Is weak form efficiency an illusion? Evidence from a market for state contingent claims

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Abstract

This paper examines the importance of methodology and market ecology when examining the extent to which a market is weak form efficient. Existing literature points to most markets being weak form efficient. However, it is argued here that this may be an artefact of the methodology employed in these studies and the fact that market ecology is often neglected when examining weak form efficiency. We demonstrate how big data from a market for state contingent claims can be used to examine weak form efficiency and to predict market prices employing conditional logit analysis. These are used, together with various trading strategies to yield substantial abnormal returns. Consequently, in contrast to the existing literature, the results suggest that the market is not weak form efficient. We discuss the various aspects of methodology and market ecology which can explain the reported results and the important implications for future market efficiency studies. In particular, we highlight the importance for efficiency studies of considering market characteristics which influence behavior and the importance of considering an appropriate investment strategy.

Keywords: Big data; Market efficiency; Behavioral finance

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1. Introduction

The broad conclusion to emerge from studies conducted across a range of financial markets is that they are weak form efficient. In particular, that market participants effectively discount information contained in historical prices such that it is not possible to make abnormal returns from information contained in asset prices. However, it is suggested here that this conclusion may have arisen in some markets because asset price information and differences in market ecology may not have been fully exploited.

In line with other financial markets, some pricing anomalies have been observed in markets for state contingent claims, but attempts to derive net profits from simple trading rules have not been successful. One set of markets for state contingent claims have attracted considerable interest in relation to efficiency, namely betting markets. The consensus which emerges from the literature is that betting markets are also weak form efficient. Law and Peel (2002, p. 327) argue that betting markets are ‘well suited for testing market efficiency’ since they share many characteristics with wider financial markets. In addition, Sauer (1998, p.2021) observes, that they ‘are especially simple financial markets, in which the pricing problem is reduced. As a result, wagering markets can provide a clear view of pricing issues which are complicated elsewhere. These markets therefore form the focus of this study.

It will be argued that previous weak form efficiency studies in betting markets may not have made sufficient use of information concerning differences in market ecology (between pari-mutuel and bookmaker based markets) or of information contained in final market prices (through an appropriate wagering strategy) to support the conclusion that abnormal returns can not be earned by using information contained in final odds.
Betting market efficiency has attracted considerable attention, but much of the literature addressing weak form efficiency is drawn from pari-mutuel betting markets. However, these markets operate without market makers. Bookmaker markets more closely resemble wider financial markets due to the central role played by market makers and it has been shown that in these markets there is an incentive for market makers (bookmakers) to offer distorted prices to protect themselves from trading by individuals with access to privileged information (Shin, 1991, 1992, 1993). Shin (1991) argues that this distortion manifests itself as a favorite/longshot bias in which odds on longshots/favorites over/under-estimate their chances of success. This conclusion is supported by Bruce and Johnson (2005) who confirm the presence of a significantly greater favorite/longshot bias in bookmaker markets compared with pari-mutuel markets covering the same events. Consequently, previous studies which have focused on pari-mutuel markets are likely to have under-estimated the degree of pricing anomalies (i.e. the degree of favorite/longshot bias) in bookmaker markets. As a result they may have over-estimated the degree of weak form efficiency in these markets. In addition, the investment strategies which have been employed to test for weak form efficiency in betting markets have often not fully exploited the information contained in prices. Kelly (1956) introduced an investment strategy which enables information concerning the estimated probability of success of each runner, together with their odds, to be used to determine the optimal fraction of one’s wealth to bet on each horse. This investment strategy maximizes the expected log payoff and has been shown to be asymptotically optimal (Breiman, 1961). Whilst this strategy has been employed in a few studies exploring semi-strong form efficiency in betting markets it has not been used to assess the degree of weak form efficiency in bookmaker betting markets. Consequently, it is possible that the consensus which has emerged that horserace betting markets are weak form efficient may have arisen from incomplete evidence. We explore this view by formulating and testing the hypothesis that it is possible to employ the Kelly strategy
to exploit the pricing anomalies found in horserace betting markets with market makers in order to make abnormal returns.

The paper proceeds as follows. Section II introduces the main features of the UK horserace betting market and explores to what extent these correspond to those found in wider financial markets. Section III offers a brief review of the literature that addresses the degree of weak form efficiency found in betting markets and introduces the hypothesis to be tested. Section IV outlines the data employed in this study and the procedures which are adopted to test the hypothesis. The results are presented in Section V and discussed in Section VI. Conclusions and implications for wider financial markets are summarized in Section VII.

II. The value of betting markets for revealing behavior in wider financial markets

“In its simplest formulation, the market for bets in an n-horse race corresponds to a market for contingent claims with n states in which the ith state corresponds to the outcome in which the ith horse wins the race”. (Shin, 1992, p1142). Investors in betting markets purchase assets (place bets), returns to which depend on the result of the event to which the particular market relates. In state contingent claims terms, the purchase price of a claim on competitor i in event j which pays £1 if competitor i wins and nothing if it loses, is given by 1/(1+Oij), where Oij represents competitor i’s market price (odds). It is argued that bettors will continue to place money on a competitor i in event j until the purchase price of a claim on this competitor (1/(1+Oij)) accurately reflects the market’s best estimate of the competitor’s chance of winning (Figlewski, 1979).

Betting markets share many fundamental characteristics with wider financial markets, including: large numbers of participants (market makers, the holders of private information, institutional/professional buyers and noise traders), ease of entry to the market, the availability of extensive information allowing both fundamental and technical analysis, the presence of complex and interdependent factors which influence an asset’s value/bet’s prospects, extensive market knowledge and some similar behavioral biases (e.g. Sauer, 1998; Snyder,
1978; Thaler and Ziemba, 1988). Decisions in betting and wider financial markets also share many features in common. These include the collection, interpretation and analysis of quantitative and qualitative information, including that associated with expert opinion; the consideration of precedent, the formation of expectations and the commitment of resources (Bruce and Johnson, 2005). In the light of these similarities, the value of betting markets in providing a window on decision-making in wider financial markets is well established (e.g. Hong and Chiu, 1988; Law and Peel, 2002).

In addition to the many similarities with wider financial markets, betting markets also possess a distinctive feature which makes it possible to discern behavior more clearly than in other financial markets: the generation of an unequivocal outcome (a winner) within a finite time frame. This provides a clear, objective benchmark against which to measure the appropriateness of the asset’s price (i.e. revealed as odds), which helps in the assessment and understanding of factors influencing market efficiency. The finite nature of betting markets also means that there is a large pool of markets of essentially similar type available for analysis.

