Asset Pricing Theory with an Imprecise Information Set*

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Abstract

This paper provides a novel theoretical platform for the pricing of imprecise accounting information as a systematic market risk. Our intertemporal asset pricing model shows that systematic information-quality risk is priced through a distinct market risk premium and three extra betas associated with an imprecise-information risk. Our first information-quality beta is related to the covariance between market-wide imprecise-information return error and security precise return. Together with the separate market information-quality risk premium, this beta provides the theoretical underpinning for a separate market information-quality factor in the spirit of empirical multiple-factor model prevalent in the literature. The second extra beta (linked to the covariance between firm and market-wide imprecise-information return errors), represents the commonality in information quality, which is priced by investors seeking to curtail adverse effects of imprecise accounting information on their portfolio value. Our third information-quality beta (related to the covariance between stock imprecise-information return error and overall market return), implies that – for hedging purposes – investors prefer to invest in stocks issued by firms that tend to, erroneously or deliberately, release false positive information about the firm when the market is bearish.
1. Introduction

Traditional asset-pricing models are based on the assumption that the financial market is informationally efficient and that individuals are well informed (see for example, Sharpe, 1964; Lintner 1965; Mossin, 1966; and Merton, 1973). However, there is substantial evidence indicating that information releases are noisy (see for example, Faust, Rogers, and Wright, 2000; Shapiro and Wicox, 1999; and Wang 1993). With an imprecise information set, investors may face information-quality (information-imprecision) risk. Ignoring this risk may lead to asset mispricing in traditional asset-pricing models. This paper provides a theoretical platform for the pricing of imprecise accounting information as a systematic market risk factor.

There is a growing body of literature that examines the pricing of different manifestations of an imperfect information set, both theoretically and empirically. For example, Easley and O’Hara (2004) derive a rational-expectations model under which asset returns are affected by the information asymmetry between privately informed and publically informed (uninformed) investors. This model implies that, while privately informed investors adjust their portfolios based on the arrival of private information, uninformed investors hold *ex-ante* underperforming portfolios. Therefore, uninformed investors face systematic asymmetric information risk for which they demand a risk premium. In addition, this model also implies that investors demand higher asset returns for facing imprecise information. These theoretical predictions suggest that higher accounting-information precision reduces the asset risk for uninformed investors, and thus results in a lower cost of capital. Consequently, numerous studies empirically test the relation between accounting-information precision and the cost of equity capital and/or the cost of debt capital.¹

Francis, LaFont, Olsson, and Schipper (2004) and Botosan, Plumlee, and Xie (2004) relate the cost of equity of a firm to the firm’s information quality. Consistent with the prediction of Easley and O’Hara (2004) that information precision is non-diversifiable, Francis et al. (2005) show that poorer market-wide accrual-quality factor as

¹ See for example, Francis LaFont, Olsson, and Schipper (2004, 2005), Botosan, Plumlee, and Xie (2004), Aboody, Hughes and Liu (2005), Liu and Wysocki (2006), Core et al. (2008), Ogneva (2008), and Kravet and Shevlin (2010) for studies focusing on the cost of equity capital. As for studies looking at the cost of debt capital, see Francis et al. (2005) Anderson et al. (2004), Graham et al. (2008), and Bhojraj and Swaminathan (2007). For a survey of this line of literature see Dechow, Ge, and Schrand (2010).
a measure of an information-quality factor is associated with a larger cost of equity capital. In other words, firms are exposed to information precision risk with a significantly positive factor loading with respect to their market-wide information-quality factor, after controlling for the Fama and French (1993) three factors.

A follow up study by Core, Guaya, and Verdi (2008) confirms that the time-series regressions of stock returns on cotemporaneous factor returns used by Francis et al. (2005) yield an average positive factor loading with respect to the market information-quality factor. However, after running a cross-sectional regression of stock returns on the estimated (time-series) information-quality factor loadings (while controlling for the Fama and French three factor loadings), they show that investors do not demand a return premium for this positive exposure to information imprecision. Stated differently, information quality is not a priced market factor as implied by Easley and O’Hara (2004) model.

More recent research challenges the conclusion of Core et al. (2008), and demonstrates that information precision is priced under certain market conditions. Kravet and Shevlin (2010) use the Fama and French (1993) three-factor model, augmented by two information precision factors: a market innate accrual quality factor and market discretionary component of accrual quality factor. Their findings indicate that during a short period following accounting restatement, higher discretionary precision factor loadings yield a higher cost of equity capital at the cross section of restatement firms. I.e., information precision risk is priced for restatement firms following the restatement announcement. Moreover, they show that the discretionary component of information risk is also priced at the cross section of firms in the same industries as the restatement firms.

Ogneva (2008) claims that Core et al.’s (2008) rejection of information precision as a priced market factor comes about because lower information-quality firms suffer from negative future cash flow shocks which depress future returns. Overall, this may offset the higher returns one would expect for low information-quality firms.2 Her

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2 When controlling for cash flow shocks in the Core et al.’s (2008) regression model, Ogneva (2008) finds that stock returns become significantly and positively related to the information-quality factor loadings at the cross section. She introduces an additional test, which replaces the standard accrual-quality measure of Dechow and Dichev (2002) by a measure of accrual quality, scaled by the average of absolute accruals
findings indicate that, after properly controlling for the impact of accrual quality on future cash-flow shocks, investors demand a return premium for a positive exposure (factor loading) with respect to information imprecision. I.e., the information-quality risk factor is priced.

While there is some empirical evidence that an information-precision factor is systematic and priced, there is still lack of theoretical underpinning for its inclusion in an asset-pricing model as a separate market factor. To the best of our knowledge, the only two papers incorporating accounting-information precision in an asset-pricing model are those of Lambert, Leuz, and Verrecchia (2007) and Lambert, Leuz, and Verrecchia (2012). The former paper is a pioneer theoretical study on the pricing of imprecise information, and the latter presents the first model to show that imprecise accounting information alters systematic risk and affects asset prices at the cross section. Note that, neither paper provides a theoretical justification for the pricing of an information-quality factor separate from the Capital Asset Pricing Model (CAPM) market factor. Our paper complements this literature by providing a theoretical foundation for the pricing of imprecise accounting information as a systematic market factor.

Of these two papers, our paper is most closely related to that of Lambert et al. (2007) who derive a one-period CAPM-based model to study the effects of imprecise accounting information on the cost of capital. In their model, lower precision of accounting information about the firm’s future cash flow increases its conditional covariance with market cash flows (or the firm’s conditional cash-flow beta) and consequently increases the cost of equity. Therefore, the cross-sectional pricing of information risk in the Lambert et al. (2007) model is manifested through the imprecise-information induced measurement error in the estimation of the firm’s cash-flow beta. 3

3 In somewhat related finance literature on estimation risk, the source of information about a firm comes from its historical time-series of returns (see for example, Barry and Brown, 1985; and Coles et al., 1995). Lambert et al. (2007) note that the reliance in this literature on the time-series of returns as the source of information affects a significant portion of the covariance structure. Different from their model, in this stream of research “new information is correlated conditionally with contemporaneous observations and conditionally independent of all other information” (p. 398). Lambert et al.’s (2007) model, as well as our model, presents a structure for the information set, which allows different covariance structures.
Lambert et al. note that this result has strong implications for empirical asset-pricing studies incorporating imprecise information. However, their model does not provide the theoretical underpinning for an extra separate “information risk” factor in an asset-pricing model. For this reason, they suggest that empirical research based on their theory should be directed at the relation between information quality and their market beta.

