loving have higher outlays for duration-matching hedging strategies (e.g., immunization technique). Besides, this hedging expensiveness is amplified by the rise in the exposure to default risk.

Panel C considers the relation between risk-free interest rate and the changes in debt value. Observe that the attitude toward ambiguity displayed by lenders has direct impacts on the reaction of bond price to a change in interest rate. The return on debt displays a quite weak sensitivity to interest rate under ambiguity aversion, but a strong sensitivity under ambiguity loving. Since a wider range of variation in bond price due to interest rate risk requires a more complicated hedging technique, ambiguity-loving lenders bear higher hedging costs, compared to ambiguity-averse lenders. As a result, in view of hedging efficiency, lenders prefer ambiguity aversion to ambiguity loving.

Next move the attention to the comparison of agency conflict effects between the two cases (Panel D). From the figure, note that both the value-to-risk sensitivities of debt and equity in the ambiguity-averse case are clearly weaker. Such discrepancies in the sensitivities disappear as asset's aggregate risk approaches an extremely high level. These findings reflect a fact that the asset substitution-based agency problem faced by ambiguity-loving debt holders is more prevalent than that by ambiguity-averse debt holders. Thus the intention to reduce the costs on agency conflict naturally motivates potential lenders to be ambiguity-averse.

7. Conclusion

This paper investigates the implications of ambiguity aversion for the features of corporate financing. To do so, I construct a novel utility-free multiple-prior model by using the probabilistic misspecification within good-deal pricing bounds of Cochrane and Saa-Requejo (2000). The proposed model represents a significant departure from the traditional ambiguity theories in two dimensions. First, it exogenously determines the magnitude of ambiguity using the quasi no-arbitrage condition without specifying the preferences of decision makers. In this way my model is much more tractable for empirical analyses due to the free of concerning about heterogeneities among various types of preference, as well as the free of estimating those abstract parameters in the utility function (e.g., intertemporal substitution elasticity and risk aversion). Second, it measures informational constraint as the systematic risk proportion, independent with ambiguity. Thus analyzing the comparative statics of ambiguity aversion effect on the decision-making with respect to informational constraint is achievable. Such a feature enables my model to be applied to explaining the relation between the cross section of corporate decision-making and systematic risk exposures.

This paper applies the proposed ambiguity model to Leland’s (1994) contingent claim-based capital structure framework. The modified capital structure model helps understand how managers make the response to lenders’ aversion to ambiguity when deciding the default timing. The model goes a long way to explain the credit spreads puzzle and low-leverage puzzle about corporate debts from the arguments concerning ambiguity aversion. The model highlights the relevance of preference to ambiguity in measuring agency conflict and hedge demand beyond debt services. The comparative statics of model outputs with respect to information constraint provides an ambiguity-based explanation for a link between systematic risk exposure and financing decision.

To empirically evaluate the proposed model, this paper uses a large cross section of S&P 500 firms covering 1992 to 2011. The results show that on average, ambiguity aversion explains about 22.8% of under uses of financial leverage, generates 185 bps yield spread without raising leverage ratio, modifies 8.9% (4.5%) of over-predictions on the equity (debt) value-to-risk elasticity, cuts one third net tax benefits, and lowers 21% default boundary. The magnitude of ambiguity aversion effect is large enough to improve the goodness-of-fit of the model with respect to leverage, yield spread, and value-risk elasticity significantly.

Much work remains to be done. For example, how the preference to ambiguity affects the endogenous interaction between firms’ financing and investment will be an important future research topic. More difficult extensions include capital restructuring under ambiguity aversion and dynamic utility-free multiple-priors theory.

Appendix A. Proof of Model Solution

To incorporate the smooth-pasting condition (21) into decision-making, I begin the proof by rewriting equations (14), (19), and (20) with $V_B = V_B^*$ as

$$D(\boldsymbol{V}_h; h, c, \rho) = cr^{-1}[1-(c/\boldsymbol{V}_h)^{a(h,\rho)+b(h,\rho)} k(h,\rho)] \tag{A.1}$$

$$TV_L(\boldsymbol{V}_h; h, c, \rho) = \boldsymbol{V}_h + \tau c r^{-1}[1-(c/\boldsymbol{V}_h)^{a(h,\rho)+b(h,\rho)} l(h,\rho)] \tag{A.2}$$

$$E(\boldsymbol{V}_h; h, c, \rho) = \boldsymbol{V}_h - (1-\tau) c r^{-1}[1-(c/\boldsymbol{V}_h)^{a(h,\rho)+b(h,\rho)} j(h,\rho)]. \tag{A.3}$$

Recall from subsection 2.4 that the objective of managers is to solve a static max -min infinite value program as (10). Given the form of net leverage value function as expression (24), using equation (A.2) thus can rewrite the control problem (10) as

$$\max_{c \in \mathbb{R}_+} \min_{\mathbb{Q}h \in \mathcal{H}(\phi)} [\tau c r^{-1}(1-(c/\boldsymbol{V}_h)^{a(h,\rho)+b(h,\rho)} l(h,\rho))]. \tag{A.4}$$

Because the objective function (A.4) is time-independent, I simply solve it with two-stage method without using dynamic programming technique. At the first stage, the contaminating drift h^* that represents the most pessimistic belief hold by managers about model's uncertainty is derived. Then, under this worst-case belief, the second stage chooses an optimal coupon level c_A^* that maximizes total leverage value.

The set of multiple-priors $\mathbb{Q}h \in \mathcal{H}(\phi)$ indirectly defines an inequality constraint $h^2 \le (\eta - h_S) h_S$. Kuhn-Tucker theorem is thus applicable. However, differentiating the firm's value function (A.2) with respect to the contaminating drift *h* is mathematically intractable. For this reason, I search h^* invoking numerical technique, which gives $h^* = (\eta h_S - h_S^2)^{0.5}$. Then plugging h^* into the max-min optimization (A.4) and using the first-order differentiation condition $\partial[\tau c r^{-1}(1-(c/\boldsymbol{V}_h)^{a(h^*,\rho)+b(h^*,\rho)} l(h^*,\rho))] / \partial c = 0$ can solve the optimal coupon choice as

$$c_A^* = \boldsymbol{V}_h [\,(1+a(h^*,\rho)+b(h^*,\rho))\,l(h^*,\rho)\,]^{-1/(a(h^*,\rho)+b(h^*,\rho))}$$

The "maximum" property of c_A^* is easily verified using the second-order condition

$$-\tau(r\boldsymbol{V}_h)^{-1} l(h^*,\rho)(1+a(h^*,\rho)+b(h^*,\rho))(a(h^*,\rho)+b(h^*,\rho))(c_A^* / \boldsymbol{V}_h)^{a(h^*,\rho)+b(h^*,\rho)-1} < 0\,.$$

Thus this completes the proof.

Appendix B. Extensions

In this appendix, I extend the base-case model in two ways: dynamic capital re-structuring and managers-outside lenders informational asymmetry.

B.1. Informational Asymmetry

I build the base-case model on a standard assumption that managers and potential lenders equally have the perfect access to information on Sharpe ratios in the market. Yet, such an assumption could be too tight, especially for a market having incomplete information structure. The idea of combining informational asymmetry with a setting of incomplete market in fact has been recognized by much of literature on information incompleteness (see, e.g., Duffie and Lando, 2001). For this reason, I discuss how and whether informational asymmetry between managers and potential lenders affects the decision-making on debt financing subject to ambiguity preference.

We are easily aware from equation (3) that the incompleteness of information on Sharpe ratios in the market naturally lowers the observed upper limit on Sharpe ratios as well as the volatility of stochastic discount factor. Thus information asymmetry can be modeled as the difference in subjective discount factor. Based on this idea, I redefine the dynamics of stochastic discount factor for managers Λ_M and lenders Λ_L as:

$$d\Lambda_M(t)/\Lambda_M(t) = -r\,dt - h_S\,dB^{\mathbb{P}}(t) + \gamma\sqrt{\mathcal{A}_M^2 - h_S^2}\,dW^{\mathbb{P}}(t),\quad \mathcal{A}_M^2 \equiv \eta_M h_S\,;$$

and

$$d\Lambda_L(t)/\Lambda_L(t) = -r\,dt - h_S\,dB^{\mathbb{P}}(t) + \gamma\sqrt{\mathcal{A}_L^2 - h_S^2}\,dW^{\mathbb{P}}(t)\,,\quad \mathcal{A}_L^2 \equiv \eta_L h_S\,.$$

Only if managers and lenders share the same Sharpe-ratio information, their discount factors will be indifferent; namely, $\eta_M = \eta_L$. Normally, a longer distance between η_L and η_M implies a stronger degree of informational asymmetry.

Next discuss the impacts of informational asymmetry on the decision-making of financing. Two possible cases should be considered: (i) $\eta_M > \eta_L$ and (ii) $\eta_M < \eta_L$. On the one hand, if Sharpe-ratio information possessed by managers is superior to that by lenders ($\eta_M > \eta_L$), managers must conceal their information about higher Sharpe ratios to act in the best interest of equity holders, in view of an inverse relation between the firm's net leverage value and the upper limit on Sharpe ratios.[15] In this case managers make the financing decision at $\eta = \eta_L$ utilizing their informational advantage. On the other hand, when lenders obtain richer Sharpe-ratio information ($\eta_M < \eta_L$), managers are forced to take the debt-issuing price with $\eta = \eta_L$. Thus summarizing the above two statements concludes that the upper Sharpe-ratio bound used by managers for decision making only depends on how much Sharpe ratios in the open market outside lenders observe, no matter whether their information structure is symmetric or not. Also, it is understandable that information asymmetry and ambiguity aversion effects within the present model will move in a same direction, if lenders have better access to Sharpe-ratio information, and vice versa.