In summary, betting markets share many features in common with wider financial markets and they also possess features which may help provide insights into the manner and degree to which information is used. In addition, it has been argued that participants in betting markets are likely to use this information efficiently as the conditions (quick, repeated feedback) are those thought to facilitate learning (Johnson and Bruce, 2001).

In this paper we employ data from the UK horserace betting market. Two parallel betting markets operate at racetracks in the UK: the pari-mutuel market and the bookmaker market. In the pari-mutuel market, odds are determined (as in the pari-mutuel markets worldwide) solely by the pattern of relative bettor demand, according to a pre-determined formula, and bets are settled at the odds prevailing at the close of the market. However, odds in the bookmaker market, which forms the setting for this study, are determined by the decisions of market...
makers (bookmakers) and bettors; bets being settled at the odds available in the market at the
time the bet is struck. Consequently, bettors in bookmaker markets (unlike parimutuel markets)
can secure their return, without the danger of a bandwagon effect eroding their gains.
Bookmaker markets are therefore more likely to attract the more serious bettors and those with
access to privileged information (Crafts, 1985; Sauer, 1998; Schnytzer and Shilony 1995).

At the start of the market for a given race independent bookmakers at the racetrack (10-50
bookmakers at most racetracks) post odds, which represent their considered opinion of each
horse’s chance of success. Bookmakers may subsequently change their odds as a result of
receiving information which alters their subjective view of the horses’ relative prospects and to
reflect bettors’ opinions, manifested in the relative weight of demand for bets on different
horses. Bettors are free to bet with the bookmaker offering the best odds on their selection up
to the start of the race.

Bookmakers base their odds on the information they receive concerning horses’ relative
prospects from a variety of sources, including their own agents who are employed to gather and
analyse such data. They also use information derived from the relative and absolute volume of
bets on each horse as the market proceeds. Bettors also employ information from a wide
variety of sources, including data concerning horses’ previous performances, professional
advice, and movements in the bookmaker and parimutuel odds.

Bookmaker markets clearly incorporate information from a number of sources and the next
section will explore to what extent existing research evidence suggests that information
contained in odds and odds movements is fully discounted in final odds.

III. Weak form efficiency in horserace betting markets

Previous studies in horserace betting markets across the world conclude that the markets are
weak form efficient, in that wagering strategies based on historical prices do not yield positive
expected returns. Prices which emerge in these markets appear to provide a good guide to the
horses’ relative prospects of success, as measured by their relative win frequencies (Hoerl and
Fallin, 1974). Research studies exploring the extent to which historical price information is efficiently incorporated into final odds, fall into three broad categories: favourite/longshot studies, which investigate the degree to which odds under/over estimate the winning probabilities of horses with the lowest/highest odds; price movement studies, which examine whether information contained in odds movements can be used to make positive expected returns; arbitrage strategies, which explore if it is possible to profitably exploit information revealed in one market within another related market.

The overwhelming consensus to emerge from favorite/longshot studies in horserace betting markets, dispersed in time and across the world, is that the odds of favorites (low odds) consistently under-estimate their true chances and the odds of longshots over-estimate their chances\(^1\) (e.g. USA: Ali, 1977; Asch, Malkiel and Quandt, 1982; McGlothlin, 1956; Snyder, 1978; Thaler and Ziemba, 1988; Ziemba and Hausch, 1986; Australia: Bird and McRae, 1994; Tuckwell, 1983; New Zealand: Gander, Zuber and Johnson, 2001\(^2\); UK: Bruce and Johnson, 2000; Dowie, 1976; Vaughan-Williams and Paton, 1997). Despite this widespread pricing discrepancy the vast majority of evidence suggests that this is not sufficiently large to enable a profitable trading strategy based on the bias to be constructed (e.g. Levitt, 2004; Snowberg and Wolfers, 2005). Two exceptions have been documented. The first is in a study by Ziemba and Hausch (1986) in the US pari-mutuel market and the second is by Dowie (1976) in the UK bookmaker market. Both these studies show that a small expected profit could be earned by betting extreme favorites (odds < \(\frac{3}{10}\) and \(<\frac{1}{2}\) respectively). However, betting opportunities with these strategies are very limited, it is unlikely that returns would cover fixed costs and subsequent studies suggest that the margin may no longer be available (Snowberg and Wolfers, 2005).

In summary, the conclusion to emerge from the vast majority of favorite/longshot studies is that the bias is not sufficient to contradict the view that the betting market is weak form efficient.
The second set of studies exploring weak form efficiency in horserace betting markets examine the extent to which information contained in odds movements can be used to construct profitable trading strategies. These studies consistently suggest that information is revealed in price movements during the course of the market. Dowie (2003), for example, estimates that approximately 25% more information is incorporated into the final odds (compared with opening odds). However, there is general agreement that this cannot be employed by those without its prior knowledge to make positive expected returns (Asch, Malkiel and Quandt, 1982; Crafts (1985); Schnytzer and Shilony, 2002; Schnytzer, Shilony and Thorne, 2003; Tuckwell, 1983). For example, Dowie (1976) found a close correlation between initial and final odds, suggesting little scope for discerning information from odds movements. Crafts (1985) focused on horses whose final and initial odds differed significantly. A strategy based on backing/avoiding horses whose odds fell/lengthened significantly produced a better return than a random betting strategy, but it did not produce a positive expected return. In a subsequent paper, Crafts (1994) identified a very specialized trading rule which could be employed to produce profits from price movement information. This involved only betting on horses who had not raced before in a given year when the subjective probabilities inherent in final odds were at least 1.5 times greater than the subjective probabilities inherent in opening odds. In addition, bets were restricted to horses whose final odds were less than 7/1. Crafts (1994, p.15) himself cautions that ‘in a period of five years only 88 bets would have resulted from a very substantial investment of time and effort.’ This, together with the very specialized nature of this trading rule suggests that it does not offer a major challenge to weak form efficiency in racetrack betting markets. In an extensive review of this literature Sauer (1998, pp. 2048–49) concludes that ‘the evidence suggests that an informed class of bettors is responsible for altering prices in these markets . . . (but) the opinions of ‘experts’ appear to be fully discounted in market prices’.
The third set of weak form efficiency studies conducted in horserace betting markets examines arbitrage opportunities between parallel markets. For example, Ali (1977) compares the odds on horses in ‘the daily double’ pool (a pari-mutuel bet paying a return only if a bettor selects the winners of two races) with the odds in the win pools of the two races. His analysis confirms that the win pool odds provide efficient estimates of the daily double pool odds; a result consistent with the hypothesis that the betting market is weak form efficient. Hausch, Ziemba and Rubinstein (1981) use probabilities revealed by win pool betting to construct probabilities of horses finishing second and third. They then construct profitable wagering strategies based on differences between these probabilities and the odds available in the place and show pools. Some studies (Hausch and Ziemba, 1990; Schnytzer and Shilony, 1995; Leong and Lim, 1994) demonstrate that positive returns are possible by exploiting the degree to which bets placed in independent win pools (e.g. operated in different locations) on a given race produce differences in odds. However, the number of opportunities for profitable betting employing the above strategies is limited, in some cases the information required to exploit the opportunity is not available in real time, and in others the degree of inefficiency is small. In addition, there is evidence that publicity concerning arbitrage opportunities has lead to a change in betting behavior which has removed the inefficiency (Ritter, 1994). Consequently, this third set of studies offer little more than a “crease in what is predominantly a smooth pattern of efficiency in the racetrack betting market” (Sauer, 1998, p.2048).