The second paper addressing asset pricing with imprecise accounting information is the model of Lambert et al. (2012). They derive a noisy rational-expectations model and study the relation between information asymmetry and the cost of equity capital. They show that with perfect competition, information asymmetry is not directly priced but the firm’s cost of equity capital is still affected by the average precision of investors’ information. However, similar to the result of Lambert et al. (2007), Lambert et al. (2012) conclude that the pricing effect of average precision does not justify a separate information-related risk factor in a pricing model.

Our paper is first to provide a theoretical model that is consistent with empirically modelling both the standard CAPM market premium and market-wide information quality as separate priced risk factors in the context of a multiple-regression model. Different from the model of Lambert et al. (2007), our model distinguishes between the standard CAPM (precise) systematic risk (beta) and systematic risk associated with imprecise accounting information. Furthermore, we decompose the firm’s total systematic risk into the standard CAPM beta and three additional betas associated with imprecise-information risk.

A related theoretical study by Hughes, Liu, and Liu (2007) examines the impact of asymmetric information on asset pricing. The source of information friction in their study comes exclusively from asymmetric information, rather than information imprecision. They use an Arbitrage Pricing Theory (APT) framework to derive their model. Their model implies that, at the limit, information asymmetry impacts factor risk premiums, not factor sensitivities. This means that asymmetric-information risk is not priced at the cross section.

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5 We further discuss Hughes et al. in section 2.1 and 2.4.
In the spirit of Merton (1973), we derive an intertemporal asset-pricing model that examines the pricing of risk associated with imprecise accounting information. We model the impact of imprecise information on asset returns with an Ornstein-Uhlenbeck mean-reverting process under which the information-related return error fluctuates around its long-term mean, to account for reversals in these errors. In the static version of our imprecise-information-adjusted asset-pricing model, information-quality risk has systematic and idiosyncratic components, and only the former is priced. Our model further demonstrates that systematic information-quality risk is priced through three extra asset betas as well as through the alteration of the market risk premium. With an imprecise information set, these three distinct systematic risk effects are measured by covariance (beta) terms between firm-specific and market-wide imprecise-information measures and the precise (fundamental) returns on the asset and the market.

The first component of systematic imprecise-information risk is a function of the covariance of the security’s precise return with the information-imprecision return error on the market portfolio. At times when the overall market information-imprecision return error is negative, investors prefer to hold securities that pay a higher precise return. Therefore, investors demand a premium for this covariance. This covariance provides the theoretical underpinning for empirical models testing the cross sectional pricing of the factor loading of an imprecise-information factor (see for example, Francis et al., 2005; Core et al., 2008; Ogneva, 2008; and Kravet and Shevlin, 2010). In this line of empirical literature the imprecise-information factor loading is also related to the covariance of the security return with a market information factor, normally measured by the return on a long-short mimicking portfolio. Thus, we provide further theoretical support for a separate information-quality factor by showing that in equilibrium this market factor exists and the extra risk premium required is distinct from the CAPM market risk premium.

In addition to the above manifestation of systematic risk that has been empirically researched in recent years, our model introduces two novel aspects of theoretically priced systematic imprecise-information risk. First, we have the covariance of the security’s imprecise-information return error with the information-imprecision return error on the market portfolio. We call the beta related to this covariance the commonality in
information quality beta. Assets with a negative commonality covariance provide a hedge against the risk of a negative return due to information imprecision on the market portfolio, and therefore have lower expected returns. Thus, investors expect a higher return premium for a security with a positive commonality in information quality beta.

The second novel form of priced systematic imprecise-information risk in our model is the covariance between the security information-imprecision return error and the market precise return. Here too, investors prefer a negative covariance as it corresponds to higher security information-imprecision return error during a bear market. The implication of this preference is that investors may choose to invest in stocks of firms that erroneously, or even intentionally, release false positive information about the firm at times of a down market. In equilibrium, investors demand a risk premium for stocks that do not provide this hedge due to this covariance term being positive.

The remainder of this paper is organized as follows: in Section 2 we derive an imprecise-information-adjusted intertemporal asset-pricing model to obtain an analytical asset-pricing framework in the presence of an imprecise-information set. Section 3 discusses the theoretical and empirical implications of imprecise accounting information for asset pricing, while discussing the different channels through which systematic information-imprecision risk affects asset pricing. Section 4 provides summary and conclusions.

2. An Intertemporal Asset-Pricing Model with Imprecise Accounting Information

In the current section we formulate a theoretical asset-pricing model with information-quality risk. In particular, we revisit Merton’s (1973) intertemporal CAPM by incorporating an imprecise information structure and explore the various channels through which information risk may affect expected asset returns.

We maintain Merton’s (1973) assumptions of continuous trading, and that the returns and the changes in the opportunity set (the transition probabilities for returns on each asset over the next trading interval) are well explained by continuous-time stochastic processes. The vector set of stochastic processes describing the investment opportunity
set and its changes follow a time-homogeneous Markov process. In the subsection below, we define the imprecise return structure in our model, based on imprecise cash-flow information available to investors. Based on this imprecise return structure, we make two additional assumptions modifying Merton’s framework to allow for imperfect information quality.

2.1. The Imprecise Information Set

Merton (1973) describes the equity expected return over a period of length $s$ as:

$$\frac{E[CFPS(t,t+s)] + UCPS(t + s) - P(t)}{P(t)},$$

where $CFPS(t,t+s)$ is cash flow per share generated by a firm between time $t$ and time $t + s$, $UCPS(t + s)$ is the balance of undepreciated capital per share (calculated under physical capital depreciation) held by the firm at time $t + s$, $P(t)$ is the beginning-of-period stock price per share.

Accounting information released by the firm is the principal source of information for estimating the cash flow component of the expected return, and the above definition of expected equity return relies on precise accounting information. Since accounting information is noisy by nature, expected cash-flow estimates, and therefore expected returns, are imprecise. Formally, we follow Lambert et al.’s (2007) definition of the noisy measure of imprecise cash flow as the sum of the firm’s precise cash flow per share, $CFPS(t,t+s)$, and an error term that represents the random imprecise cash flow component per share misestimated for the period between time $t$ and time $t + s$, $\varepsilon(t,t+s)$. This implies the following imprecise expected return on equity over a period $s$:

$$\frac{E[CFPS(t,t+s)] + E[\varepsilon(t,t+s)] + UCPS(t + s) - P(t)}{P(t)},$$

Merton assumes that all assets have limited liability, and there are no transactions costs, taxes, or problems with indivisibilities of assets. There are a sufficient number of investors with comparable wealth levels so that each investor can buy and sell unlimited amounts at the market prices, and there exists an exchange market for borrowing and lending at the same rate of interest. Investors have homogeneous expectations with respect to asset returns. Short-sales of all assets, with full use of the proceeds are allowed. Finally, it is assumed that trading in assets takes place continually in time. For specific details see Merton (1973).
Given Merton’s definition of the precise (fundamental) expected return, the imprecise expected asset return is the sum of the precise expected return and the expected return due to cash flow imprecision:

\[
\frac{E[CFPS(t,t+s)] + UCPS(t+s) - P(t)}{P(t)} + \frac{E[\epsilon(t,t+s)]}{P(t)},
\]

We denote the \textit{ex-ante} return due to cash flow imprecision for the period between time \( t \) and time \( t + s \) with: \( \psi(t,t+s) \), such that: \( \psi(t,t+s) \equiv \frac{\epsilon(t,t+s)}{P(t)}. \) In the continuous time framework, for an infinitesimally small time interval \( s \), we denote the information-imprecision return error on asset \( i \) by \( \psi_i \) (for convenience, we drop the time subscript). The inclusion of imprecise accounting information about cash flow leads to the two additional assumptions below, which modify Merton’s (precise-information) framework.