B.2. Dynamic Capital Restructuring

Appendix B.2 shows how the proposed model can be extended to allow dynamic upward capital restructuring briefly. To do so, let ψ_i represents the *i*-th restructuring

[15] The inverse relation between a firm's net leverage value and the upper boundary of observed Sharpe ratios is available from Table 1.

point where $i=1,2,\cdots,\infty$ and $\psi_0=0$. By following Leland (1998), Hackbarth et al. (2006), and Chen (2010), assume that the perpetual debt sold by representative firm at time point ψ_i is callable at par P_i^A once the subjectively-expected asset value V_h^i firstly reaches the upper restructuring level V_U^i before the lower bankruptcy-triggered boundary V_B^i. At next restructuring time point, a new issue of another perpetual debt $D_{i+1}^A(V_h^{i+1};c_{\psi_{i+1}},V_U^{i+1},h_{\psi_{i+1}},\rho)$ is optimally issued, and a proportional transaction cost for this new debt issuance qP_{i+1}^A is incurred. Introducing such a transaction cost into the model helps prevent the firm from restructuring continuously. Therefore, the dynamic version of max-min infinite value program is given as

$$\sum_{i=0}^{\infty}\max_{c_{\psi_i}\in\mathbb{R}_+}\min_{\mathbb{Q}h_{\psi_i}\in\mathcal{H}(\phi)}\mathrm{E}_0^{h_{\psi_i}}\Big[\int_{\psi_i}^{\psi_{i+1}}\tilde{\Lambda}(t)\big(\tau c_{\psi_i}1_{(\xi_i>t)}-\alpha V_B(c_{\psi_i},h_{\psi_i},\rho)1_{(\xi_i=t)}-qP_i^A 1_{(\psi_i=t)}\big)dt\Big] \quad \text{(B.1)}$$

subject to

$$dV_h^i(t)\big/V_h^i(t)=[r+(h_V-h_S\rho-h_{\psi_i}\sqrt{1-\rho^2})\sigma_V]dt+\rho\sigma_V dB^{\mathbb{Q}h}(t),\ V_h^i(\psi_i)\equiv V_U^{i-1},$$

where

$$\xi_i:=\inf\big(t>\psi_i:V_h^i(t)\le V_B^i(c_{\psi_i},h_{\psi_i},\rho),\max_{\psi_i<s<t}V_h^i(s)\le V_U^i\big);$$

$$\psi_i:=\inf\big(t>\psi_{i-1}:V_h^{i-1}(t)\ge V_U^{i-1},\min_{\psi_i<s<t}V_h^{i-1}(s)\ge V_B^{i-1}(c_{\psi_{i-1}},h_{\psi_{i-1}},\rho)\big).$$

The program above seems much more complicated relative to the static program (10). An effective way to resolve it is to utilize "scaling property". The reason lies in a key model feature that the path of optimal contaminating drift chosen by managers is invariant to the capital restructuring, because ambiguity factors are constant; namely, $h_{\psi_0}^*=h_{\psi_1}^*=h_{\psi_2}^*=\cdots=(\eta h_S-h_S^2)^{0.5}$. The path of optimal coupon chosen by managers in fact is only dependent with assets' subjectively-expected value. Goldstein et al. (2001) prove that if two firms are identical except that their initial values differ by a factor, their levels of optimal coupon, optimal default-triggered thresholds, and all the claims will differ by the same factor. This argument confirms the scaling feature inherent in my model, because it also holds for a single firm that first issues debt when its value is $V_h^0(\psi_0) = \boldsymbol{V}_h$, and later restructures its capital by retiring all outstanding debt at face value and then issuing a new larger debt when its value rises to $V_h^1(\psi_1) \equiv V_U^0$. In brief, the scaling property ensures that all period-ψ_i claims to firm will scale upwardly by the same factor ϖ^i consistently where $\varpi = V_U^0 \big/ \boldsymbol{V}_h$.

Now derive the model solution. Using the scaling property, program (B.1) can be reduced as

$$\max_{c_{\psi_0}, V_U^0 \in \mathbb{R}_+} \min_{\mathbb{Q}h \in \mathcal{H}(\phi)} tb(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho) - bc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho) - tc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho) \qquad \text{(B.2)}$$

where $tb(\cdot) \equiv \mathrm{E}_0^h [\sum_{i=0}^{\infty} \int_{\psi_i}^{\psi_{i+1}} \tilde{\Lambda}(t) \tau \varpi^i c_{\psi_0} 1_{(\xi_i > t)} dt]$, $tc(\cdot) \equiv \mathrm{E}_0^h [\sum_{i=0}^{\infty} \tilde{\Lambda}(\psi_i) q \varpi^i P_0^A]$, and $bc(\cdot) \equiv \mathrm{E}_0^h [\sum_{i=0}^{\infty} \tilde{\Lambda}(\xi_i) \alpha \varpi^i V_B(c_{\psi_0}, h, \rho)]$ represents the present value of a contingent claim to the firm's tax benefits, transaction costs, and bankruptcy costs respectively. Notably, since the time when either boundary is first hit is random, the prices of these claims at any point before either boundary is hit do not depend explicitly on time. So I can apply the general solution (12) to deriving their explicit forms.

Consider first the total value of tax benefits. Two boundary conditions should be imposed on the general solution:

$$\lim_{V_h \downarrow V_B} tb(V_h; c_{\psi_0}, V_U^0, h, \rho) = 0 \quad \text{and} \quad \lim_{V_h \uparrow V_U^0} tb(V_h; c_{\psi_0}, V_U^0, h, \rho) = tb(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi .$$

The first defines the condition where the firm declares bankruptcy as usual; while the other, based on the scaling property, shows that the total tax shield at the restructuring point V_U^0 should be ϖ as large as it is at $\boldsymbol{V}_h$. Thus a straightforward derivation has:

$$tb(V_h; c_{\psi_0}, V_U^0, h, \rho) = \tau c_{\psi_0} r^{-1} + \lambda_1 V_h^{-[a(h,\rho)+b(h,\rho)]} + \lambda_2 V_h^{-[a(h,\rho)-b(h,\rho)]} \qquad \text{(B.3)}$$

where

$$\lambda_1 = \Sigma^{-1}(-(V_U^0)^{-[a(h,\rho)-b(h,\rho)]}\tau c_{\psi_0} r^{-1} - V_B^{-[a(h,\rho)-b(h,\rho)]}(tb(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi - \tau c_{\psi_0} r^{-1}))\,,$$

$$\lambda_2 = \Sigma^{-1}(-(V_U^0)^{-[a(h,\rho)+b(h,\rho)]}\tau c_{\psi_0} r^{-1} - V_B^{-[a(h,\rho)+b(h,\rho)]}(tb(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi - \tau c_{\psi_0} r^{-1}))\,,$$

$$\Sigma = V_B^{-[a(h,\rho)+b(h,\rho)]}(V_U^0)^{-[a(h,\rho)-b(h,\rho)]} - (V_U^0)^{-[a(h,\rho)+b(h,\rho)]}V_B^{-[a(h,\rho)-b(h,\rho)]}\,,$$

and $tb(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho) = \overline{tb}(\boldsymbol{V}_h)(1-\Xi)^{-1}$, where

$$\overline{tb}(\boldsymbol{V}_h) = \tau c_{\psi_0} r^{-1}(1 + \Sigma^{-1}(\boldsymbol{V}_h^{-[a(h,\rho)+b(h,\rho)]} V_B^{-[a(h,\rho)-b(h,\rho)]} - V_B^{-[a(h,\rho)+b(h,\rho)]}\boldsymbol{V}_h^{-[a(h,\rho)-b(h,\rho)]}$$
$$+ (V_U^0)^{-[a(h,\rho)+b(h,\rho)]}\boldsymbol{V}_h^{-[a(h,\rho)-b(h,\rho)]} - (V_U^0)^{-[a(h,\rho)-b(h,\rho)]}\boldsymbol{V}_h^{-[a(h,\rho)+b(h,\rho)]}))\,,$$

$$\Xi = \varpi\,\Sigma^{-1}(\boldsymbol{V}_h^{-[a(h,\rho)-b(h,\rho)]} V_B^{-[a(h,\rho)+b(h,\rho)]} - \boldsymbol{V}_h^{-[a(h,\rho)+b(h,\rho)]} V_B^{-[a(h,\rho)-b(h,\rho)]})\,.$$

Consider next the total value of bankruptcy and transaction cost. Similarly, these two claims have the corresponded boundary conditions:

$$\lim_{V_h \downarrow V_B} bc(V_h; c_{\psi_0}, V_U^0, h, \rho) = \alpha V_B\,,\quad \lim_{V_h \uparrow V_U^0} bc(V_h; c_{\psi_0}, V_U^0, h, \rho) = bc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi\,,$$

$$\lim_{V_h \downarrow V_B} tc(V_h; c_{\psi_0}, V_U^0, h, \rho) = 0\,,\quad \lim_{V_h \uparrow V_U^0} tc(V_h; c_{\psi_0}, V_U^0, h, \rho) = tc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi\,.$$

Their explicit forms are the following

$$bc(V_h; c_{\psi_0}, V_U^0, h, \rho) = \lambda_3 V_h^{-[a(h,\rho)+b(h,\rho)]} + \lambda_4 V_h^{-[a(h,\rho)-b(h,\rho)]} \tag{B.4}$$

$$tc(V_h; c_{\psi_0}, V_U^0, h, \rho) = \lambda_5 V_h^{-[a(h,\rho)+b(h,\rho)]} + \lambda_6 V_h^{-[a(h,\rho)-b(h,\rho)]} \tag{B.5}$$

where

$$\lambda_3 = \Sigma^{-1}((V_U^0)^{-[a(h,\rho)-b(h,\rho)]}\alpha V_B - V_B^{-[a(h,\rho)-b(h,\rho)]} bc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi)\,,$$

$$\lambda_4 = \Sigma^{-1}(V_B^{-[a(h,\rho)+b(h,\rho)]} bc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi - (V_U^0)^{-[a(h,\rho)+b(h,\rho)]}\alpha V_B)\,,$$

$$\lambda_5 = -\Sigma^{-1} V_B^{-[a(h,\rho)-b(h,\rho)]} tc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi\,,\quad \lambda_6 = \Sigma^{-1} V_B^{-[a(h,\rho)+b(h,\rho)]} tc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho)\varpi\,,$$

$$bc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho) = \overline{bc}(\boldsymbol{V}_h)(1-\Xi)^{-1}\,,\quad tc(\boldsymbol{V}_h; c_{\psi_0}, V_U^0, h, \rho) = \overline{tc}(\boldsymbol{V}_h)(1-\Xi)^{-1}\,,$$

$$\overline{bc}(\boldsymbol{V}_h) = \alpha V_B\,\Sigma^{-1}((V_U^0)^{-[a(h,\rho)-b(h,\rho)]}\boldsymbol{V}_h^{-[a(h,\rho)+b(h,\rho)]} - (V_U^0)^{-[a(h,\rho)+b(h,\rho)]}\boldsymbol{V}_h^{-[a(h,\rho)-b(h,\rho)]})\,,$$

and $\overline{tc}(\boldsymbol{V}_h) = qP_0^A$.