**Hypothesis generation**

The conclusion which emerges from previous studies is that horserace betting markets worldwide are weak form efficient. However, there are some clues in the methods and results of the preceding studies which suggest that weak form inefficiency may be found in UK bookmaker markets, provided that a investment strategy is employed which utilizes all the information in the prices.
The vast majority of studies exploring efficiency have been conducted in pari-mutuel markets. Bookmaker markets, which more closely resemble wider financial markets due to the central role played by market makers, have received relatively little attention. This is surprising, as it can be argued that pricing in these markets is likely to be less efficient, since the favorite/longshot bias may be expected to be larger than in pari-mutuel markets. This arises since factors associated with both bettors’ behavior and market maker’s behavior may contribute to the bias.

The favorite/longshot bias has been attributed to various demand-side explanations, arising from the characteristics of bettors, including, their risk-loving preference, (Quandt, 1986; Rossett, 1965), their tendency to discount only a proportion of their losses (Henery, 1985), their lack of relevant information (Sobell and Raines, 2003) and the importance of motivations which are not related to financial return (Thaler and Ziemba, 1988). Within bookmaker markets, in addition to these potential influences, there is also the possibility of price distortion arising from market makers’ behavioral responses to an adverse selection problem. In bookmaker markets, ‘insiders’, those, such as owners and trainers with access to privileged information concerning the likely performances of their horses, are likely to bet with bookmakers rather than the pari-mutuel operator. This arises because bets are settled at the odds prevailing at the time the bet is struck. Consequently, ‘insiders’ can insulate themselves from subsequent falls in odds as other market participants plunge onto horses which they perceive to have been bet by those with access to privileged information (e.g. Law and Peel, 2002). Shin (1991, 1992, 1993) argues that bookmakers, believing they face a population of bettors which includes insiders, deliberately reduce the odds available on longshots. This has the effect of minimizing their exposure to large potential losses from ‘insiders’ who know that their horse has a better prospect of success than the odds indicate. Odds on longshots (c.f. favorites) are shortened since a relatively modest bet on a longshot could expose bookmakers to a relatively large loss. Vaughan-Williams and Paton (1997) offer support for the view that
the favorite/longshot bias is exacerbated by the presence of bookmakers. In addition, Bruce and Johnson (2005) demonstrate that within the UK horserace market, where bookmaker and pari-mutuel markets co-exist, the bias is significantly more pronounced in the former. Consequently, it is expected that market inefficiency is more likely to be found in bookmaker rather than pari-mutuel markets.

Previous studies also suggest that markets which are less likely to attract professional bettors are those most vulnerable to market inefficiency. Professional bettors are distinguished from insiders by the fact that they use all publicly available information in a sophisticated manner to assess the prospects of each horse, rather than simply being aware that their own horse is miss-priced. Studies by Busche and Hall (1988) and Busche (1994) found no favourite longshot bias in horserace markets in Hong Kong and Japan. Busche and Walls (2000) suggest that this may arise because these relatively high betting volume markets attract professional bettors, since these markets offer reasonable compensation for the time and effort expended in their pursuit of profit. Busche and Walls (2000) argue that the presence of significant numbers of professional bettors in these markets removes the traditional favorite/longshot bias, as the odds are driven to a level which more accurately reflects the prospects of each horse in the race.

In summary, there are some clues in previous research to suggest that market inefficiency is most likely to be found in moderate volume bookmaker betting markets such as those found in the UK.

Previous studies in racetrack betting markets have almost exclusively tested for weak form efficiency by assessing the expected returns available for various odds categories (e.g. Busche and Hall, 1988; Dowie, 1976; Gabriel and Marsden, 1990, 1991; Snyder, 1978) and/or for groups of horses associated with particular trends in odds (e.g. Crafts, 1994); it being argued that if positive expected returns cannot be identified for horses in any particular category then the market is weak form efficient. The betting strategy these studies employ is a simple one of
assuming a level stake is bet on each horse in a particular category. However, such a strategy fails to take full account of the information contained in prices, since it does not differentiate between wagers which are expected to be extremely profitable and those which are likely to be less profitable. In addition, the strategies adopted in some previous papers focus on betting on certain horses (e.g. those whose odds fall significantly: Crafts, 1994; Law and Peel, 2002) without reference to the odds of the remaining runners; whose odds may have drifted to a level which enables positive expected returns to be realized. To address this problem, the approach adopted here, which is discussed in more detail below, is to develop a model to predict winning probabilities based on historic prices and to make full use of the information contained in the predicted probabilities for all horses in the race, by adopting a betting strategy which maximizes the long run rate of growth of wealth (Kelly, 1956). Using this ‘Kelly strategy’, when the probability of winning is greater (for the same expected return) and when the expected return is greater (for the same winning probability) the bet size is larger. Consequently, the most ‘attractive’ wagers are weighted most heavily. This betting strategy identifies those races to bet on, as well as how much to bet on each horse. More than one horse can be bet on in each race, although bets are usually restricted to horses that give a positive expected return.

In summary, the results and methodology of previous studies suggest the following hypothesis: *U.K bookmaker based horserace betting markets exhibit weak form inefficiency.* The next section outlines the data and the procedures which are adopted to test this hypothesis.

**IV. Data and procedures**

The data was supplied by Raceform (2000, 2001), and consists of the closing bookmaker market odds and finishing positions of each 60,568 runners in 5,558 races run at 35 different racetracks across the UK between 16 June 1999 and 13 Aug 2000. The number of horses in each race varies from 2 to 34, with a mode of 12. The closing market odds for horses in the sample range from 0.04 to 100 with a mean value of 16.42.
In order to test the hypothesis the following procedures are adopted. Firstly, a popular approach to test weak-form efficiency is employed, namely, determining if positive returns are achieved for horses in any odds group. Horses are grouped by market closing odds and an expected rate of return \( R_i \) is computed for each odds-group \( i \) as follows:

\[
R_i = \frac{V_i (O_i + 1) - N_i}{N_i}
\]

(1)

where \( V_i \) is the \textit{ex post} number of winning horses and \( N_i \) is the total number of horses in odds group \( i \), with odds \( O_i \).