\textbf{Assumption 1:} The information-imprecision return error, \( \psi_i \), follows an Ornstein-Uhlenbeck process as follows: \( d\psi_i = \kappa(\mu_{\psi_i} - \psi_i)dt + \sigma_{\psi_i}dz_i \), for every asset \( i \) (\( i = 1, 2, \ldots n \)), where \( \kappa, \mu_{\psi_i}, \sigma_{\psi_i} \) are constants, \( z_i \) is standard Wiener process, and \( E[dz_idz_j] = \rho_{\psi_i,\psi_j}dt \), for every asset \( i \) (\( i = 1, 2, \ldots n \)) and asset \( j \) (\( j = 1, 2, \ldots n \)).

The drift term \( \kappa(\mu_{\psi_i} - \psi_i) \) gives the expected change in the information-related return error. With a positive speed of mean reversion (\( \kappa > 0 \)), the level of information-related return error, \( \psi_i \), fluctuates around a long-term steady-state mean, \( \mu_{\psi_i}, \) which is constant for security \( i \). The parameter \( \sigma_{\psi_i} \) measures the magnitude of the innovation in \( \psi_i \). We assume that the information-related return error follows a mean-reverting process in order to account for reversals in these imprecise-information induced errors.
**Assumption 2:** Market participants observe the stochastic instantaneous *imprecise* return, \( \tilde{r}_i = r_i + \psi_i \), where \( r_i \) is the precise return on asset \( i \). This precise return follows a Gaussian process: \( dr_i = \mu_r \, dt + \sigma_r \, d\omega_i \), for every asset \( i \) \((i = 1, 2, \ldots n)\).

Applying Itô’s lemma we write the mean of the instantaneous imprecise return as \( \mu_{\tilde{r}_i} = \mu_r + \mu_{\psi_i} \), which is the sum of the mean instantaneous precise return and the long-term mean of the return error. The instantaneous imprecise return variance is given by: \( \sigma_{\tilde{r}_i}^2 = \sigma_r^2 + \sigma_{\psi_i}^2 + 2\sigma_{r,\psi_i} \), where \( \sigma_{r,\psi_i} \) is the instantaneous covariance between the precise return on asset \( i \) and the return due to information imprecision related to asset \( i \): \( \sigma_{r,\psi_i} = \rho_{r,\psi_i} \sigma_r \sigma_{\psi_i} \). The term \( \rho_{r,\psi_i} \) denotes the instantaneous correlation between the precise return on stock \( i \) and the information imprecision return error for stock \( i \).

We further denote the instantaneous correlation coefficient between \( d\omega_i \) and \( d\omega_j \) (for two different assets \( i \) and \( j \)) with \( \rho_{\omega_i,\omega_j} \), and the instantaneous correlation coefficient between \( d\omega_i \) and \( dz_j \) with \( \rho_{\omega_i,\psi_j} \). That is, \( E(d\omega_i d\omega_j) = \rho_{\omega_i,\omega_j} \, dt \) and \( E(d\omega_i dz_j) = \rho_{\omega_i,\psi_j} \, dt \). The instantaneous covariance between the imprecise returns on any two assets \( i \) and \( j \) is given by: \( \sigma_{\tilde{r}_i,\tilde{r}_j} = \text{Cov}(r_i + \psi_i, r_j + \psi_j) = \sigma_{r,r_j} + \sigma_{\psi_i,\psi_j} + \sigma_{r,\psi_j} + \sigma_{\psi_i,\psi_j} \), where \( \sigma_{r,r_j} \) is the instantaneous covariance between the precise returns on the two assets.

The above assumptions imply the following Itô processes for the instantaneous imprecise return on the asset \( i \) (\( \tilde{r}_i \)): \[
\begin{align*}
\, d\tilde{r}_i &= (\mu_r + \kappa(\mu_{\psi_i} - \psi_i))dt + \sqrt{\sigma_r^2 + \sigma_{\psi_i}^2 + 2\sigma_{r,\psi_i}} \, d\omega_i, 
\end{align*}
\]
where $\sigma_i$ is a standard Brownian Motion. 7 Equation (1) implies that 
\[
\{ (\mu_i + \kappa(\mu_{\psi_i} - \psi_i)), \sqrt{\sigma_{\psi_i}^2 + 2\sigma_{\psi_i,\psi_i}, \rho_{\psi_i}} \}
\]
is a sufficient set of statistics for the imprecise opportunity set at any given point in time.

Note that in the current setup all investors face the same imprecise information set. This is similar to the assumption of Lambert et al. (2007) that investors hold homogeneous beliefs about future cash flows (and imprecise return moments in our model), but different from the setup of Lambert et al. (2012) that allows for asymmetric information. In addition, similar to Lambert et al. (2007), our setup facilitates studying the pricing effect of firm-specific information imprecision. In Hughes et al. (2007) the information structure is based on the firm’s cash-flow component that stems from a factor common to all firms. Different from Lambert et al. (2007) and from the current paper, the idiosyncratic cash-flow portion is cross-sectionally independent in Hughes et al. (2007). Lambert et al. (2007) note that this difference in the information structure leads to different results related to the cross-sectional effects on the expected return on equity. The same difference applies to our model.

2.2. The Investor’s Problem

Following Merton (1973), we assume that there are $L$ investors who maximize their expected lifetime utility of wealth given an imprecise information set:
\[
J[W(t),\psi(t),t] = \max E_0 \left[ \int_0^{T_l} U^l[c^l(s),s]ds + B^l[W^l(T^l),\psi(T^l),T^l] \right], \quad l = 1,2,...,L,
\]
where “$E_0$” is the expectation operator, conditional on the current value of the $l^{th}$ investor’s wealth and the imprecise information set. $U^l$ is the von Neumann-Morgenstern utility function of consumption for the $l^{th}$ investor, which is strictly concave.

The initial value of investor $l$’s wealth is given by $W^l(0) = W^l$. $T^l$ is the $l^{th}$ investor’s

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7 Application of Itô’s Lemma implies that: 
\[
d\sigma_i = \sigma_{\psi_i} d\omega_i + \sigma_{\psi_i} dz_i \quad \text{and} \quad E(d\sigma_i dz_i) = \frac{\sigma_{\psi_i} \rho_{\psi_i,\psi_i} + \sigma_{\psi_i}^2}{\sqrt{\sigma_{\psi_i}^2 + 2\sigma_{\psi_i,\psi_i}}} dt.
\]
horizon and \( c'(t) \) is the instantaneous consumption flow at time \( t \). Finally, \( B^t \) denotes a strictly concave utility function of terminal wealth. The terminal value of lifetime utility in equation (2) is given by: 

\[
J[W(T),\psi(T),T] = B(W(T),\psi(T),T).
\]

With \( n \) risky assets and one instantaneously riskless asset, and with imprecise accounting information incorporated in equation (1), the wealth accumulation equation for the \( l \)th investor is given by:

\[
dW = \sum_{i=1}^{n} q_i W d\tilde{r}_i + (1 - \sum_{i=1}^{n} q_i) W r_j - c dt ,
\]

where \( q_i \) is the proportion of the investor’s wealth invested in the \( i \)th asset. Following Assumptions 1 and 2, the imprecise-information wealth-accumulation process is given by (see derivation in Appendix A):

\[
dW = \left[ \sum_{i=1}^{n} q_i (\mu_{r_i} + \kappa (\mu_{w_i} - \psi_j) - r_j) + r_j \right] W dt + \sum_{i=1}^{n} q_i W \sqrt{\sigma_{r_i}^2 + \sigma_{w_i}^2 + 2\sigma_{r_i,w_i} d\sigma_i} - c dt ,
\]

where \( r_j \) is an exogenous instantaneous interest rate on a risk-free bond, and \( \sum_{i=1}^{n+1} q_i = 1 \), where \( q_{n+1} \) is the weight of the riskless asset. Using the above assumptions and wealth-accumulation process, we solve for an investor’s consumption-investment optimal choice which results in the following Hamilton-Jacobi-Bellman (HJB) equation:

\[
0 = \max[U(c,t) + J_r + J_w [(\sum_{i=1}^{n} q_i (\mu_{r_i} + \mu_{w_i} - r_j) + r_j) W ]