Consider finally the value of debt. Applying the two boundary conditions,

$$\lim_{V_h \downarrow V_B} D_0^A(V_h\ ; c_{\psi_0}, V_U^0, h, \rho) = (1-\alpha)V_B\,, \quad \lim_{V_h \uparrow V_U^0} D_0^A(V_h\ ; c_{\psi_0}, V_U^0, h, \rho) = P_0^A\,,$$

we have

$$D_0^A(V_h\ ; c_{\psi_0}, V_U^0, h, \rho) = c_{\psi_0} r^{-1} + \lambda_7 V_h^{-[a(h,\rho)+b(h,\rho)]} + \lambda_8 V_h^{-[a(h,\rho)-b(h,\rho)]} \tag{B.6}$$

where

$$\lambda_7 = \Sigma^{-1}((V_U^0)^{-[a(h,\rho)-b(h,\rho)]}((1-\alpha)V_B - c_{\psi_0} r^{-1}) - V_B^{-[a(h,\rho)-b(h,\rho)]}(P_0^A - c_{\psi_0} r^{-1}))\,,$$

$$\lambda_8 = \Sigma^{-1}(V_B^{-[a(h,\rho)+b(h,\rho)]}(P_0^A - c_{\psi_0} r^{-1}) - (V_U^0)^{-[a(h,\rho)+b(h,\rho)]}((1-\alpha)V_B - c_{\psi_0} r^{-1}))\,.$$

As is typical in practice, I require the debt to satisfy *sell-at-par* condition. Thus setting $D_0^A(\boldsymbol{V}_h\ ; c_{\psi_0}, V_U^0, h, \rho) = P_0^A$ solves P_0^A as

$$\begin{aligned} P_0^A = {} & ((1-\alpha)V_B - c_{\psi_0} r^{-1})(\boldsymbol{V}_h^{-[a(h,\rho)+b(h,\rho)]}(V_U^0)^{-[a(h,\rho)-b(h,\rho)]} - \boldsymbol{V}_h^{-[a(h,\rho)-b(h,\rho)]}(V_U^0)^{-[a(h,\rho)+b(h,\rho)]}) \\ & \times(\Sigma + \boldsymbol{V}_h^{-[a(h,\rho)+b(h,\rho)]} V_B^{-[a(h,\rho)-b(h,\rho)]} - \boldsymbol{V}_h^{-[a(h,\rho)-b(h,\rho)]} V_B^{-[a(h,\rho)+b(h,\rho)]})^{-1} + c_{\psi_0} r^{-1}. \end{aligned}$$

Using equations (B.3)-(B.6) has the equity value after the debt is issued, equaling the unlevered firm value, plus tax shields, less bankruptcy costs, less transaction costs, less the value of debt,

$$\begin{aligned} E_0^A(V_h\ ; c_{\psi_0}, V_U^0, h, \rho) = {} & (\lambda_1 - \lambda_3 - \lambda_5 - \lambda_7) V_h^{-[a(h,\rho)+b(h,\rho)]} + (\lambda_2 - \lambda_4 - \lambda_6 - \lambda_8) V_h^{-[a(h,\rho)-b(h,\rho)]} \\ & + V_h + (\tau - 1) c_{\psi_0} r^{-1}. \end{aligned}$$

The limited liability of the equity must satisfy the smooth-pasting condition:

$$\begin{aligned} & [a(h,\rho)+b(h,\rho)](\lambda_1 - \lambda_3 - \lambda_5 - \lambda_7) V_B^{-[a(h,\rho)+b(h,\rho)+1]} + [a(h,\rho)-b(h,\rho)] \\ & \times(\lambda_2 - \lambda_4 - \lambda_6 - \lambda_8) V_B^{-[a(h,\rho)-b(h,\rho)+1]} - 1 = 0 \end{aligned} \tag{B.7}$$

Equation (B.7) endogenously determines the optimal bankruptcy-triggering boundary as a function of coupon and capital restructuring level. Thus plugging the equilibrium bankruptcy-triggering threshold into program (B.2) rewrites the decision program that can be used to solve optimal coupon and upward restructuring level numerically.

Appendix C. Calibrating the Interest-Expense Adjustment Factor

I adopt a backward recursive method to calibrate the interest-expense adjustment factor. The calibration is achieved by the following steps.

Step 1: I collect the merging data on credit rating, financial leverage, and yield spread from Huang and Huang (2003) and Chen et al. (2007). The merging data reveals that on average, firms rated at AAA have yield spread of 133 bps and 13.08% leverage; at AA have 21.18% leverage and 152 bps yield spread; at A have 31.98% leverage and 183 bps yield spread; at BBB have 242 bps yield spread and 43.28% leverage; at BB have 53.53% leverage and yield spread of 437 bps; and at B have 65.7% leverage and yield spread of 681 bps. The reason I use the yield spread data from Chen et al. (2007) is that the horizon of their long-maturity (15-40 years) data can well match the present model of consol bonds without term premium-based modifications.

Step 2: I calibrate the final firm-year adjustment factor for each sample firm κ_{i20} at which the modified book yield spread matches a historical average level corresponded to their own credit rating. Specifically, for each firm *i* with current rating *j*, I solve

$$\begin{aligned} YS(\kappa_{i,20}) &\equiv Interest\ expense\ total_{i,20}\kappa_{i,20} \big/ Book\ debt_{i,20} - Riskless\ interest\ rate_{20} \\ &= Rating\text{-}j\ avg.\ yield\ spread \end{aligned} \qquad \text{(C.1)}$$

For little firms without credit rating, I artificially assign them to rating AAA, AA, A, BBB, BB, and B, if their last firm-year market leverage is within the range from 0% to 17%, from 17% to 26%, from 26% to 37%, from 37% to 48%, from 48% to 59%, or from 59% to 100% respectively. Then the last firm-year adjustment factors of these firms can be similarly solved using equation (C.1).

Step 3: I use the formula of S&P 500 yield spread changes from Collin-Dufresne et al. (2001) to backwardly calibrate each period's adjustment factor $\kappa_{i,19}, \cdots, \kappa_{i,1}$. The goal is to recursively solve the following system of equation with terminal condition (C.1): for $t = 2, 3, \cdots, 19$,

$$
\begin{cases}
\Delta YS_{i,t} = -0.095\,\Delta r_t^{10} - 0.014\,SP_t, \text{ if } Lev_{i,t-1} \in [0\%, 15\%] \\
\Delta YS_{i,t} = -0.161\,\Delta r_t^{10} + 0.057\,\Delta (r_t^{10})^2 - 0.028\,\Delta\, slope_t - 0.015\,SP_t, \text{ if } Lev_{i,t-1} \in [15\%, 25\%] \\
\Delta YS_{i,t} = -0.156\,\Delta r_t^{10} + 0.056\,\Delta (r_t^{10})^2 - 0.035\,\Delta\, slope_t - 0.012\,SP_t, \text{ if } Lev_{i,t-1} \in [25\%, 35\%] \\
\Delta YS_{i,t} = -0.2\,\Delta r_t^{10} + 0.055\,\Delta (r_t^{10})^2 - 0.017\,SP_t, \text{ if } Lev_{i,t-1} \in [35\%, 45\%] \\
\Delta YS_{i,t} = 0.015\,\Delta\, Lev_{i,t} - 0.21\,\Delta r_t^{10} - 0.013\,SP_t, \text{ if } Lev_{i,t-1} \in [45\%, 55\%] \\
\Delta YS_{i,t} = 0.013\,\Delta\, Lev_{i,t} - 0.211\,\Delta r_t^{10} + 0.143\,\Delta (r_t^{10})^2 - 0.088\,\Delta\, slope_t - 0.017\,SP_t, \; otherwise
\end{cases}
\tag{C.2}
$$

In (C.2) $\Delta YS_{i,t} \equiv YS(\kappa_{i,t}) - YS(\kappa_{i,t-1})$ is the single-period changes in the aggregate yield spread; $\Delta\, Lev_{i,t}$, Δr_t^{10}, $\Delta (r_t^{10})^2$, and $\Delta\, slope_t$ respectively denotes the single-period changes in market leverage, 10-year treasury rate, the square of 10-year treasury rate, and the slope of treasury yield curve; and SP_t is *t*-year return on S&P 500 index.

The system of equations above is a little bit different with Collin-Dufresne et al. (2001). I do not take the changes in the magnitude of downward jumps in the firm's value into account, because the impact of jump risk on capital structure choices is not in the primary attempts of this research. Also, I drop the variables whose explanation powers are statistically insignificant from formulas (*t*-stat. within the range from -1.96 to 1.96). While there may exist under biases in the variance of yield spread under such tractable procedures, my cross-sectional estimates of yield spreads hold unbiasedness, so long as (i) the final year-end book yield spread matches its historical average level; and (ii) actual changes in the yield spread are symmetrically distributed.

There are some indications that corporate spread changes approximately follow a symmetric distribution. Using the descriptive statistics, Campbell and Huisman (2003) report that shifts in the U.S. corporate spread have -0.052 skewness. Huang and Kong (2003) particularly discover that the skewness of long-maturity and high-rating spread changes is closer to zero. The weighted-average skewness of 15-year-and-more yield spread changes of investment grade bonds equals -0.293; while the spread changes of junk bonds are right-skewed weakly, and have the average skewness of 1.035. Similar to Huang and Kong (2003), Avramov et al. (2007) show that the mean and median of changes in investment-grade corporate spread match perfectly, but the mean of spread changes of junk bonds (0.06) is slightly larger than the corresponded median (0.05).

It is noteworthy that a positive skewness of yield spread changes implies that the probabilities of negative changes are more than those of positive changes. Hence, my backward calibration for interest-expense adjustment factor would underestimate the cross section of yield spread on low-rated firms, if the case of right-skewed junk-bond spread changes holds true. This indirectly explains why my target yield spreads in the case of highly-levered firms (equaling 311 bps) are slightly lower than those from the modified capital structure model (equaling 354 bps). Even though such under biases weaken (improve) the performance of my (benchmark) model on fitting yield spreads, the present results on prediction-error tests still support the hypothesis that my model has better goodness-of-fit with respect to yield spreads (see Panel C in Table 7).