Secondly, a model is developed, based solely on the closing market odds of each horse, to estimate the winning probabilities of each horse in the sample. These estimated probabilities are then employed with a Kelly betting strategy to determine if positive expected returns can be obtained.

In order to develop the model a ‘winningness’ index, \( W_{ij} \), for horse \( i \) in race \( j \), is defined as follows:

\[
W_{ij} = \ln(p_{ij}^n) + \varepsilon_{ij}
\]

(2)

where \( p_{ij}^n \) represents the normalized, implied probability of winning of a horse with odds \( O_{ij} \) (i.e. \( p_{ij}^n = \frac{1}{1+O_{ij}} \sqrt{\frac{1}{\sum_i 1/(1+O_{ij})}} \)) and \( \varepsilon_{ij} \) is an independent error term. \( W_{ij} \) is defined such that the horse which is observed to win a particular race has the largest winningness index of all runners in that race. Consequently, the probability of horse \( h \) winning race \( j \) (\( p_{hj} \)) is given as follows:

\[
p_{hj} = \Pr(W_{hj} > W_{ij}, i = 1,2,...,n_j, i \neq h)
\]

(3)

where \( n_j \) = number of runners in race \( j \). Consequently,

\[
p_{hj} = \Pr(\ln(p_{hj}^n) + \varepsilon_{hj} > \ln(p_{ij}^n) + \varepsilon_{ij}, i = 1,2,...,n_j, i \neq h)
\]

(4)

The \( W_{ij} \) cannot be observed directly. However, whether horse \( i \) wins race \( j \) can be observed and a win/lose variable \( t_{ij} \) is defined such that:
\[ t_{ij} = 1 \text{ if } W_{ij} = \text{Max} (W_{i1}, W_{i2}, \ldots, W_{ijn}) ; \quad t_{ij} = 0 \text{ otherwise.} \]

Consequently, the probability of horse \( h \) winning race \( j \) given in equation (4) can be represented as follows:

\[ p_{hj} = \Pr(t_{hj} = 1 | \ln(p^n_{ih}), i = 1,2,\ldots,n_j) \]  

(5)

McFadden (1974) demonstrates that, if it is assumed the error terms \( \varepsilon_{ij} \) in equation (4) are independent and distributed according to the double exponential distribution, the probability of horse \( i \) winning race \( j \) is given by a conditional logit function, as follows:

\[ p_{ij} = \frac{\exp(\alpha \ln(p^n_{ij}))}{\sum_{i=1}^{n_j} \exp(\alpha \ln(p^n_{ij}))} \]  

(6)

where \( \alpha \) is a parameter which measures the importance of normalized probability derived from odds in determining the likelihood of horse \( i \) winning race \( j \).

Equation (6) simplifies to:

\[ p_{ij} = \frac{\exp\left((p^n_{ij})^\alpha\right)}{\sum_{i=1}^{n_j} \exp\left((p^n_{ij})^\alpha\right)} \]  

(7)

The conditional logit model has been successfully employed for a range of discrete choice problems (McFadden, 1974) including a number of studies estimating the probability of success of racehorses (e.g. Figlewski, 1979; Bolton & Chapman, 1986; Edelman, 2003). The parameter \( \alpha \), is estimated by maximizing the joint probability of observing the results of all \( J \) races in the sample. Consequently, the value of the following log-likelihood (LL) function is maximized:

\[ LL = \sum_{i=1}^{J} \sum_{j=1}^{n_i} t_{ij} \ln p_{ij} \]  

(8)

If the estimated value of \( \alpha \) in equation (7) is greater than one then this will demonstrate the traditional favorite/longshot bias; i.e. the favorites/longshots odds are generally under/over estimated. In order to test the hypothesis that abnormal returns are achievable using...
information contained in final odds the model represented by equation (7) is estimated using a test sample of races. The model is then employed to estimate winning probabilities for a holdout sample. These probabilities are used as inputs to a betting strategy for the holdout sample, based on maximizing the expected log payoff (Kelly, 1956), and the returns are analyzed. A positive return indicates that abnormal returns can be made by simply employing historical price information.

The Kelly strategy requires that a fraction $f_j(i)$ of current wealth is bet on horse $i$ in race $j$. Let $f_j = (f_j(1), \ldots, f_j(k))$ be the total fraction of wealth bet on race $j$. Betting fraction $f_j$, if horse $x$ wins race $j$ then current wealth will increase by a factor of $1 - \sum_{i=1}^{k} f_j(i) + f_j(x)(O_{sj} + 1)$. The Kelly strategy consists of choosing $f_j$ to maximize the expected log payoff, $F(f_j)$ where

$$F(f_j) = \sum_{x=1}^{k} p_{sj} \log\left(f_j(x)(O_{sj} + 1) + 1 - \sum_{i=1}^{k} f_j(i) \right). \tag{9}$$

This betting strategy has been shown to be asymptotically optimal by Breiman (1961), in the sense that it maximizes the asymptotic rate of growth for wealth, with 0 probability of ruin. Using the Kelly criterion, total wealth grows at an exponential rate, though the standard deviation remains proportional to total wealth and thus also grows exponentially. This strategy gives zero probability of ruin if arbitrarily small bets are allowed. The strategy also identifies which races to bet on, as well as how much to bet on each horse. The strategy therefore allows bets on more than one horse in a race, though bets are usually restricted to horses that give a positive expected return. Employing the Kelly strategy means that potentially attractive wagers are weighted more heavily.

A conditional logit model is developed with the $p_{sj}$ as inputs from 3648 races (40,514 runners) which were run between 16 June 1999 and 15 May 2000. Winning probabilities are estimated using this model for the holdout sample of races run between 16 May 2000 and 13 August 2000 (i.e.1910 races, 20054 runners). These probabilities are used as inputs to
implement a Kelly betting strategy for the holdout sample races. If positive returns are identified using the Kelly betting strategy this would imply that the market is not weak form efficient.

V. Results

Expected rates of return

The expected rate of return (equation (1)) and the cumulative rate of return for each odds category resulting from betting at level stakes on every horse in the 5558 races in the database between 16 June 1999 and 13 August 2000, are plotted in figures 1 and 2 respectively.

Figure 1 clearly demonstrates that there are only a few odds categories (all with odds less than 3.33/1) where positive rates of return are possible, and it is clear from figure 2 that the cumulative rate of return is negative at all odds levels (other than for one horse in the sample with odds less than 0.04/1). The results reported in figure 2 also show that there is a tendency for the cumulative rate of return to decrease as odds increase. This confirms the existence of the traditional favorite/longshot bias in the UK bookmaker based horserace betting market.