+ \frac{1}{2} J_w \sum_{i=1}^{n} q_i W \left( \sigma_{r_i}^2 + \sigma_{w_i}^2 + 2\sigma_{r_i,w_i} W^2 \right)

+ \frac{1}{2} \sum_{i=1}^{n} J_w \sum_{j=1}^{n} q_i q_j \sigma_{r_i,r_j} \rho_{w_i,w_j}

+ \sum_{j=1}^{n} J_w \sum_{j=1}^{n} q_i W \sigma_{w_i,w_j} + \sigma_{w_i,w_j} ) .
\]  

(4)

The \( n+1 \) first-order conditions for each investor derived from (4) are given by:

\[
0 = U_c(c,t) - J_w(W,t,\psi) ,
\]  

(4.1)

\[
0 = J_w (\mu_{r_i} + \mu_{w_i} - r_j) W + J_w \sum_{j=1}^{n} q_j W \left( \sigma_{r_i,r_j} + \sigma_{w_i,w_j} + \sigma_{r_i,w_j} + \sigma_{r_i,w_j} \right)

+ \sum_{j=1}^{n} J_w \sum_{j=1}^{n} W (\sigma_{w_i,w_j} + \sigma_{w_i,w_j} ) ,
\]  

(4.2)

\[ \forall i = 1,2,..n \text{ and } j = 1,2,..n \text{, where } c^* = c(W,t,\psi) , \quad q_i^* = q_i(W,t,\psi) \text{ represent the optimal level of consumption and the optimal weights for assets in portfolio.} \]
2.3. The Optimal Portfolio Choice

The assumption of constant risk-free rate in our model allows us to simplify our analysis and focus on the market for risky equity securities. Using matrix notation, we rewrite equation (4) for the \( n \) risky assets:

\[
0 = J^W (\mu_r + \mu_\psi - r_f I) + J_{WW} \Sigma_{\tilde{r},\tilde{r}} q + \Sigma_{\tilde{r},\psi} J_{\psi W},
\]

(5)

where \( \mu_r \) is the vector of mean precise returns of the \( n \) risky securities, \( \mu_\psi \) is the vector of long-term means of information-imprecision return error (which are also the long-term return spreads between the precise returns and imprecise returns), \( \psi \) is the vector of firm-specific information-related return error, \( \Sigma_{\tilde{r},\tilde{r}} \) is the variance-covariance matrix of imprecise return vectors \( \tilde{r} \) with elements \( \sigma_{\tilde{r}_{ij}} = \sigma_{r_{ij}} + \sigma_{\psi_{ij}} + \sigma_{\tilde{r}_{ij}}, \) and \( \Sigma_{\tilde{r},\psi} \) is the covariance matrix between the observed imprecise return vector \( \tilde{r} \) and the information-related return-error vector \( \psi \) on risky assets, with components given by \( \sigma_{\tilde{r}_{ij}} = (\sigma_{r_{ij}} + \sigma_{\psi_{ij}}) \forall i = 1,2,...n, \) and \( j = 1,2,...n. \) From (5), we obtain the vector of optimal portfolio weights,

\[
q^* = -\frac{J^W}{WJ_{WW}} \Sigma_{\tilde{r},\tilde{r}}^{-1} (\mu_r + \mu_\psi - r_f I) - \frac{\Sigma_{\tilde{r},\psi}}{WJ_{WW}} J_{\psi W}.
\]

(5.1)

We rewrite (5.1) for every asset \( i \) as follows:

\[
q^*_i = -\frac{J^W}{WJ_{WW}} \sum_{j=1}^{n} \nu_{\tilde{r}_{ij}} (\mu_r + \mu_\psi - r_f I_j) - \sum_{j,k=1}^{n} \nu_{\tilde{r}_{ij}} \sigma_{\tilde{r}_{jk}} \nu_{\psi_{jk}} J_{\psi W_k},
\]

(5.2)

\( \forall i = 1,2,...n, \) \( j = 1,2,...n, \) and \( k = 1,2,...n. \) \( \nu_{\tilde{r}_{ij}} \) denotes an element in the inverse of the variance covariance matrix, \( \Sigma_{\tilde{r},\tilde{r}}^{-1}. \)

Equations (5.1) and (5.2) give the optimal weights (demand) for asset \( i \) in the presence of imprecise accounting information. The optimal portfolio weight, \( q^*_i \), in equation (5.1) is the combination of the tangency (market) portfolio with \( n \) hedge portfolios. This optimal portfolio hedges against imprecise-information risk related to each of the \( n \) assets, which causes unfavorable changes in the noisy investment opportunity set. Note that, the optimal portfolio composition is altered in the presence of
imprecise information not only through the hedge portfolios, but also through the definition of the tangency portfolio, which is altered as well by imprecise return errors.  

2.4. The Equilibrium Pricing Equation

The equilibrium asset-pricing equation derived in this section shows that expected asset risk premiums with imprecise accounting information arise due to three elements: (i) the sensitivity of asset returns to the precise excess market return (as in the standard CAPM); (ii) a return sensitivity to a market-wide imprecise information factor; and (iii) the asset return sensitivity with respect to \( n \) hedge portfolios. The expected excess return on a security takes the following form (see derivation in Appendix B):

\[
\mu_n + \mu_{\psi_i} - r_f = \beta_i^m (\mu_{r_n} - r_f) + \beta_i^m \mu_{\psi_i} + H_i,
\]

where \( \mu_{r_n} \) is the mean precise market return, \( \beta_i^m = \frac{\sigma_{\tilde{r}_m \tilde{r}_i}}{\sigma_{r_m}^2} \) reflects the \( i \)th security return sensitivity to the imprecise-information-adjusted market excess returns, \( \tilde{r}_m \) is the imprecise return on the market portfolio, \( \tilde{r}_m = \sum_{i=1}^{n} x_i \tilde{r}_i \). In the intertemporal model, investors hold \( n \) hedge portfolios, represented by a vector \( h \), to hedge against the fluctuations in imprecise returns that cause random shifts in the imprecise information set. This result implies the following extra term in equation (6):

\[
H_i = \sum_{k=1}^{n} \sum_{j=1}^{n} \left( \beta_i^m \sigma_{\tilde{r}_j \tilde{r}_i} - \sigma_{\tilde{r}_i \tilde{r}_j} \right) b_{h_{i,j} \tilde{r}_k} \left( \pi_i^m - \beta_k^h \pi_m \right)
\]

that represents the risk premium associated with the \( n \) hedge portfolios. The term: \( \pi_m = (\mu_{r_n} - r_f) + \mu_{\psi_m} \) is the market risk premium adjusted for imprecise accounting information, \( \beta_k^h = \frac{\sigma_{\tilde{r}_i \tilde{r}_k}}{\sigma_{r_i}^2} \) is the \( k \)th hedge portfolio’s return sensitivity to imprecise-information-adjusted excess market returns, \( \tilde{r}_{i,k} \) is the imprecise return on hedge portfolio \( k \), \( \pi_k^h = (\mu_{\tilde{r}_k} - r_f) + \mu_{\psi_{i,k}} \) is the imprecise information adjusted risk premium on the \( k \)th hedge portfolio, and \( b_{h_{i,j} \tilde{r}_k} \) denotes an element in the inverse matrix \( \Gamma_{h_{i,j} \tilde{r}_k}^{-1} \), which is presented in Appendix B.