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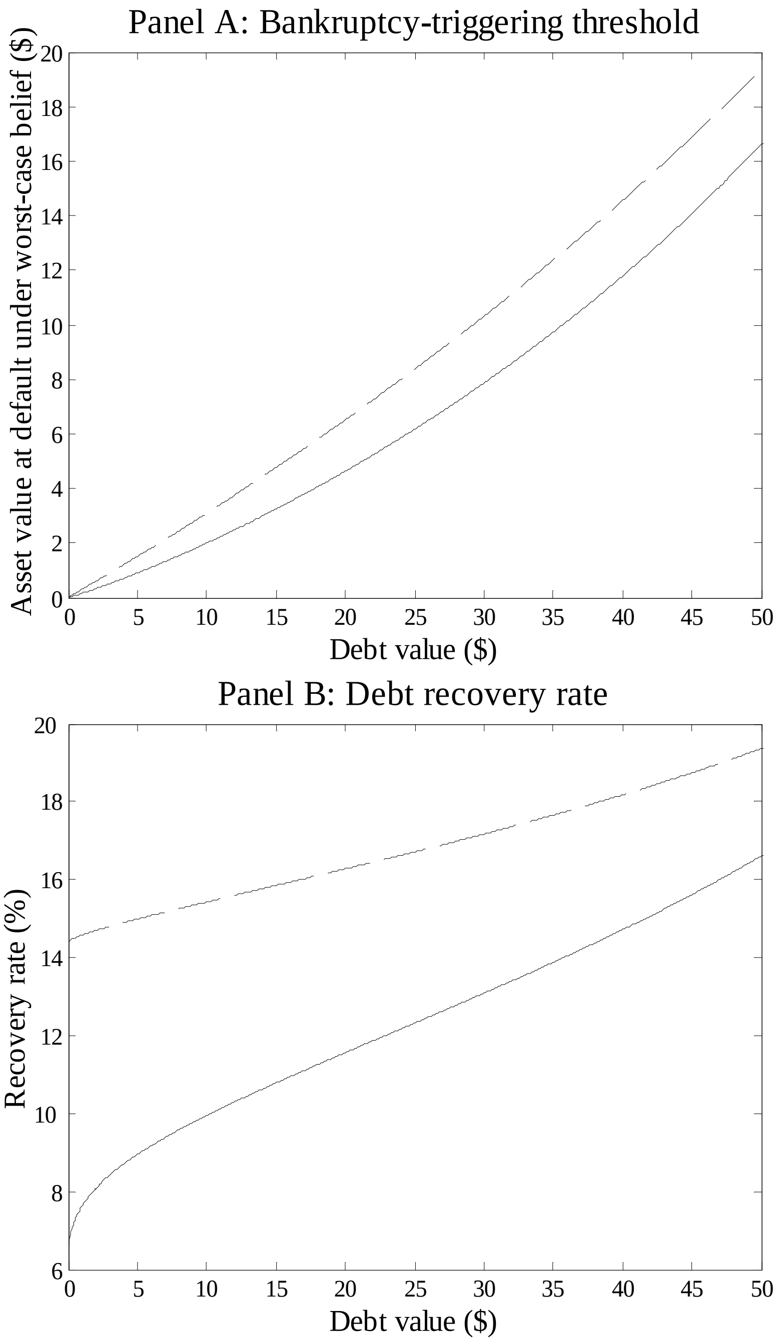
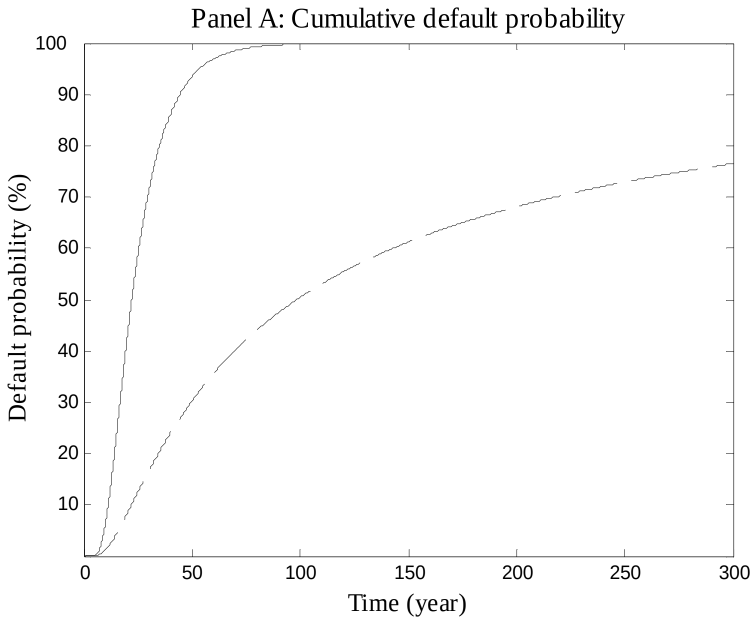


Figure 1. Bankruptcy-triggering threshold and expected debt recovery rate as a function of debt value. Panel A plots the bankruptcy-triggering threshold as a function of debt value for the case where managers display rational expectation (dashed line) and ambiguity aversion (solid line). Panel B plots expected debt recovery rate as a function of debt value for the case where managers display rational expectation (dashed line) and ambiguity aversion (solid line). The values of parameters are chosen at their baseline level.



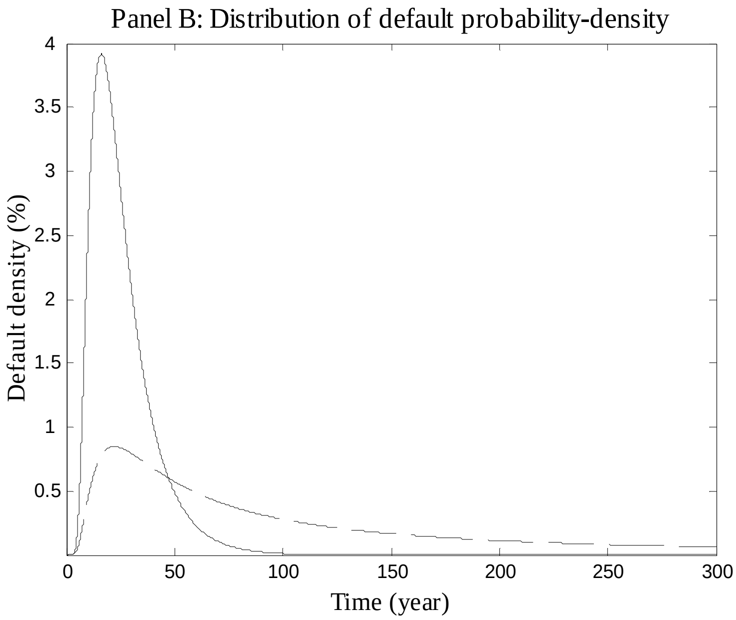


Figure 2. Subjective cumulative default probabilities and probability-density distribution. Panel A plots the subjective cumulative default probabilities as a function of time for the case where managers display ambiguity aversion (solid line) and rational expectation (dashed line). Panel B plots the default probability-density as a function of time for the case where managers display ambiguity aversion (solid line) and rational expectation (dashed line). The coupon is chosen to match the debt value at \$50. The values of parameters are chosen at their baseline level.

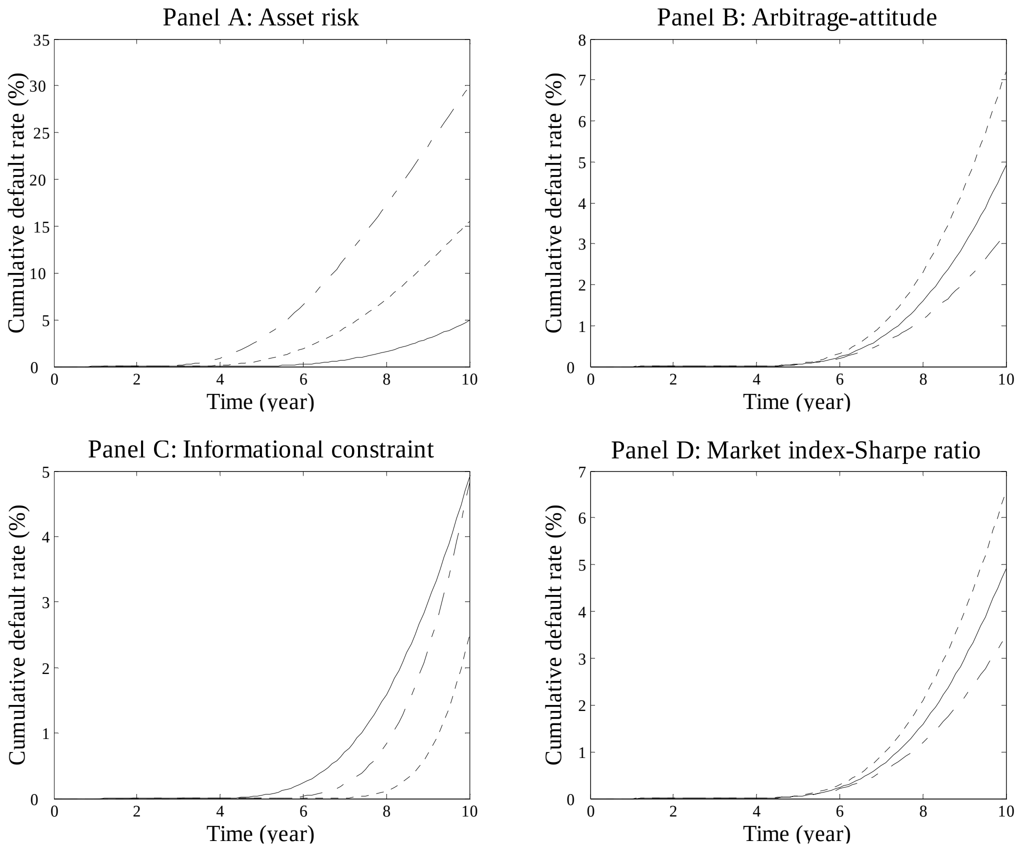


Figure 3. Cumulative default probabilities as a function of time at optimal leverage. The lines in each panel plot the cumulative default probabilities at optimal leverage given a range of time from 0 to 10 years. Panel A plots the probability curve for firm with three different levels of asset risk σ_V: 20% (solid line), 25% (dotted line), and 30% (dash-dotted line). Panel B plots the probability curve for firm with three different levels of investors' intention to arbitrage η: 1 (dash-dotted line), 2 (solid line), and 3 (dotted line). Panel C plots the probability curve for firm with three different levels of the assets' correlation with market ρ: 0.9 (solid line), 0.7 (dash-dotted line), and 0.5 (dotted line). Panel D plots the probability curve for firm facing three different levels of market index Sharpe ratio h_S: 0.55 (dotted line), 0.15 (dash-dotted line), and 0.33 (solid line). The values of parameters are chosen at their baseline level. The coupons are chosen to maximize the firm's total levered value under a worst-case belief about the asset return.