Conditional logit model

The results of estimating a conditional logit model with a single predictor variable, (log of normalized odds probability) using data from 3648 races run between 16 June 1999 and 15 May 2000, are given in table 1. The estimated coefficient of the predictor variable is highly significant (t= 47.81, p< 0.0001) suggesting, as expected, that market prices are an important indicator of which horse will win a race. This is confirmed by the highly significant LL ratio statistic for the model (LL ratio=11,444.85, \( \chi^2_{1}(0.01) = 6.64 \)) and the adjusted R-square statistic, which suggests that log of normalized odds explains 44.86% of the variance in the outcome of races. A null hypothesis that the coefficient of log of normalized odds equals 1 is rejected using a t test (t=9.18, p< 0.0001), confirming the results of the expected returns analysis, that the odds of longshots/favorites overstate/understate their chances of success.
**Kelly wagering strategy**

The estimated conditional logit model (represented by equation (7)) is then used to predict winning probabilities for horses running in races in the ‘out of sample period’, between 16 May and 13 August 2000. These winning probabilities are used, together with the horses’ odds, as inputs to a Kelly wagering strategy. This strategy suggests the fraction of the current wealth which should be bet on each horse. Such a strategy leads to bets in 492 of the 1910 holdout races (565 separate bets) and positive returns are obtained in 224 races. A conservative Kelly strategy is adopted, whereby the level of wealth is not updated after each race; this approach is recommended by Hausch and Ziemba (1990) and Leong and Lim (1994), since it does not jeopardize previous winnings to the same extent as a full Kelly strategy. An initial wealth of $1000 is assumed and bets totaling $14050 are recommended, leading to a net profit of $360; a positive return of 2.58 per cent. This strategy is compared with a number of naïve strategies, namely betting $1 on each horse in the holdout races, betting enough on each horse in the holdout races to return $1 (to help counter the favorite/longshot bias) and to bet 1$ on the favorite of each of the holdout races (to benefit from the favorite/longshot bias). The results of applying these various betting strategies are given in table 2 and clearly indicate that none of the naïve betting strategies yield positive returns. However, the Kelly strategy, incorporating winning probabilities derived from the estimated conditional logit model, does enable a positive net profit to be earned; this is achieved despite a large bookmaker profit margin of 31.3 per cent. This result suggests that the UK bookmaker based betting market is not weak form efficient. This conclusion is in contrast to that arrived at using the method which has usually been employed to test for weak form market efficiency, namely the level stakes, expected returns based approach represented in figures 1 and 2.

Table 2 about here

The Kelly strategy can recommend that large proportions of one’s current wealth should be wagered on horses with high predicted win probabilities. However, the conditional logit model
used to estimate these probabilities is based on data which contains relatively few cases of horses with very low odds (and hence high predicted win probabilities). The chances of estimation errors in this tail of the distribution are therefore high. In order to counter this problem it is sensible to restrict the proportion of one’s wealth bet on low odds horses. A grid search is conducted in the out-of-sample period (16 May-13 Aug 2000) to investigate the impact of a restricted Kelly strategy of this sort on the overall levels of profitability. The maximum fraction of one’s wealth which is permitted on any bet is varied between 0.01 and 0.1 and the maximum odds at which the restriction applies is varied from less than or equal to 0.5/1 to less than or equal to 4/1. A conservative Kelly strategy is adopted such that the wealth level is not updated after each bet. The results are displayed in table 3 and clearly indicate that positive rates of return are obtainable for each of the odds/fraction of wealth restriction categories. An optimal return of 8.66 per cent is obtained if, for horses with odds of 2/1 or less, we restrict the maximum percentage of our wealth on a single bet to 1 per cent.

Hausch, Ziemba and Rubenstein (1981) recommend that bets using the Kelly criterion should be restricted to those where the expected return from a given bet exceeds a pre-selected minimum level. This approach reduces the chance of inaccuracies in the predicted probabilities leading to bets on horses where the true expected return is negative. In order to explore the impact of applying such a strategy in the out of sample period (16 May–13 August 2000), the maximum fraction of bankroll invested on short odds horses was limited to the optimal value indicated in table 3 (i.e. a maximum of 1 percent of bankroll on horses with odds less than or equal to 2/1) and the minimum expected profit level on a bet was varied from 0 to 7 per cent; beyond this point the number of bets recommended was too small for meaningful conclusions to be drawn. Once again a conservative Kelly criterion was adopted, whereby the initial bankroll of $1000 is not updated after each bet. The results of this analysis are provided in table 4 and indicate that limiting bets to those where the expected profit exceeds some positive level generally increases the expected rate of return. In particular, when the minimum expected
profit level per bet is set at 7 per cent an impressive 44.69 per cent rate of return is achievable over the 3 month out of sample period.

Table 4 about here

The above results are obtained without reinvestment of winnings. To identify the extent to which the market inefficiency could be exploited to maximize the long run growth rate of wealth, a Kelly strategy with reinvestment of winnings is explored. Once more the conditional logit model (represented by eq. (7)), developed for the training period (16 June 1999-15 May 2000), is used to predict winning probabilities for horses running in races between 16 May and 13 August 2000. An initial wealth level of $1 is updated after each bet and once again bet sizes are restricted on horses with odds of less than or equal to 2/1 (i.e. to a maximum of 1 per cent of wealth). The log of cumulative wealth arising from the application of this strategy over the 3 month out-of-sample period, for a variety of minimum levels of expected profit per bet, are shown in table 4. With no restriction on the minimum expected profit per bet the log of cumulative wealth increases from 0 (initial wealth of $1) to 0.38, indicating an increase in wealth of about 46 per cent (see figure 3). This substantial increase in wealth results from bets on 492 of the 1,910 races in the out of sample period. If, when applying the Kelly strategy, bets are restricted to those where the expected profit level exceeds 7 per cent, log of cumulative wealth increases to 0.297; representing an increase in wealth of 35 per cent (see figure 4). This is achieved from only 46 bets, whereas the 46 per cent increase in wealth obtained without any restriction on the expected profit per bet results from 554 bets. To permit a clearer comparison between strategies involving different minimum expected profit levels per bet, the cumulative wealth levels are divided by the number of bets over which they are achieved. A comparison of these ratios, shown in table 4, clearly indicates the value of enforcing a minimum expected profit per bet. The ratio increases as higher minimum expected profit levels are applied, from 0.0026 where no minimum expected profit level is applied to 0.0293 when a 7 per cent
minimum expected profit per bet threshold is applied; suggesting that a given target wealth level can be achieved in fewer bets the greater the minimum profit per level which is imposed.