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8 The optimal portfolio choice varies across investors as reflected in the different derivatives of the investor’s expected lifetime utility of wealth, \( J \).
Equation (6) is the equilibrium intertemporal capital asset pricing equation with information imprecision. It describes the equilibrium relation between the asset risk premium and three types of risk: market (systematic) risk, systematic information imprecision risk, and the risk of unfavorable shifts in the stochastic investment opportunity set. Equation (6) further shows that imprecise accounting information affects the risk premium associated with the asset betas (risks). In addition to the standard CAPM market risk premium, \((\mu_{m} - r_{f})\), investors also demand a premium for asset return sensitivity to the market-wide information-quality risk factor, \(\mu_{\psi_{m}}\).

Systematic risk is measured by the sensitivities (betas) with respect to the market portfolio, information-quality factor, and the hedge portfolios. The imprecise-information related adjustment in \(\beta_{i}^{m}\) and \(\beta_{k}^{h}\) highlights a significant difference from the APT model of Hughes et al.’s (2007). In their model, risk related to asymmetric information only affects factor risk premiums, not factor sensitivities. Unique to our model, the risk associated with the imperfect-information attribute we consider (information imprecision) affects the risk premiums as well as factor sensitivities. As a result, information imprecision is systematic and cross-sectionally priced. We further explore this point and explain the way in which imprecise accounting information alters factor sensitivities in Section 3.

3. Empirical and Theoretical Implications of Imprecise Accounting Information

In this section, we further analyze the implications of our model using a more tractable static version of equation (6). To arrive at the static version, we assume that every \(k^{th}\) hedge portfolio is correctly priced by its return sensitivity to the noisy market factor (in other words, \(H_{k} = 0\), for every \(k\)).\(^9\) Under this assumption we can write equation (6) as follows for every \(k^{th}\) hedge portfolio:

\[
\mu_{r_{a,k}} + \mu_{\psi_{r_{k}}} - r_{f} = \beta_{k}^{m} (\mu_{m} - r_{f}) + \beta_{k}^{h} \mu_{\psi_{m}}.
\]

\(^9\) Alternative to this assumption, there are two additional assumptions that lead to the static version of our model in equation (7). First, one can assume the derived utility function, \(J\), is either additive in wealth and return errors. Second, imprecise returns have a factor structure.
With the notations used following the discussion of equation (6), this is equivalent to:
\[ \pi^h_k = \beta^h_k \pi^m \]
for every \( k \). 10 Thus the pricing error on hedge portfolio return, represented by \( \pi^h_k - \beta^h_k \pi^m \), becomes zero. This implies that:
\[ H_i = \sum_{k=1}^n \sum_{j=1}^n (\beta^m_i \sigma_{t,i,j} - \sigma_{t,i,j} \beta^m_j) (\pi^h_k - \beta^h_j \pi^m) = 0 \]
for every asset \( i \), which leads to the static version of our imprecise-information-adjusted asset-pricing model:
\[ \mu_i + \mu_{(i)} = \beta^m_i (\mu_{r,n} - r_f) + \beta^m_i \mu_{(i)} \equiv \beta^m_i \mu_{(i)} \]
where:
\[ \beta^m_i = \frac{\sigma_{t,i,r_m} + \sigma_{t,i,r_m} + \sigma_{t,i,r_m} + \sigma_{t,i,r_m}}{\sigma_{t,i,r_m} + \sigma_{t,i,r_m} + 2\sigma_{t,i,r_m}} \]
and \[ \sigma^2_{(i)} = \sigma^2_{(i)} + \sigma^2_{(i)} + 2\sigma^2_{(i)} \]. The ensuing discussion is based on this static version.

3.1. Empirical Implications of the Imprecise-Information-Adjusted CAPM

Equation (7) provides the theoretical underpinning for incorporating an information-imprecision risk factor in a regression model as a separate market factor. Recall that Lambert et al. (2007) do not provide a theoretical justification for the pricing of an information-quality factor. In their one-period CAPM-based model, the cross-sectional pricing of imprecise information manifests through the estimation of the firm’s cash-flow beta. They emphasize that their model does not provide the theoretical foundation for an extra separate “information risk” factor in an asset-pricing model. However, our imprecise-information-adjusted model provides a pricing relation that is consistent with empirically modeling both market-wide information-quality risk premium( \( \mu_{(i)} \)) and the standard CAPM market risk premium ( \( \mu_{r,n} - r_f \)) as separate priced risk factors in the context of a multiple-regression model (as in the empirical work of Francis et al., 2004, 2005; Botosan et al. (2004); Aboody et al. (2005); Liu and Wysocki (2006); Core et al., 2008; Ogneva, 2008; and Kravet and Shevlin.) In addition, in our model imprecise accounting information also affects asset prices cross-sectionally through its impact on the firm’s imprecise-return beta (rather than the Lambert et al. cash-flow beta).

---

10 Note that, \( \beta^m_k \) and \( \beta^h_k \) refer to the same thing: hedge portfolios’ return sensitivities to market excess returns.
In the next subsection, we decompose our imprecise-return beta and investigate the three different channels through which imprecise information is manifested as a systematic risk factor in our model, two of which are unique to this paper. We further show that one of these channels is closely related to the factor loading of the information-quality factor that appears in the above body of empirical work. This provides further theoretical support for empirically modeling a distinct information-quality factor.

3.2. Theoretical Implications of the Imprecise-Information-Adjusted CAPM

Under the static version of the imprecise-information-adjusted CAPM given in equation (7), the three-fund separation of the intertemporal model collapses to the standard equation that reflects a two-fund separation in the static CAPM, adjusted for imprecise accounting information. The two mutual funds (the riskless asset and the market portfolio) allow the investor to create a risk-return profile comparable to that of asset \( i \) on an instantaneously efficient frontier. Thus, the imprecise-return beta in equation (7) measures the risk contribution of asset \( i \) (\( \sigma_{i,\tilde{r}_m} \)) to the total risk of holding the market portfolio (\( \sigma_{\tilde{r}_m}^2 \)), which consists of a systematic component of imprecise-information risk.

After expanding the covariance term in equation (7) we can write the asset pricing equation as follows:

\[
\mu_i = \mu_i + \pi^m = r_f + \pi^m \frac{\sigma_{r_f, \tilde{r}_m}}{\sigma_{\tilde{r}_m}^2} + \pi^m \frac{\sigma_{\tilde{r}_m, \tilde{r}_m}}{\sigma_{\tilde{r}_m}^2} + \pi^m \frac{\sigma_{\psi, \tilde{r}_m}}{\sigma_{\tilde{r}_m}^2} + \pi^m \frac{\sigma_{\psi, \tilde{r}_m}}{\sigma_{\tilde{r}_m}^2} + \pi^m \frac{\sigma_{\psi, \tilde{r}_m}}{\sigma_{\tilde{r}_m}^2}.
\]

Recall that \( \pi^m = (\mu_{\tilde{r}_m} - r_f) + \mu_{\psi, \tilde{r}_m} \), is the imprecise-information-adjusted risk premium on the market. The long-term equilibrium imprecise-information-adjusted asset-pricing model in Equation (8), provides the framework for understanding the various channels through which imprecise-information risk may affect asset returns.