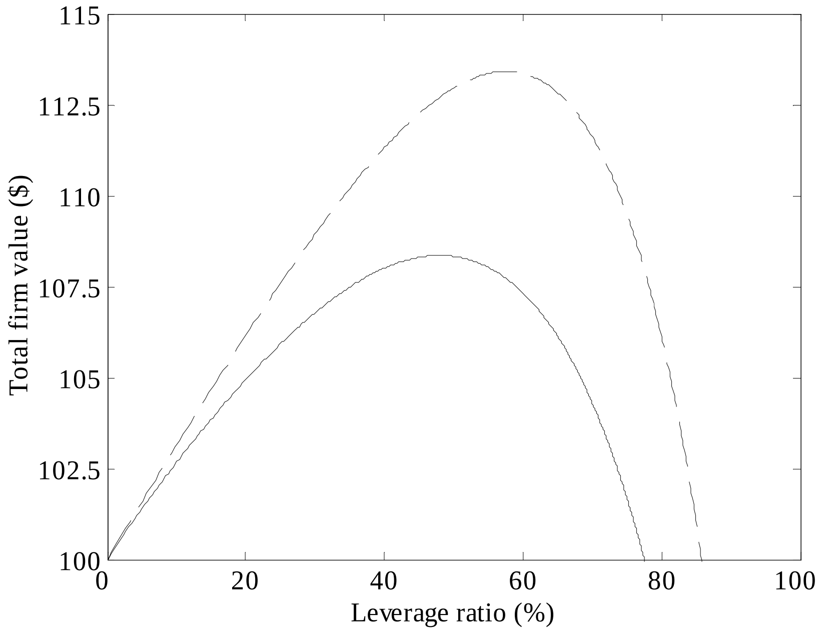


Figure 4. Total firm value as a function of leverage. The lines plot the total firm value as a function of leverage ratio for the case where managers display rational expectation (dashed line) and ambiguity aversion (solid line). The values of parameter are assumed to be chosen at their baseline level.

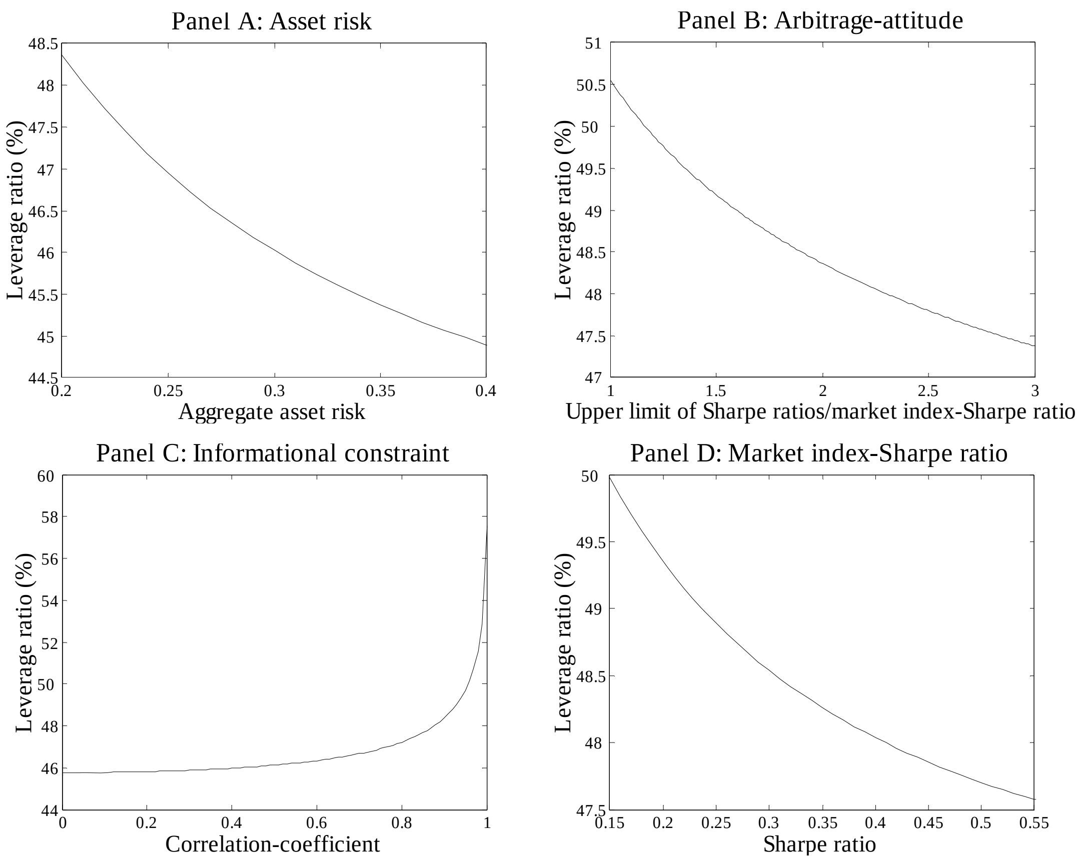


Figure 5. Optimal leverage ratio as a function of model parameters. The lines plot optimal leverage ratio as a function of aggregate asset risk σ_V in Panel A, investors' intention to arbitrage η in Panel B, asset's correlation with market ρ in Panel C, and market index Sharpe ratio h_S in Panel D. The values of parameters are chosen at their baseline level.

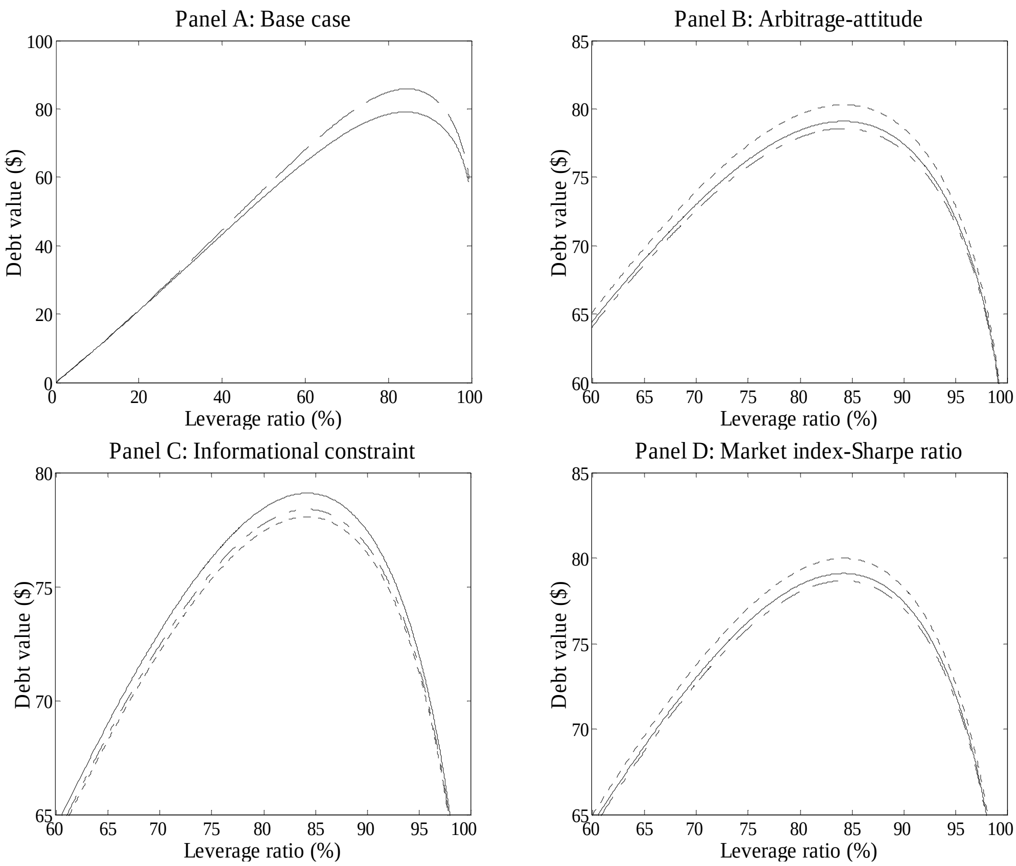


Figure 6. Debt value as a function of leverage with different combination of model parameters. Lines in each panel plot the debt value as a function of leverage ratio. Panel A plots the debt value for the case where managers display ambiguity aversion (solid line) and rational expectation (dashed line). Panel B plots the debt value with three different levels of investors' intention to arbitrage η: 1 (dotted line), 2 (solid line), and 3 (dash-dotted line). Panel C plots the debt value given three different levels of the asset's correlation with market ρ: 0.9 (solid line), 0.8 (dash-dotted line), and 0.7 (dotted line). Panel D plots the debt value with three different levels of market index Sharpe ratio h_s: 0.15 (dotted line), 0.33 (solid line), and 0.55 (dash-dotted line). The parameter values are chosen at their baseline level.