Figure 3 and figure 4 about here

VI. Discussion

Taken together, the results concerning expected returns from level stake betting at each odds level (see figures 1 and 2) and the value of the coefficient of log of normalized odds probabilities in the conditional logit model (estimated for races run between 16 June 1999 and 15 May 2000: see table 1), confirm the findings of previous studies, which have pointed to the existence of the traditional favorite/longshot bias (e.g. Ali, 1977; Ziemba and Hausch, 1986; Vaughan-Williams and Paton, 1997; Levitt, 2004; Snowberg and Wolfers, 2005). To exploit any inherent inefficiency caused by the favorite/longshot bias it might be suggested that one could restrict bets to horses whose odds are less than a given value. However, the cumulative returns (to level stake betting) results, presented in figure 2, suggest that abnormal returns are not achievable by restricting bets in this manner. Previous studies which have examined weak form efficiency have pursued this line of enquiry and have reached the same conclusion (e.g. Gander, Zuber and Johnson, 2001; Snowberg and Wolfers, 2005). In addition, other simple means of accounting for the bias, such as betting only on favorites or betting enough on each horse to return a $1 (hence restricting the size of bets on longshots) improve the expected return over a simple strategy of betting level stakes on each horse (see table 2). However, these strategies still do not enable net profits to be achieved.

The estimation of a conditional logit model with log of normalized odds probabilities as a single independent variable for the period 16 June 1999 to 15 May 2000 permits us to predict winning probabilities for races run in the test period, 16 May to 13 August 2000. The model estimation enables a subtle account to be taken of any favorite/longshot bias in the underlying odds; and this is removed from the predicted probabilities. To capitalize fully on the information contained within the predicted probabilities, together with information concerning
the favorite/longshot bias in the underlying odds, a Kelly wagering strategy (Kelly 1956) is adopted. As indicated above, this strategy weighs more attractive wagers more heavily and the results reported in table 2 show that a cautious Kelly strategy (without reinvestment of wealth) leads to positive expected returns in the out of sample period (2.58 per cent). Applying restrictions to the proportion of wealth to bet on low odds horses further enhances these returns (8.66 per cent; see table 3). Furthermore, if bets are only made when the expected return on a given bet exceeds 7 per cent, the profits increase to 44.69 per cent. Applying a full Kelly wagering strategy with reinvestment, but restricting the maximum proportion of wealth to invest on short odds horses and only placing bets with an expected return greater than 7 per cent, enables cumulative wealth to increase by 35 per cent in less than fifty bets. Overall these results provide strong evidence that abnormal returns can be earned by effectively employing information contained in final odds in UK bookmaker markets.

One of the key questions arising from the results of the current study is why they contradict the consensus to emerge from the literature over the last 40 years; namely, that the horserace betting market is weak form efficient. It is suggested that the answers fall into two broad categories: differences in methodology and differences in market ecology.

The methodology employed here enables good predictions of winning probabilities to be obtained; the conditional logit model generating these predictions having first accounted for any inherent favorite/longshot bias. The Kelly wagering strategy enables full use to be made of the predicted probabilities together with the underlying biases in prevailing odds. This strategy, designed to maximize the long run growth rate of wealth, weighs attractive wagers more heavily than unattractive wagers. On the other hand, previous papers exploring weak form efficiency have largely relied on identifying groups of odds where level stakes betting produces net profits (e.g. Ziemba and Hausch, 1986). Not surprisingly, these have failed to identify opportunities for earning abnormal returns. In fact, when mirroring this methodology, the current study also fails to find an odds level below which positive net profits can be achieved.
In addition to the methodology, the nature of the betting market examined may well have a significant influence on the results. Two particular features of the market examined are highlighted: first, the fact it is a bookmaker not a pari-mutuel market and second that betting volumes in this market are moderate in comparison to markets which have been the focus of most weak form efficiency studies.

The vast majority of existing weak form studies has been conducted in pari-mutuel markets (e.g. Ali, 1977; Asch, Malkiel and Quandt, 1982; McGlothlin, 1956; Snyder, 1978; Thaler and Ziema, 1988; Ziema and Hausch, 1986; Gander, Zuber and Johnson, 2001). Within these markets the favorite/longshot bias can only arise from the behavior of bettors, whether this be caused by their risk-loving preferences (Quandt, 1986; Rossett, 1965), their tendency to discount only a proportion of their losses (Henery, 1985), their lack of relevant information (Sobell and Raines, 2003) or from their non-financial motivations (Thaler and Ziema, 1988). Within bookmaker markets the favorite/longshot bias can arise both from the demand and supply side. Consequently, in addition to the factors which can lead to biased behavior on the part of bettors, bookmakers’ behavior may exacerbate the bias. It has been suggested (Shin 1991, 1992, 1993) that bookmakers, fearing they may be faced by some insiders who have access to privileged information concerning a horse’s prospects of success, artificially reduce the odds on longshots. They do this, since a relatively small bet on a longshot could expose the bookmaker to significant losses. This problem is of particular concern for bookmakers since bookmaker markets are those most likely to attract insiders, since their bets will be settled at the odds prevailing in the market at the time the bet is struck. As a result, their return will not be eroded by subsequent betting ‘plunges’ on the horse by those who seek to capitalize on others’ insider knowledge (Law and Peel, 2002). Shin (1991, 1992, 1993) argues that the process by which bookmakers minimize their exposure to large potential losses from insiders leads to them artificially shortening the odds on longshots. However, as Tuckwell (1983) observes, bookmakers also value revenue to help balance their books, and in an effort to
stimulate revenue from those without access to inside information they may artificially increase the odds available on favorites. Vaughan-Williams and Paton (1997) offer support for the view that the favorite/longshot bias is exaggerated in this manner in bookmaker markets and Bruce and Johnson (2005) demonstrate that within the UK market, where bookmaker and pari-mutuel markets co-exist, the favorite/longshot bias is significantly more pronounced in the former.

The second feature of the market examined in the current study, which can provide some explanation for the results, relates to betting volumes. The UK on-course betting market involves relatively small betting volumes compared to markets such as Hong Kong and the USA, where the presence of professional bettors is well established (Benter, 1994) and where the majority of market efficiency studies have been conducted. For example, the mean on-course betting volume in 1999/2000 in the UK was approximately £55,000 ($87,571) per race with bookmakers and £13,000 ($20699) with the pari-mutuel operator4. This contrasts with a per race betting volume during the same period of $127.5 HK million ($16.4 million) in Hong Kong and $254,627 in the USA with the monopoly pari-mutuel operators.