Unlike the model of Lambert et al. (2007), our model distinguishes between the standard CAPM (precise) systematic risk and systematic risk associated with imprecise information. This is reflected in equation (8) where we decompose the firm’s total systematic risk into the standard CAPM beta and three additional betas associated with imprecise-information risk. These four betas are related to the following covariance terms: \( \sigma_{r_f, \tilde{r}_m} \), \( \sigma_{\psi, \tilde{r}_m} \), \( \sigma_{\psi, \psi, \tilde{r}_m} \), and \( \sigma_{\tilde{r}_m, \psi, \tilde{r}_m} \). The first term, \( \sigma_{r_f, \tilde{r}_m} \), is the standard CAPM
covariance of the precise returns on the individual asset \((r_i)\) and on the market \((r_m)\). With an imprecise information set the three distinct betas representing systematic information-quality risk are related to the following covariance terms: \(\sigma_{r_i, \psi_m}\), \(\sigma_{\psi_i, \psi_m}\), and \(\sigma_{r_m, \psi_i}\). Below, we show that the first covariance term captures the effect of the information-quality factor of Francis et al. (2005). The other two covariance terms reveal two novel channels that manifest the pricing of systematic risk related to information imprecision.

The first systematic imprecise-information risk term, \(\sigma_{r_i, \psi_m}\), is the covariance between the security’s precise return \((r_i)\) and the overall market imprecise-information return error \((\psi_m)\). In the CAPM world investors hold the market portfolio. Investors prefer securities for which this covariance has a negative sign so that the security tends to pay a positive precise return at times when the market portfolio suffers from a negative imprecise-information return error.\(^{11}\) Stated differently, when this covariance is negative, the security hedges against market losses due to information imprecision. This means that investors demand a risk premium when this covariance term is positive. Therefore, this covariance term provides the theoretical framework for the cross-sectional pricing of the Francis et al. (2005) information-quality factor loading, which is a function of \(\sigma_{r_i, \psi_m}\). Note that the two remaining betas (covariance terms) represent two new channels that are unique to our model, through which systematic imprecise-information risk affects asset prices.

The second imprecise-information beta is related to the term \(\sigma_{\psi_i, \psi_m}\), which is the covariance between the security’s information-related return \((\psi_i)\) and the overall market imprecise-information return error \((\psi_m)\). Investors holding the market portfolio prefer securities for which this covariance is negative so that the security tends to pay a positive imprecise-information return error at times when the market portfolio suffers from a negative imprecise-information return error. In other words, when this covariance is negative, the favorable imprecise accounting information about this security hedges against market losses due to information imprecision. We call the beta related to this

\(^{11}\) The analysis in this paper assumes that the market risk premium, \(\pi^m\), is positive, which is empirically consistent.
covariance \( (\sigma_{\psi_{i}, \mu_{n}} / \sigma_{\mu_{n}}^2) \) the commonality in information quality beta. Investors will demand a risk premium for securities with a positive commonality in information quality beta.

The last component of systematic risk related to imprecise information, \( \sigma_{\psi_{i}, \mu_{n}} \), is the covariance between the security’s return due to information imprecision (\( \psi_{i} \)) and the overall market precise return (\( \mu_{n} \)). In this case, investors prefer to hold securities that hedge with a positive imprecise-information return error against a bearish market. This means that investors holding the market portfolio prefer securities for which this covariance is also negative, so that the security tends to pay a positive imprecise-information return error at times when the market portfolio suffers from a negative precise return. This implies that – for portfolio hedging purposes – investors may prefer to invest in stocks of firm’s that erroneously, or even deliberately, release false positive information about the firm at times of a down market. Here again, investors demand a risk premium when this last covariance term is positive.

Finally, two-fund separation implies that investors hold the market portfolio. In our model, the risk involved in holding the market when information is imprecise is given by: \( \sigma_{\mu_{n}}^2 = \sigma_{\mu_{n}}^2 + \sigma_{\psi_{n}}^2 + 2\sigma_{\mu_{n}\psi_{n}} \). This means that the total systematic risk consists of three components: the precise market-portfolio risk (\( \sigma_{\mu_{n}}^2 \)), risk related to imprecise-information market return (\( \sigma_{\psi_{n}}^2 \)), and the comovement of market precise return and imprecise-information market return (\( \sigma_{\mu_{n}\psi_{n}} \)). Therefore, the presence of \( \sigma_{\psi_{n}}^2 \) and \( \sigma_{\mu_{n}\psi_{n}} \) shows explicitly that imprecise-information risk cannot be diversified away, even in the context of a very large portfolio such as the market portfolio.

The above discussion highlights that our static version of the imprecise-information-adjusted asset-pricing model presents three distinct channels through which this systematic imprecise-information risk is priced. Equation (8) implies that investors demand a higher risk premium due to the three additional information-related betas representing systematic imprecise-information risk they face. This risk is priced because, even when one holds security \( i \) within a large portfolio such as the market portfolio, one still faces the systematic information-quality risk that security \( i \) contributes to the market.
portfolio. Unique to our model, this risk is priced through an altered market risk premium as well as through the factor loadings of the security.

4. **Summary and Conclusions**

We derive an intertemporal asset-pricing model in the spirit of Merton (1973) that allows us to investigate different channels through which systematic risk associated with imprecise accounting information affects asset prices. We show that in our model imprecise-information risk has systematic and idiosyncratic components, and only the former is priced. A static version of our model demonstrates that systematic information-quality risk is priced through the alteration of the market risk premium as well as through three extra asset betas. In our model, a market-wide required excess return due to imprecise information is distinct from the CAPM market risk premium. This addresses criticism of the lack of theoretical underpinning for the inclusion of a separate information-quality factor in empirical multiple-factor models such as that of Francis et al. (2005).

The three extra betas in our model represent three distinct systematic risk effects of imprecise accounting information for which investors demand an extra risk premium. Our first information-imprecision beta reflects the security-return sensitivity with respect to market-wide information imprecision. This beta provides further theoretical support for the cross-sectional pricing of the Francis et al. (2005) information-quality factor loading. In the multiple-regression model in their paper, the security return is regressed on the market-wide information-quality factor, which means that, like our first information-imprecision beta, their information-quality factor loading is a function of the covariance between the security return and the market information-quality factor.

The remaining two information-precision betas are unique to our paper. The commonality in information quality beta is the second channel through which imprecise-information risk affects asset prices in our model. This beta reflects the co-movement between the information-imprecision return error of the individual asset and that of the market. To hedge against adverse imprecise-information effects on their portfolio, investors prefer to include an asset with a negative commonality beta, so that when information-imprecision depresses market returns it inflates the asset return.
The third and last channel, through which risk associated with imprecise information affects asset pricing in our model, is the beta that reflects the relation between the asset return errors due to imprecise-information and the precise market return. To protect their portfolio at times of a bearish market, investors prefer a security for which this relation is negative, so that when their portfolio is down the security tends to pay a positive imprecise-information return error. This unique result implies that investors may have a preference for investing in stocks issued by firms that erroneously or intentionally, release false positive information about the firm at times of a down market. This implication of our model raises the question of whether, empirically, at times of a depressed market investors prefer to invest in firms performing earnings management or management manipulation of management earnings forecast that improve expectations about the firm’s future performance. This empirical question is left for future research.
Appendix A – Derivation of the Wealth-Accumulation Process

Recall that the wealth accumulation equation for the \( i \)th investor is given by:

\[
dW = \sum_{i=1}^{n} q_i Wdr_i + (1 - \sum_{i=1}^{n} q_i) Wr_f - cdt,
\]  

(3)

Substituting for \( dr_i \) from equation (2) we rewrite equation (3) as:

\[
dW = \left[ \sum_{i=1}^{n} q_i (\mu_{r_i} + k(\mu_{\psi_i} - \psi_i) - r_f) + r_f \right] Wdt + \sum_{i=1}^{n} q_i W \sqrt{\sigma_{r_i}^2 + \sigma_{\psi_i}^2 + 2\sigma_{r_i,\psi_i}} d\omega_i - cdt.
\]  

(3.1)

where \( r_f \) is an exogenous instantaneous interest rate on a risk-free bond, and \( \sum_{i=1}^{n+1} q_i = 1 \), where \( q_{n+1} \) is the weight of the riskless asset. The assumption of constant risk-free rate in our model allows us to simplify our analysis and focus on the stock market.