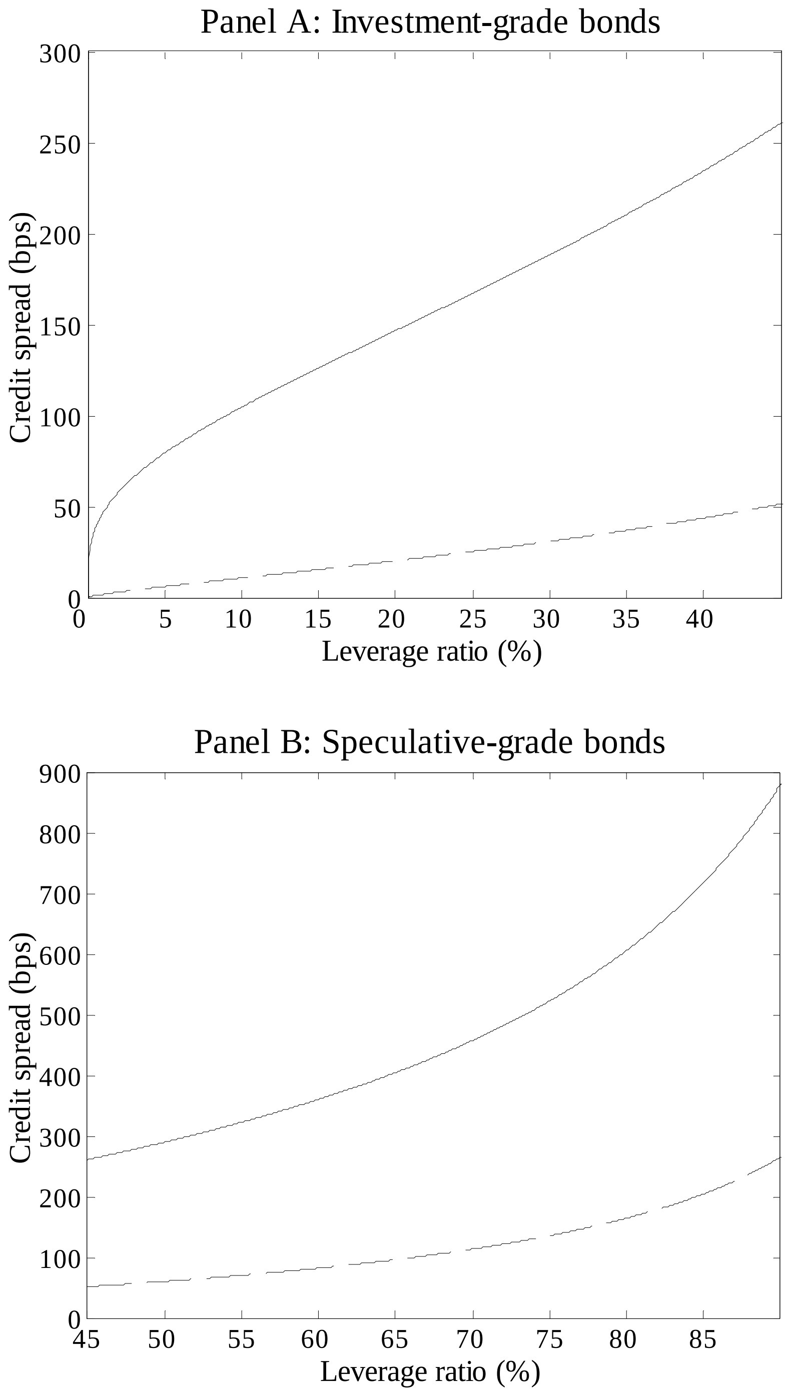


Figure 7. Credit spread as a function of leverage ratio. The lines plot credit spread as a function of leverage for the case where managers display ambiguity aversion (solid line) and rational expectation (dashed line). The values of parameters are chosen at their baseline level.

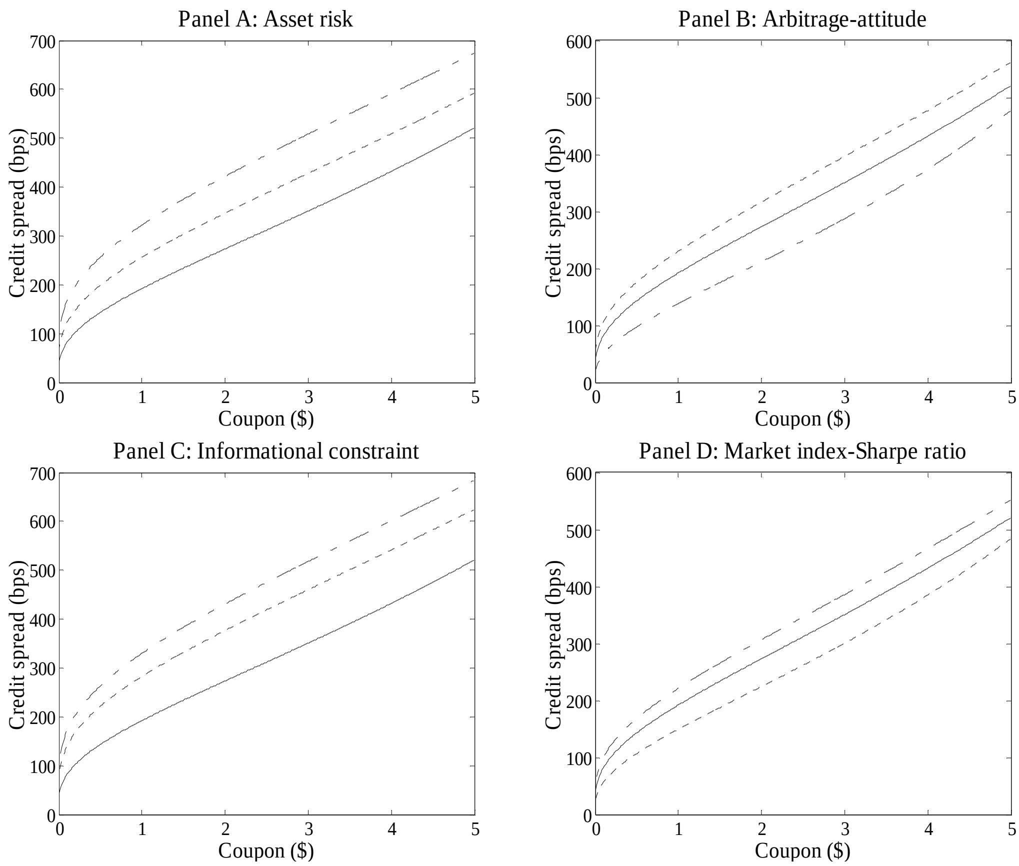
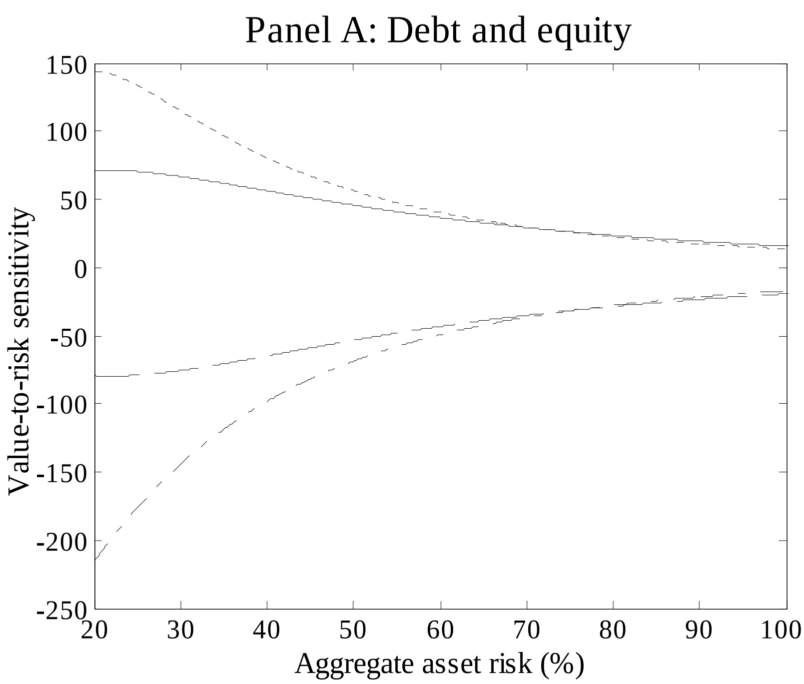


Figure 8. Credit spread curves and model parameters. The lines in each panel plot credit spread as a function of coupon. Panel A plot the spread curve for firm with three different levels of asset risk σ_V: 20% (solid line), 25% (dotted line), and 30% (dash-dotted line). Panel B plots the spread curve for firm facing three different levels of investors' intention to arbitrage η: 1 (dash-dotted line), 2 (solid line), and 3 (dotted line). Panel C plots the spread curve with three different levels of the assets' correlation with market ρ: 0.9 (solid line), 0.7 (dotted line), and 0.5 (dash-dotted line). Panel D plots the spread curve for firm facing three different levels of market index Sharpe ratio h_S: 0.15 (dotted line), 0.33 (solid line), and 0.55 (dash-dotted line). The values of parameters are chosen at their baseline level.



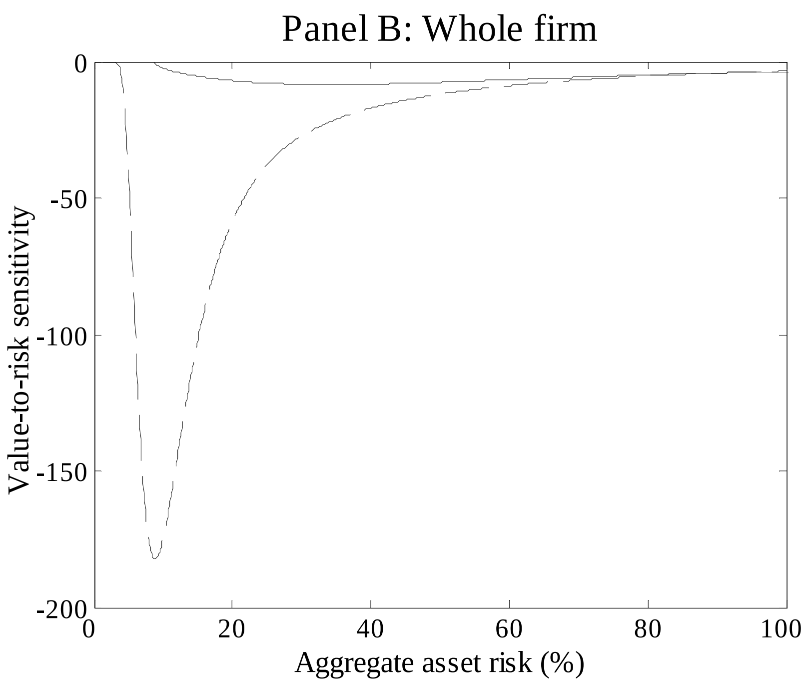


Figure 9. Effects of a change in asset risk on equity, debt, and firm value at optimal leverage. The lines in Panel A plot the partial derivative of equity value (solid and dotted lines) and partial derivative of debt value (dashed and dash-dotted lines) with respect to aggregate asset risk σ_V. The solid and dashed lines are based on the case with ambiguity aversion; while the dotted and dash-dotted lines are based on the standard case without ambiguity. The lines in Panel B plot the partial derivative of firm value with respect to aggregate asset risk where the solid line is for the base case and the dashed line is for the benchmark case. Coupons are chosen to maximize the firm's total levered value under a worst-case belief given 20% aggregate asset risk. The values of parameters are chosen at their baseline level.