One of the reasons for the betting volume disparities between the UK and other countries is that in the UK (unlike Hong Kong and the USA) betting is permitted with bookmakers in off-track betting offices. This market is relatively large compared with the on-track market, with mean per race betting volumes of approximately £687,500 ($1.09 million). However, the declared odds for a race are solely determined by betting activity at the track. Professional bettors are not generally attracted to the off-course market since average bet sizes are considerably smaller than those placed at the track. As a result, professional bettors are too easily detected and their bets refused or severely limited in size by the off-track bookmakers. In addition, the majority (80 per cent) of off-course bettors elect to have their bets settled at the final odds prevailing in the market. This strategy is not favored by professional bettors who fear erosion of their returns as plunges can develop on horses which are perceived to have attracted informed bets. Consequently, whilst the professional may desire to bet at the
prevailing odds, such a strategy is less common amongst off-course betting office clientele and it would lead to the professionals being easily spotted by off-course bookmakers and their bets being refused or restricted. Professional bettors in the UK are therefore more likely to be attracted to the on-course market, but the low betting volumes here also deter their activities. It is clear that bets from individual bettors in the US and Hong Kong markets are likely to have less impact on the final odds than bets of equivalent size in the UK market. Consequently, professional bettors who wish to recoup research costs and the cost of sophisticated procedures to profit from market inefficiencies in real time are more likely to be attracted to markets where substantial bets can be placed without driving the final odds to levels where profits cannot be achieved (Busche and Walls, 2000). Professional bettors are likely to drive out inefficiencies in large volume markets and it is no surprise that the favorite/longshot bias is absent in the very large betting volume markets, such as that in Hong Kong (e.g. Busche 1994; Busche and Hall 1988; Busche and Walls, 2000). Consequently, the relatively low on-course betting volume market in the UK is significantly less attractive to professional bettors and it is their absence which may permit the inefficiency identified here to persist.

It appears, therefore, that a combination of differences in market ecology and differences in the methodology employed to test for weak form efficiency may explain the disparity between the results of the current study and those previously conducted. The clear evidence which emerges from the application of a methodology designed to make full use of the information contained in final odds is that the UK bookmaker betting market is not weak form efficient.

**VII. Conclusion**

This paper investigates the degree of weak form efficiency in the horserace betting market. The motivation for the study lies in the observation that previous studies addressing this issue have largely failed to employ methodologies which enable the full degree of information contained in final odds to be exploited. In addition, previous weak form studies have mainly been conducted in markets with features which are likely to create conditions conducive to
weak form efficiency (i.e. large betting volume, pari-mutuel markets). Specifically, the paper explores whether predicted win probabilities, derived from a conditional logit model incorporating final odds drawn from the UK bookmaker betting market, can be used, with an appropriate wagering strategy, to earn abnormal returns. The results of the analysis demonstrate that using techniques normally employed to test for weak form efficiency in betting markets, abnormal returns cannot be earned. However, employing the methodology outlined above for predicting win probabilities in future races, based on the final odds in bookmaker markets in the UK, it is possible to earn abnormal returns. In fact, if appropriate restrictions are placed on the wagering strategy (to offset any possible inaccuracies in the predicted probabilities), returns of nearly 45 per cent are achievable without reinvestment of winnings in a 3 month period. When reinvestment of winnings is permitted, rates of growth of cumulative wealth in excess of 2.9 per cent per bet are achievable. These results provide strong evidence that the UK bookmaker horserace betting market is not weak form efficient.

The contrast between the results of the current study and that of previous explorations of weak form efficiency in racetrack betting markets is explained in terms of differences in the methodology employed and in the markets which form the focus of the studies. It is argued that the methodology employed here enables fuller account to be taken of the information contained in final odds. Furthermore, it is demonstrated that the wagering strategy selected can have a significant impact on the returns. This suggests that the wagering strategy should be an important feature of future investigations of weak form efficiency.

Differences between the results of the current study and those obtained in earlier investigations are also explained in terms of variations in the nature of the markets examined. In particular, it is argued that the favorite/longshot bias is likely to be larger in bookmaker markets than pari-mutuel markets (the focus of the majority of previous weak form studies) and that this can be exploited to earn abnormal returns. In addition, the markets which many previous studies have investigated (e.g. USA and Hong Kong) involve significantly greater on-
course betting volumes than UK markets. It is argued that these large volume markets are more likely to attract professional bettors, who drive out market inefficiency.

The importance of the results reported here goes beyond an enhanced understanding of the characteristics and processes of betting markets. We specifically point to the similarities between betting markets and wider financial markets. Consequently, in identifying weak form efficiency in betting markets, the current study has two important implications for weak form efficiency studies conducted in wider financial markets: firstly, that particular attention needs to be paid to ensure that the methodology employed makes full use of the data contained in asset prices, including the investment strategy employed. Secondly, that differences in market ecology may have an important impact on the degree of market efficiency. These conclusions may provide a basis for further work of this type in wider financial markets.

References


Press, 65-78.


Endnotes

1. The only exceptions to the prevalence of the traditional favorite/longshot bias appear to be high betting volume markets in Hong Kong and Japan (Busche, 1994; Busche and Hall, 1988; Busche and Walls, 2000) and for a small racetrack in the USA (Swidler and Shaw, 1995).

2. The favorite-longshot bias was found for bets placed close to the opening of the market but less so for bets placed near to the close of the market.

3. The precise distribution of $\varepsilon_g$ is open to question, and in circumstances involving binary choice, there is little theoretical justification for choosing one formulation over another (Judge, Garrett and Griffiths, 1985). However, there is a large number of independent factors which are likely to influence the result of a horserace and, consequently, the central limit theorem would suggest that the errors are likely to be normally distributed. This assumption would lead to a multinomial generalization of the probit model. However, this model requires non-linear estimation and is computationally difficult. The double exponential distribution is a good approximation to the normal distribution, except at the extreme tails (Judge, Garrett and Griffiths, 1985). In addition, it is computationally more appealing and logit based models have been demonstrated to produce similar predictions to probit based models (Maddala, 1993). Consequently, it is assumed that the $\varepsilon_g$ are distributed according to the double exponential distribution.

The resulting conditional logit model predicts the probability of a given horse winning a race, but it is only possible to compare this probability with the race result (i.e. horse wins or loses). Consequently, as Cranmer (1991) observes: ‘even if we have residuals … they can not be used for an estimate of the unexplained disturbances, because there is no such thing in
probability models……it also explains why there are no analogous (measures of error fit) to the linear regression measures.’ However, this does not cause concern since, as indicated above, the conditional logit model has been successfully employed in a number of horseracing studies, and there is no reason to doubt that it is not applicable to the sample employed in this study.

4. Figures obtained from the Horserace Betting Levy Board, the 1999-2000 Annual Report of the Totalisator Board and estimates from Ladbrokes, the UK’s largest bookmaking organization. Dollar equivalent values given as at 31st March 2000.