The necessary instantaneous optimality condition for solving for an investor’s consumption-investment optimal choice is as follows:

\[
0 = \max_{c,q} \left[ U(c,t)dt + J_i dt + J_W E_i (dW) + \frac{1}{2} J_{WW} E(dW)^2 + \sum_{i=1}^{n} J_{\psi_i} E_i (d(\psi_i)) \right]  
+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} J_{\psi_i,\psi_j} E(d(\psi_i)d(\psi_j)) + \sum_{i=1}^{n} J_{W\psi_i} E(dW d(\psi_i)) + o(dt) \].

(3.2)

Equation (3.2) implies a Gaussian process of wealth accumulation. Thus the variance and covariance of the instantaneous change in wealth and the instantaneous change in the observable noisy return are given by:

\[
E_i (dW) = \left[ \sum_{i=1}^{n} q_i (\mu_{r_i} + \mu_{\psi_i} - r_f) + r_f - c \right] dt, 
\]  

(3.3)

\[
E(dW)^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} q_i q_j W^2 (\sigma_{r_i,r_j} + \sigma_{r_i,\psi_j} + \sigma_{r_j,\psi_j} + \sigma_{r_j,r_j}) dt, 
\]  

(3.4)

\[
E(d\psi_i,d\psi_j) = \sigma_{\psi_i,\psi_j} \rho_{\psi_i,\psi_j} dt, 
\]  

(3.5)

\[
E(dW d\psi_j) = \sum_{i=1}^{n} q_i W (\sigma_{r_i,\psi_j} + \sigma_{\psi_i,\psi_j}) dt. 
\]  

(3.6)

Substituting equations (3.3) to (3.6) into equation (3.2), we get the following equation,
The $n+1$ first-order conditions for each investor derived from equation (4) are given by:

$$0 = U_c(c, t) - J_w(W, t, \psi), \quad (4.1)$$

where $c^* = c(W, t, \psi)$ and $q_i^* = q_i(W, t, \psi)$ are optimal solutions for equations (4.1) and (4.2) as functions of the perceived state variables.

Using matrix notation, we rewrite equation (4.2) for the $n$ risky assets as follows:

$$0 = J_w(\mu_r + \mu_\psi - r_f)W + J_{ww} \sum_{j=1}^n q_j W^2 (\sigma_{r,r_j} + \sigma_{\psi,\psi_j} + \sigma_{r,\psi_j} + \sigma_{\psi,\psi_j}) + \sum_{j=1}^n J_{w,\psi_j} W(\sigma_{r,\psi_j} + \sigma_{\psi,\psi_j}). \quad (4.2)$$

where $\mu_r$ is the vector of mean precise returns of the $n$ risky securities, $\mu_\psi$ is the vector of long-term means of information-imprecision return error (which are also the long-term return spreads between the precise returns and imprecise returns), $\psi$ is the row-vector of firm-specific information-related return error, $\varrho$ is the variance-covariance matrix of imprecise returns with elements $\varrho_{r,r_j} = \sigma_{r,r_j} + \sigma_{\psi,\psi_j} + \sigma_{r,\psi_j} + \sigma_{\psi,\psi_j}$, and $\Sigma_{r,\psi}$ is the matrix of covariance terms between observed noisy return variables and the information-related return errors on risky asset, with components given by $\varrho_{r,\psi_j} = (\sigma_{r,\psi_j} + \sigma_{\psi,\psi_j})$,

$$\forall \ i=1,2,...,n, \ and \ j=1,2,...,n.$$ Solving equation (5), we obtain the vector of optimal portfolio weights for the $n$ risky securities:

$$q^* = -\frac{J_w}{WJ_{ww}^{-1}} (\mu_r + \mu_\psi - r_f I) - \Sigma_{r,\psi}^{-1} \frac{J_{w,\psi}}{WJ_{ww}^{-1}}.$$ 

$$\quad (5.1)$$
Equation (5.1) can be written for every asset $i$ as follows:

$$q_i^* = -\frac{J_{WW}}{W_{WW}} \sum_{j=1}^{n} v_{r_{i},r_{j}} (\mu_{r_{j}} - \mu_{r_{j}}) - \sum_{j,k=1}^{n} v_{r_{i},r_{j}} (\sigma_{r_{i},r_{j}} + \sigma_{r_{j},r_{k}}) \frac{J_{W_{i},r_{j}}}{W_{WW}} ,$$

(5.2)

$\forall i=1,2,..,n, \ j=1,2,..,n, \text{and} \ k=1,2,..,n$. $v_{r_{i},r_{j}}$ denotes an element in the inverse of the variance covariance matrix, that is $\Sigma_{r_r}^{-1}$. Equation (5.2) gives the optimal weights (demand) for risky assets in the presence of imprecise accounting information.

**Appendix B – Derivation of Equilibrium Pricing Equation (6)**

Equation (5) can be rearranged as follows:

$$a^l(\mu_{r} + \mu_{r} - r_{j} I) = -W^{l} \Sigma_{r,r} q^{l} - \Sigma_{r,r} b^{l} ,$$

(5.3)

where $a^l = (\frac{J_{WW}}{J_{r_{r}}})^l$ and vector $b^l = (\frac{J_{WW}}{J_{r_{r}}})^l$, for investor $l, l = 1, 2,.., L$. Summing across the $L$ investors and dividing by $\sum_{l=1}^{L} a^l$, we obtain:

$$\mu_{r} + \mu_{r} - r_{j} 1 = -A \Sigma_{r,r} \frac{\sum_{l=1}^{L} W_{l}^{T} q^{l}}{\sum_{l=1}^{L} W_{l}^{T}} - \Sigma_{r,r} B = -A \Sigma_{r,r} x_{m} - \Sigma_{r,r} B ,$$

(5.4)

where $A = \frac{\sum_{l=1}^{L} W_{l}^{T}}{\sum_{l=1}^{L} a^{l}}$ is a scalar, $B = \frac{\sum_{l=1}^{L} b^{l}}{\sum_{l=1}^{L} a^{l}}$ is an $n \times 1$ vector, and $x_{m} = \frac{\sum_{l=1}^{L} W_{l}^{T} q^{l}}{\sum_{l=1}^{L} W_{l}^{T}}$ is a vector of market equilibrium security weights. This notation allows us to write:

$$\Sigma_{r,r} x_{m} = \sigma_{r_{r},r_{m}} = (\sigma_{r_{r},r_{m}} + \sigma_{r_{r},r_{m}} + \sigma_{r_{r},r_{m}} + \sigma_{r_{r},r_{m}}), \quad x_{m}^{T} \Sigma_{r,r} = \sigma_{r_{m},r_{m}} = (\sigma_{r_{m},r_{m}} + \sigma_{r_{m},r_{m}}), \quad \text{and} \quad x_{m}^{T} x_{m} = \sigma_{r_{m}}^{2} .$$