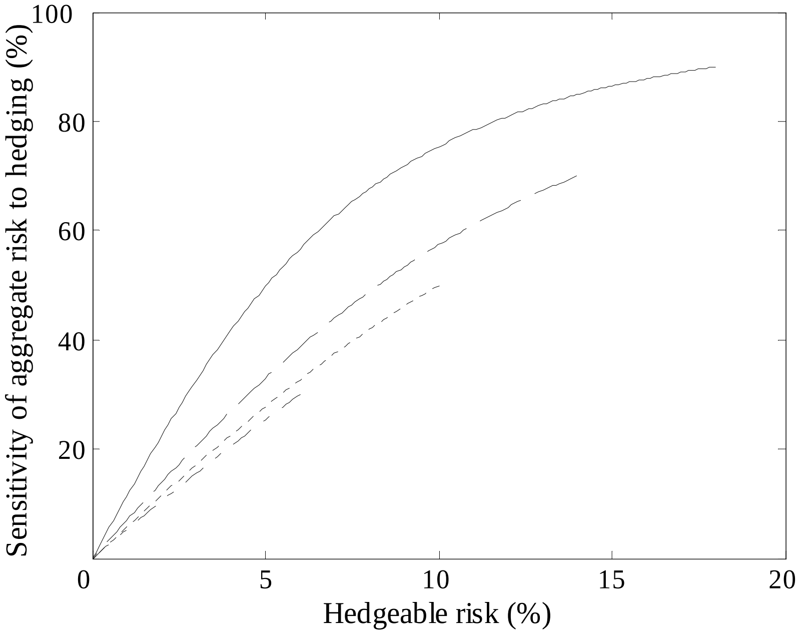


Figure 10. Hedging efficiency with different levels of informational constraints. The lines plot the partial derivative of aggregate asset risk σ_V with respect to firm's hedgeable (systematic) risk σ_{VB} under four different levels of asset's correlation with market ρ: 0.9 (solid line), 0.7 (dashed line), 0.5 (dotted line), and 0.3 (dash-dotted line). The value of non-hedgeable risk σ_{VW} is chosen to match the initial aggregate asset risk at 20%.

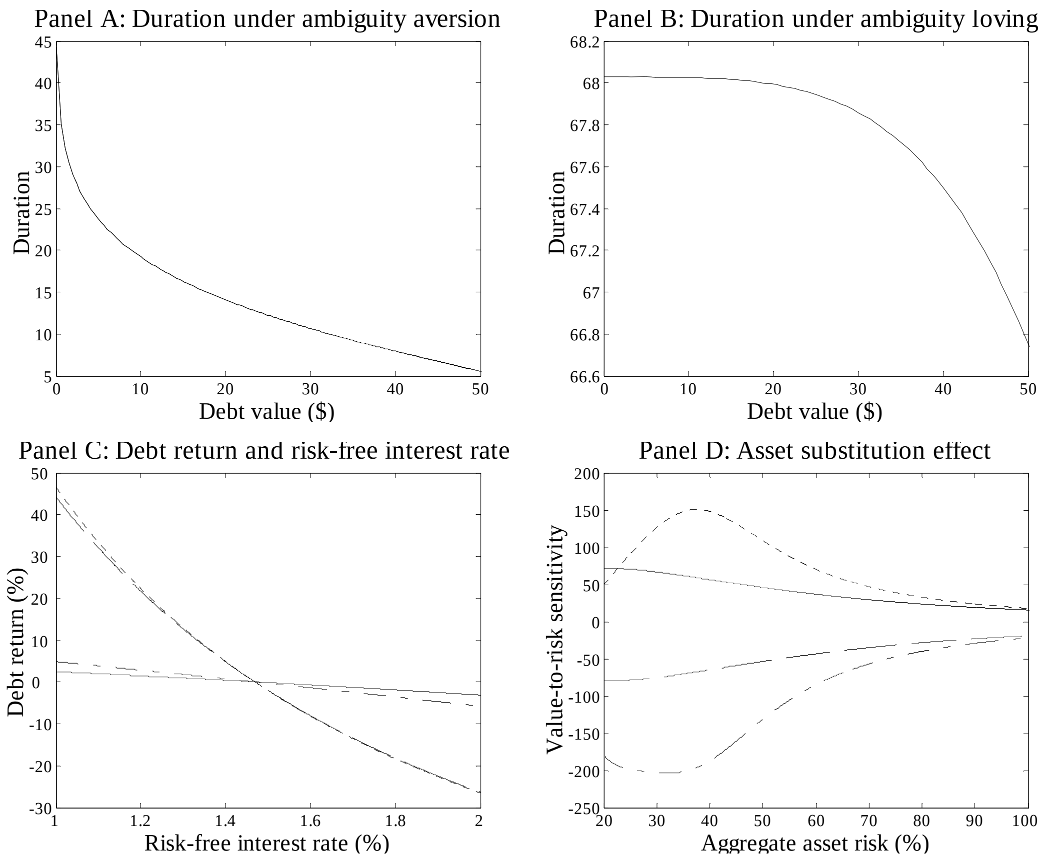


Figure 11. Bond hedging and agency problem under different ambiguity attitudes. The solid line in Panel A and B plots the duration as a function of debt value under ambiguity aversion and ambiguity loving respectively. Panel C plots the relation between the returns on debt value and risk-free interest rate. The solid line is based on a coupon choice to match debt value at $50 under ambiguity aversion; the dashed line is based on a coupon choice to match debt value at $50 under ambiguity loving; the dotted line is based on a coupon choice to match debt value at $30 under ambiguity loving; and the dash-dotted line is based on a coupon choice to match debt value at $30 under ambiguity aversion. Panel D plots the partial derivative of equity value (solid and dotted lines) and debt value (dashed and dash-dot -ted lines) with respect to aggregate asset risk at optimal leverage. The solid and dashed lines are based on the ambiguity-averse case. The dotted and dash-dotted lines are based on the ambiguity-loving case. Coupons are chosen to maximize the firm's value under the worst/best-case (ambiguity-averse/loving) belief given 20% asset risk. The values of parameters are assumed to be chosen at their baseline level.

Table 1: Model Outputs at Optimal Capital Structure

This table presents the comparative statics of model outputs at optimal capital structure with respect to the factors of ambiguity (panels B, C, F, and G) and information constraint (panels D and E). Columns from left to right respectively report the type of models, the ratio of debt to total firm value, the credit spread of debt over risk-free rate, the ratio of expected recovery value to debt par, the debt's face value, the market value of equity, the value of net tax benefits, and 10-year cumulative default probabilities.

Ambiguity-attitude	Leverage ratio	Yield spread	Debt recovery rate	Debt value	Equity value	Net tax benefits	10-yr default rate
Panel A: Base case							
Aversion	48.36%	279 bps	17.19%	\$52.47	\$56.02	8.49%	4.91%
N/A	57.55%	76 bps	21.83%	\$65.27	\$48.14	13.41%	5.56%
Panel B: Conservative arbitrage-attitude $\eta = 3$ (Base: $\eta = 2$)							
Aversion	47.38%	348 bps	16.68%	\$51.18\$	\$56.85	8.04%	7.23%
N/A	57.55%	76 bps	21.83%	\$65.27\$	\$48.14	13.41%	5.56%
Panel C: Ambitious arbitrage-attitude $\eta = 1$ (Base: $\eta = 2$)							
Aversion	50.55%	188 bps	18.31%	\$55.38	\$54.17	9.55%	3.26%
N/A	57.55%	76 bps	21.83%	\$65.27	\$48.14	13.41%	5.56%
Panel D: Low degree of informational constraint $\rho = 0.7$ (Base: $\rho = 0.9$)							
Aversion	46.67%	416 bps	16.32%	\$50.27	\$57.45	7.72%	4.83%
N/A	57.55%	76 bps	21.83%	\$65.27	\$48.14	13.41%	5.56%
Panel E: Moderate degree of informational constraint $\rho = 0.5$ (Base: $\rho = 0.9$)							
Aversion	46.12%	486 bps	16.04%	\$49.58	\$57.91	7.48%	2.50%
N/A	57.55%	76 bps	21.83%	\$65.27	\$48.14	13.41%	5.56%
Panel F: High market-index Sharpe ratio $h_S = 0.55$ (Base: $h_S = 0.33$)							
Aversion	47.58%	332 bps	16.79%	\$51.44	\$56.68	8.13%	6.59%
N/A	57.55%	76 bps	21.83%	\$65.27	\$48.14	13.41%	5.56%
Panel G: Low market-index Sharpe ratio $h_S = 0.15$ (Base: $h_S = 0.33$)							
Aversion	49.98%	207 bps	18.02%	\$54.61	\$54.65	9.26%	3.48%
N/A	57.55%	76 bps	21.83%	\$65.27	\$48.14	13.41%	5.56%

Table 2: Data Definitions

Variable (Data Source)	Variable Definition
Financial Indicators (*Compustat*):	
Book debt	Long term debt + Debt in current liabilities
Book equity	Assets total – Book debt
Leverage	Book debt / (Assets total – Book equity + Market value)
Tangibility	0.715 * Receivables + 0.547 * Inventory + 0.535 *Capital
Effective tax rate	Total tax payments / Net pretax income
Interest payments	Total interest expenses * adj. factor (see footnote 15)
Volatility, Sharpe ratio, Value-Risk Elasticity, and Beta (*3000 Xtra, Reuters/Datastream*):	
Equity volatility	Standard deviation of monthly equity return
Index volatility	Standard deviation of monthly S&P 500 index return
Sharpe ratio of market index	Excess index return / Standard deviation of index return
Market model beta	Market model regression beta on monthly equity return
Equity value-risk elasticity	OLS equity return rate-percentage changes in the firm's risk regression coefficient
Debt value-risk elasticity	OLS debt return rate-percentage changes in the firm's risk regression coefficient
Economic indicator (*FED*):	
Term premium	Difference between 30-year and 5-year government bond yield
Slope of treasury yield curve	Difference between 10-year and 2-year government bond yield

Table 3: Summary Statistics

The table presents the descriptive statistics for main variables used in estimations. The sample of S&P 500 firms is based on Compustat, Datastream and 3000 Xtra of Thomson Reuters, covering the period of January 2, 1992 to December 31, 2011. Last 3-year average and current credit rating data is acquired from Compustat. The detailed definition of other variables is available in Table 2.