5. Data obtained from the Hong Kong Jockey Club. Dollar equivalent values given as at 31st March 2000.

Figure 1: Rate of return from level stakes bets at given odds
Figure 2: Cumulative rate of return from level stakes bets at odds less than or equal particular values
Figure 3: Log of cumulative wealth resulting from the application of a Kelly wagering strategy with reinvestment for the holdout races (16 May – 13 August 2000); with the restriction that the maximum fraction to bet on horses with odds $\leq 2/1$ is 1%, but with no restriction concerning the minimum profit level required per bet.
Figure 4: Log of cumulative wealth resulting from the application of a Kelly wagering strategy with reinvestment for the holdout races (16 May – 13 August 2000); with the restriction that the maximum fraction to bet on horses with odds ≤ 2/1 is 1% and each individual bet has an expected profit of greater than 7%.
Table 1: Coefficient and test statistics of the conditional logit model with predictor variable, log of normalized odds probability; estimated using 3,648 races run between 16 June 1999 and 15 May 2000 (40,514 runners).

<table>
<thead>
<tr>
<th>Explanatory variable</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log of normalized odds probability</td>
<td>1.2377</td>
<td>0.0259</td>
<td>47.81</td>
</tr>
</tbody>
</table>

Summary statistics

\[
\begin{align*}
L(\theta = 0) & \quad L(\theta = \hat{\theta}) & \quad LL \text{ ratio statistic} & \quad \text{Adj} R^2 \\
-12,755.26 & \quad -7,032.84 & \quad 11,444.85 & \quad 0.4486
\end{align*}
\]
Table 2: A comparison of returns from different wagering strategies on the 1,910 races in the holdout period (16 May-13 August 2000).

<table>
<thead>
<tr>
<th>Wagering strategy</th>
<th>No. of bets</th>
<th>No. of races bet</th>
<th>No. of races with profit</th>
<th>Amount bet ($)</th>
<th>Profits ($)</th>
<th>Rate of return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 bet on each horse</td>
<td>20,054</td>
<td>1,910</td>
<td>313</td>
<td>20,054</td>
<td>-6,282</td>
<td>-31.3</td>
</tr>
<tr>
<td>Return $1 on each horse if horse wins</td>
<td>20,054</td>
<td>1,910</td>
<td>0</td>
<td>2,326</td>
<td>-415.96</td>
<td>-17.9</td>
</tr>
<tr>
<td>$1 bet on favorite in each race¹</td>
<td>2,081</td>
<td>1,910</td>
<td>673</td>
<td>2,081</td>
<td>-97.08</td>
<td>-4.67</td>
</tr>
<tr>
<td>Kelly strategy²</td>
<td>565</td>
<td>492</td>
<td>224</td>
<td>14,050</td>
<td>360</td>
<td>2.58</td>
</tr>
</tbody>
</table>

¹ Where a race has joint favorites, $1 is bet on each favorite.
² A Kelly strategy can involve betting on more than one horse in a race.
Table 3: Rates of return using a Kelly wagering strategy for horses in the holdout period (16 May to 13 August 2000) with restrictions on the maximum proportion to bet on low odds horses

<table>
<thead>
<tr>
<th>≤SP Max(f)</th>
<th>≤0.5/1</th>
<th>≤1/1</th>
<th>≤2/1</th>
<th>≤3/1</th>
<th>≤4/1</th>
<th>Mean return for each fraction level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>6.85</td>
<td>5.36</td>
<td>8.66</td>
<td>4.04</td>
<td>4.34</td>
<td>5.85</td>
</tr>
<tr>
<td>0.03</td>
<td>5.87</td>
<td>4.71</td>
<td>6.93</td>
<td>6.23</td>
<td>6.34</td>
<td>6.02</td>
</tr>
<tr>
<td>0.05</td>
<td>4.93</td>
<td>4.33</td>
<td>7.07</td>
<td>7.08</td>
<td>7.11</td>
<td>6.10</td>
</tr>
<tr>
<td>0.07</td>
<td>3.96</td>
<td>3.73</td>
<td>5.64</td>
<td>5.65</td>
<td>5.65</td>
<td>4.93</td>
</tr>
<tr>
<td>0.10</td>
<td>2.92</td>
<td>2.61</td>
<td>3.82</td>
<td>3.82</td>
<td>3.82</td>
<td>3.40</td>
</tr>
<tr>
<td>Mean return for each odds level</td>
<td>4.91</td>
<td>4.15</td>
<td>6.42</td>
<td>5.36</td>
<td>5.45</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Results for a restricted Kelly strategy; a maximum of 1% of bankroll (fixed at $1000) is bet on horses with odds less than or equal to 2/1 and bets are only placed on horses with expected profit greater than or equal to a given level

<table>
<thead>
<tr>
<th>Expected profit (%)</th>
<th>No. of bets</th>
<th>No. of races bet</th>
<th>No. of races won</th>
<th>Amount bet ($)</th>
<th>Profits ($)</th>
<th>Rate of return (%)</th>
<th>Ln( Cum. wealth)</th>
<th>Cum. wealth/ No. bets</th>
</tr>
</thead>
<tbody>
<tr>
<td>No restriction</td>
<td>554</td>
<td>492</td>
<td>224</td>
<td>5060</td>
<td>440</td>
<td>8.66</td>
<td>0.378</td>
<td>0.0026</td>
</tr>
<tr>
<td>≥1.0</td>
<td>397</td>
<td>377</td>
<td>169</td>
<td>4350</td>
<td>410</td>
<td>9.41</td>
<td>0.346</td>
<td>0.0036</td>
</tr>
<tr>
<td>≥2.0</td>
<td>291</td>
<td>277</td>
<td>126</td>
<td>3510</td>
<td>380</td>
<td>10.76</td>
<td>0.320</td>
<td>0.0047</td>
</tr>
<tr>
<td>≥3.0</td>
<td>206</td>
<td>200</td>
<td>89</td>
<td>2660</td>
<td>240</td>
<td>8.92</td>
<td>0.186</td>
<td>0.0058</td>
</tr>
<tr>
<td>≥4.0</td>
<td>142</td>
<td>137</td>
<td>65</td>
<td>1930</td>
<td>400</td>
<td>20.69</td>
<td>0.358</td>
<td>0.0101</td>
</tr>
<tr>
<td>≥5.0</td>
<td>90</td>
<td>88</td>
<td>43</td>
<td>1310</td>
<td>350</td>
<td>26.43</td>
<td>0.312</td>
<td>0.0152</td>
</tr>
<tr>
<td>≥7.0</td>
<td>46</td>
<td>45</td>
<td>24</td>
<td>720</td>
<td>320</td>
<td>44.69</td>
<td>0.297</td>
<td>0.0293</td>
</tr>
</tbody>
</table>