To compute $A$ and $B$ in terms of first and second moments of the tangency and hedge portfolios as well as covariance terms of individual returns with these portfolios, we pre-multiply equation (5.4) by weight vectors, $x_{m}^{T}$, and $y_{h,k}^{T}, \ k = 1,2,..,n$, respectively, to yield the following $n+1$ equations:

$$\mu_{r} + \mu_{r} - r_{j} = -A \sigma_{r_{m}}^{2} - \sigma_{r_{m},r_{m}} B ,$$

(5.5)
\[ \mu_{\tau, h, k} + \mu_{\psi, h, k} - r_f = -A \sigma_{\tau, h, k} - \sigma_{\tau, h, \psi} B, \quad k = 1, 2, \ldots, n, \] (5.6)

where \( \mu_{\tau} = \sum_{i=1}^{n} x_i \mu_{r, i} \) is the weighted expected precise return for the market portfolio, \( \mu_{\psi} = \sum_{i=1}^{n} x_i \mu_{\psi, i} \) is the weighted mean of the information-imprecision return error for the market portfolio, \( \psi_m = \sum_{i=1}^{n} x_i \psi_i \) is the market-wide weighted imprecise-information-related return error, and \( x_m \) is an \( n \times 1 \) vector of security weights in the market portfolio with elements \( x_i \). Similarly, for the hedge portfolios, \( \mu_{\tau, h, k} = \sum_{i=1}^{n} y_{i, k} \mu_{r, i} \) is the weighted expected precise return on hedge portfolio \( k \), the term \( \mu_{\psi, h, k} = \sum_{i=1}^{n} y_{i, k} \mu_{\psi, i} \) is the weighted mean of imprecise-information return error on hedge portfolio \( k \), \( \psi_{h, k} = \sum_{i=1}^{n} y_{i, k} \psi_i \) represents the weighted imprecise-information return error inherent in hedge portfolio \( k \), and \( y_{h, k} \) is an \( n \times 1 \) vector of security weights in hedge portfolio \( k \) with elements \( y_{i, k} \).

Solving for \( A \) from equation (5.5), we have:

\[ -A = \frac{\mu_{\tau} + \mu_{\psi} - r_f + \sigma_{\tau, \psi} B}{\sigma_{\tau, \tau}^2}. \] (5.7)

Next, let \( \mu_{\tau} \) denote an \( n \times 1 \) vector with elements \( \mu_{\tau, k} \), \( \mu_{\psi} \) is an \( n \times 1 \) vector with elements \( \mu_{\psi, h, k} \), and \( \psi_h \) denotes an \( n \times 1 \) vector with elements \( \psi_{h, k} \). Substituting the above solution to \( A \) into equation (5.5) and simplifying leads to:

\[ \mu_{\tau} + \mu_{\psi} - r_f = \frac{\sigma_{\tau, \tau}}{\sigma_{\tau, \tau}^2} \sigma_{\tau, \psi} B. \]

Define \( \Gamma \) and \( \delta \) as

\[ \Gamma = \beta^m \sum_{n=1}^{m} \sigma_{\tau, \psi} - \sum_{n=1}^{m} \sigma_{\tau, \psi}, \quad \text{where} \quad \beta^m = \frac{\sigma_{\tau, \tau}}{\sigma_{\tau, \tau}^2}. \]

\[ \delta = \beta^h \sum_{n=1}^{h} \sigma_{\tau, \psi} - \sum_{n=1}^{h} \sigma_{\tau, \psi}, \quad \text{where} \quad \beta^h = \frac{\sigma_{\tau, \tau}}{\sigma_{\tau, \tau}^2}. \]

Assuming that the inverse matrix \( \Gamma^{-1} \) exists, we solve for \( B \)
\[ B = \Gamma^{-1}_h \left[ \mu_{r,h} + \mu_{\psi_h} - r_j 1 - \beta^h (\mu_{r,u} + \mu_{\psi_u} - r_j) \right] \]
\[ = \Gamma^{-1}_h \pi^h - \beta^h \pi^m, \tag{5.8} \]

where \( \pi^m = (\mu_{r,u} - r_j) + \mu_{\psi_u} \) represents the imprecise-information-adjusted market risk premium, and \( \pi^h = (\mu_{r_h} - r_j) + \mu_{\psi_h} \) is a \( n \times 1 \) vector of hedge portfolio risk premiums.

Substituting solutions (5.7) and (5.8) for \( A \) and \( B \) into equation (5.4), we obtain the following result:
\[ \mu_r + \mu_{\psi} - r_j = -A \sigma_{r,\tilde{r}_u} - \sigma_{r,\psi} B \]
\[ = \frac{\sigma_{r,\tilde{r}_u}}{\sigma_{\tilde{r}_u}} \pi^m + \left( \frac{\sigma_{r,\tilde{r}_u}}{\sigma_{\tilde{r}_u}} \sigma_{\tilde{r}_u,\psi} - \sigma_{r,\psi} \right) B \]
\[ = \beta^m \pi^m + \Gamma^{-1}_{\tilde{r}} \Gamma^{-1}_{\tilde{r}} [\pi^h - \beta^h \pi^m], \tag{5.9} \]

where the term \( [\pi^h - \beta^h \pi^m] \) is the vector of mean pricing errors of the hedge portfolios, determined by hedge portfolio return sensitivities to the market portfolio.

Substituting for \( \Gamma_{\tilde{r}} \) and \( \Gamma_{\tilde{r}} \), we rewrite Equation (5.9) to obtain our imprecise-information-adjusted asset-pricing equation:
\[ \mu_r + \mu_{\psi} - r_j = \beta^m \mu_{r,u} - r_j + \beta^m \mu_{\psi,u} + H_i, \tag{6} \]

where \( \beta^m_i = \frac{\sigma_{r,\tilde{r}_u}}{\sigma_{\tilde{r}_u}} \), and \( H_i = \sum_{i=1}^n \sum_{j=1}^n (\beta^m_i \sigma_{r,u,\psi_j} - \sigma_{r,\psi_j} b_{r_j,\tilde{r}_u,\tilde{r}_u} (\pi^h - \beta^h \pi^m) \)

\( (\beta^m_i \sigma_{r,u,\psi_j} - \sigma_{r,\psi_j} \) is an element of the \( n \times n \) matrix \( \Gamma_{\tilde{r}} \), \( \forall i = 1,2,...,n \), and \( j = 1,2,...,n \);
\( b_{r_j,\tilde{r}_u,\tilde{r}_u} \) represents an element of the inverse matrix \( \Gamma^{-1}_{\tilde{r}_u} \), \( \forall j = 1,2,...,n \), and \( k = 1,2,...,n \); and
\( (\pi^h - \beta^h \pi^m) \) reflects the \( k^{th} \) hedge portfolio’s mean pricing error under CAPM, \( \forall k = 1,2,...,n \).
References


Faust, J., Rogers, J., Wright, J., 2000, News and noise in G-7 GDP announcement, 
*International Finance Discussion Paper* 680, Federal Reserve Board, 
Washington, D.C.

Francis, J., LaFond, R., Olsson, P., Schipper, K., 2004, Costs of equity and earnings 

Francis, J., LaFond, R., Olsson, P., Schipper, K., 2005, The market pricing of earnings of 

Graham, J., Li, S., Qiu, J., 2008, Corporate misreporting and bank loan contracting, 

Hughes, J., Liu, J., Liu, J., 2007, Private information, diversification, and asset pricing, 
*The Accounting Review* 82, 705-730.

Kravet, T., Shevlin, T., 2010, Accounting restatements and information risk, *Review of 
Accounting Studies* 15, 264-294.

Lambert, R., Leuz, C., Verrecchia, R.. 2007. Accounting information, disclosure and the 

Lambert, R., Leuz, C., Verrecchia, R., 2012, Information asymmetry, information 

Lintner, J., 1965, The valuation of risk assets and the selection of risky investments in 

and Massachusetts Institute of Technology.

Merton, R., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867- 
887.


Ogneva, M., 2008, Accrual quality and expected returns, the importance of controlling 
for cash flow shocks, Ph.D. Dissertation, Stanford University.

Shapiro, M., Wilcox, D., 1996, Mismeasurement in the consumer price index, an
evaluation, *NBER Macroeconomic Annual*.