Panel A: Descriptive Statistics

Quantitative variable	Mean	S.D.	25%	Median	75%	N
Equity Std. Dev. (%)	31.043	10.558	23.966	29.680	36.387	274
Effective tax rate (%)	33.377	6.381	29.967	33.783	37.589	274
Tangibility-asset ratio (%)	33.378	9.880	26.687	34.394	41.095	274
Market-model beta	1.011	0.442	0.669	0.963	1.312	274
Market leverage (%)	17.640	11.941	8.735	15.068	24.093	274
Total interest expenses-book debt ratio (%)	8.241	3.120	6.439	7.430	9.242	274
Val.-risk elasticity (equity)	-0.220	0.185	-0.323	-0.211	-0.124	274
Val.-risk elasticity (debt)	-0.029	0.037	-0.042	-0.022	-0.005	133

Panel B: Frequency (numbers of firm) distribution

Qualitative variable	AAA	AA	A	BBB	BB	B/CCC	N.A.
Last 3-yr avg. credit rating	4	14	87	104	35	6	24
Current credit rating	3	12	83	119	32	7	18

Table 4: Parameter Estimates and Target Financial Variables

The table compiles the estimates of firm-specific parameters, non-firm-specific parameters, and target financial variables in Panel A, B, and C respectively. Asset aggregate risk is estimated by the monthly volatility of equity return. Effective tax rate is estimated by using the ratio of tax payment to net pretax income. Bankruptcy cost is estimated by the formula of assets' abandonment value proposed by Berger et al. (1996). Systematic risk proportion is estimated using the ratio of the product of market beta and market risk to firm's total risk. Sharpe ratio of the market portfolio is estimated by the ratio of average excess return on S&P 500 index to its standard deviation. Riskless rate is estimated by 1-year treasury rate. Term premium on bonds is measured as the difference between 30-year and 10-year government bond yield, as in Morellec et al. (2011). The slope of treasury yield curve is estimated by the difference between 10-year and 2-year government bond yields, as in Collin-Dufresne et al. (2001). The detailed definition of target financial variables is available in Table 2.

Panel A: Firm-specific parameters by leverage group

Items	Overall	Leverage group				
		< 8%	8-16%	16-24%	24-32%	>32%
Asset aggregate risk (%)	31.043	31.087	30.090	32.937	30.455	30.624
Effective tax rate (%)	33.377	31.907	33.708	32.606	35.566	34.394
Bankruptcy cost rate (%)	57.401	51.042	57.869	58.805	61.965	60.778
Systematic risk proportion (%)	44.375	42.268	43.169	45.225	46.770	47.232

Panel B: Non-firm-specific parameters

Market-portfolio Sharpe ratio	One-year treasury rate (%)	Term premium (bps)	The slope of treasury yield curve (bps)
0.187	3.524	116	118

Panel C: Target variables

Leverage (%)	Yield spread (bps)	Debt value-risk elasticity	Equity value-risk elasticity
17.640 (mean)	262 (mean)	-0.029(mean)	-0.220 (mean)
11.941 (S.D.)	123 (S.D.)	0.037(S.D.)	0.185 (S.D.)

Table 5: The Magnitude of Ambiguity Aversion Effect on Model Outputs

This table presents the magnitude of ambiguity aversion effect on model outputs by leverage group. In each panel the outputs of my modified model/standard model are reported by the top/middle row, while changes in the model outputs due to ambiguity aversion are compiled in the bottom row. The residual leverage ratio is calculated as the difference between the observed leverage and the leverage implied by model. In Panels B, C, and G, numbers are based on the leverage calibrated to match its observed level. In Panels D-F, numbers are based on the leverage chosen to maximize the firm's total value.

Model	Leverage Group					
type	Overall	< 8%	8-16%	16-24%	24-32%	>32%
Panel A: Residual leverage ratio						
Modified	31.179%	44.149%	37.292%	26.586%	23.911%	8.771%
Standard	40.400%	53.431%	47.126%	35.528%	32.638%	17.364%
Magnitude	-22.824%	-17.372%	-20.867%	-25.169%	-26.739%	-49.487%
Panel B: Yield spread (with calibrated leverage)						
Modified	262bps	146bps	200bps	305bps	334bps	473bps
Standard	77bps	33bps	50bps	95bps	101bps	165bps
Magnitude	185bps	113bps	150bps	210bps	233bps	308bps
Panel C: 20-year cumulative default probability (with calibrated leverage)						
Modified	31.876%	9.284%	18.977%	40.856%	47.607%	71.387%
Standard	11.726%	3.126%	6.682%	14.010%	17.022%	29.713%
Magnitude	20.150%	6.158%	12.295%	26.846%	30.585%	41.674%
Panel D: Equity's value-risk elasticity (with endogenous leverage)						
Modified	0.472	0.492	0.475	0.450	0.469	0.468
Standard	0.539	0.559	0.530	0.537	0.538	0.531
Magnitude	-0.067	-0.067	-0.055	-0.087	-0.069	-0.063
Panel E: Debt's value-risk elasticity (with endogenous leverage)						
Modified	-0.570	-0.572	-0.574	-0.580	-0.557	-0.557
Standard	-0.595	-0.594	-0.585	-0.621	-0.585	-0.583
Magnitude	-0.025	-0.022	-0.011	-0.041	-0.028	-0.026
Panel F: Net tax benefits scaled by initial firm value (with endogenous leverage)						
Modified	10.571%	9.877%	10.369%	9.252%	13.182%	12.004%
Standard	16.088%	15.273%	16.307%	14.173%	18.721%	17.704%
Magnitude	-5.517%	-5.396%	-5.938%	-4.921%	-5.539%	-5.700%
Panel G: Default boundary scaled by initial firm value (with calibrated leverage)						
Modified	5.185%	1.111%	2.760%	5.098%	8.144%	15.286%
Standard	6.574%	1.681%	3.894%	6.676%	10.042%	17.910%
Magnitude	-1.389%	-0.570%	-1.134%	-1.578%	-1.898%	-2.624%

Table 6: Goodness-of-Fit for Leverage

The table presents the comparison of goodness-of-fit for leverage in two ways: moments (Panel A) and prediction errors (Panel B). Columns in the top panel reports the value of various statistical moments in the data and models, including mean, standard deviation, median, skew, and kurtosis (from left to right). In the bottom panel, MAE reports the mean of absolute error; RMSE reports the root of mean squared error; and Range shows the distance between max and min error. The *t*-statistic tests whether the errors of the chosen model are lower than its alternative model. The associated *p*-value gives the testing result.

Panel A: Moments

Items	Mean	S.D.	Median	Skew	Kurtosis
Full sample:					
Empirical data	0.176	0.119	0.151	1.180	4.563
Modified model	0.488	0.099	0.499	-0.594	8.071
Standard model	0.581	0.101	0.587	-1.015	8.430
Low-levered firms (< 17%)*:*					
Empirical data	0.095	0.042	0.095	0.007	2.005
Modified model	0.490	0.077	0.502	-0.441	3.034
Standard model	0.584	0.083	0.589	-0.501	3.584
Highly-levered firms (> 17%)*:*					
Empirical data	0.287	0.100	0.258	1.373	5.259
Modified model	0.485	0.123	0.498	-0.542	7.360
Standard model	0.575	0.123	0.583	-1.085	8.222

Panel B: Prediction errors

Items	MAE	RMSE	Range	*t*-statistic	*p*-value
Full sample:					
Modified model	0.322	0.347	1.037	-35.998	0.999
Standard model	0.410	0.433	1.008	35.998	0
Low-levered firms (< 17%)*:*					
Modified model	0.395	0.405	0.489	-27.090	0.999
Standard model	0.489	0.498	0.547	27.090	0
Highly-levered firms (> 17%)*:*					
Modified model	0.222	0.247	1.037	-23.618	0.999
Standard model	0.303	0.325	0.961	23.618	0

Table 7: Goodness-of-Fit for Yield Spread

The table gives the comparison of goodness-of-fit for yield spread in two ways: moments (Panel A) and prediction errors (Panel B). Columns in the top panel reports the value of various statistical moments in the data and models, including mean, standard deviation, median, skew, and kurtosis (from left to right). In the bottom panel, MAE reports the mean of absolute error; RMSE reports the root of mean squared error; and Range shows the distance between max and min error. The *t*-statistic tests whether the errors of the chosen model are lower than its alternative model. The associated *p*-value gives the testing result.

Panel A: Moments

Items	Mean	S.D.	Median	Skew	Kurtosis
Full sample:					
Empirical data	0.0262	0.0123	0.0231	2.5584	12.3052
Modified model	0.0262	0.0197	0.0226	1.9161	8.5451
Standard model	0.0077	0.0116	0.0044	4.1622	28.4176
Low-levered firms (< 17%)*:*					
Empirical data	0.0226	0.0079	0.0206	2.5900	13.3510
Modified model	0.0195	0.0154	0.0163	2.8064	16.1584
Standard model	0.0055	0.0108	0.0026	6.2918	55.2550
Highly-levered firms (> 17%)*:*					
Empirical data	0.0311	0.0152	0.0251	2.0043	8.3649
Modified model	0.0354	0.0214	0.0316	1.5118	6.5682
Standard model	0.0108	0.0118	0.0076	2.4770	10.1205

Panel B: Prediction errors

Items	MAE	RMSE	Range	*t*-statistic	*p*-value
Full sample:					
Modified model	0.0123	0.0169	0.1548	-3.0574	0.999
Standard model	0.0193	0.0220	0.1490	3.0574	0.001
Low-levered firms (< 17%)*:*					
Modified model	0.0105	0.0141	0.1317	-2.1501	0.984
Standard model	0.0183	0.0201	0.1314	2.1501	0.016
Highly-levered firms (> 17%)*:*					
Modified model	0.0146	0.0201	0.1290	-2.5656	0.995
Standard model	0.0206	0.0243	0.0747	2.5656	0.005

Table 8: Goodness-of-Fit for Value-Risk Elasticity

The table gives the comparison of prediction-error between my modified model and standard model for value-risk elasticity. MAE reports the mean of absolute error. RMSE reports the root of mean squared error. The *t*-statistic tests check whether the prediction errors of the chosen model are smaller than the alternative model. The associated *p*-value displays the test result. In calculating MAE and RMSE on the debt's value-to-risk elasticity, the target elasticity for firms without analyzable bond data is set at -0.029, which equals the mean of bond sample.

Items	MAE (debt)	RMSE (debt)	t-statistic (debt)	*p*-value (debt)	MAE (equity)	RMSE (equity)	t-statistic (equity)	*p*-value (equity)
Full sample:								
Modified	0.542	0.545	-3.061	0.999	0.693	0.728	-9.153	0.999
Standard	0.566	0.586	3.061	0.001	0.760	0.814	9.153	0
Low-levered firms (< 17%)*:*								
Modified	0.548	0.550	-2.221	0.987	0.693	0.723	-6.417	0.999
Standard	0.572	0.592	2.221	0.013	0.758	0.810	6.417	0
Highly-levered firms (> 17%)*:*								
Modified	0.533	0.539	-2.132	0.983	0.694	0.735	-6.711	0.999
Standard	0.556	0.579	2.132	0.017	0.763	0.819	6.711	0