Analysis of Sequential Conversions of Convertible Bonds: A Recurrent Survival Approach

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Abstract

When the capital structure for a firm has straight bonds in addition to stocks and convertible bonds (CBs), the optimal conversion strategy can be different from a block conversion as often assumed in the literature. Instead, from empirical and theoretical perspectives, conversions take places sequentially over the lifespan of a bond is more prevalent. As an early conversion will weaken the risk-mitigating effect of CBs as well as change the firm’s capital structure, thus the prices of the shares and the CBs will depend the path of these sequential conversions. This study develops an empirical model for the instantaneous rates of conversions to study the time-path of conversions sequentially over the lifespan of a CB. A recurrent survival analysis technique is used to estimate the time-varying instantaneous rates of conversions with a dataset that has longitudinal records of the timings of sequential conversions for a set of 128 convertible bonds listed on the Taiwan Stock Exchange outstanding from January 2004 to December 2009. Our analysis shows that dividend yield, percentages spread between the equity and conversion prices, and total asset values have positive effects on the instantaneous rates of conversions, while the credit rating of category 9th, current ratio, and long-term debt ratio have negative effects. The estimated instantaneous rates of conversions can be incorporated into the risky discount rates in a reduced-form pricing model and a more relevant pricing framework for the CBs can be developed.

Keywords. Block Conversion, Convertible Bond, Sequential Conversion, Recurrent Survival Analysis, Instantaneous Rates of Conversions.
1. Introduction

Convertible bonds (CBs), embedded with warrants, are often considered as a sequential financing vehicle to mitigate the asset substitution problem that curbs shareholder incentives for risk (Green, 1984; Mayers, 1998, 2000). Nevertheless, the risk-mitigating effect can be mitigated when the bondholders decide to convert the bonds strategically either before the maturity, the dividend payment dates, or a call (Francois et al., 2006). This is due to the fact that early conversion might rally the initial shareholders to restore incentives towards risk and transfer wealth at the expense of straight bondholders. The degree of CBs’ risk-mitigating effect will depend on the path these early conversions occur. Not only that, the firm’s capital structure and thus the values of the CBs remained outstanding will also be affected.

In literature, two types of CBs’ conversion paths are addressed. The first is the block conversion under which all the CBs are completely converted at the same time or not at all; and the second is the sequential conversion under which conversions are taking places sequentially over a bond’s lifespan. For price-taking competitive bondholders, if the firm has only stocks and CBs in the capital structure, the sequential conversion strategy is not optimal and the traditionally adopted block conversion assumption is justified (Constantinides and Rosenthal, 1984; Constantinides, 1984; Cox and Rubinstein, 1985; Spatt and Sterbenz, 1988). On the other hand, sequential conversions can be optimal under the following two situations: 1. There is a monopoly bondholder (Emanuel, 1983); or 2. A firm has straight bonds outstanding in addition to stocks and CBs in the capital structure (Bühler and Koziol, 2004; Koziol, 2006). Nevertheless, the literature often adopted a simplifying assumption of a block conversion for the path of CB conversions (eg., Ingersoll, 1977; Brennan and Schwartz, 1977, 1980; McConnell and Schwartz, 1986; Goldman Sachs,
However, due to market frictions, strategic early conversions will not take place in a block empirically. Instead, sequential conversions over a bond’s lifespan are often observed (Bühler and Koziol, 2002). This raises the issue of the prediction of the paths of sequential conversions over the lifecycle of an individual bond. To address the issue, this study develops an empirical model to predict the hazard rate of conversions, or equivalently, the instantaneous rate of conversions, over an individual bond’s lifespan. In our model, conversions over a bond’s lifespan are considered as recurrent events and recurrent survival analysis technique (Andersen and Gill, 1982; Prentice, Williams, and Peterson, 1981) is adopted to estimate the time-varying instantaneous rate of conversions. The prediction is based on a set of predictors including: the dividend, the percentage spread between the highest stock price and the conversion price, the credit rating categories 4 to 9, the current ratio, the operating cash flow ratio, the long-term debt ratio, the net worth, and the total asset. A dataset that contains longitudinal records of the timings of sequential conversions over the lifespan of a set of 128 convertible bonds listed on the Taiwan Stock Exchange from January 2004 to December 2009 is used. Our analysis shows that the dividend, percentage spread between the equity and conversion prices, and total asset values have positive effects on the hazard rate of a conversion, while the credit rating of category 9, current ratio, and long-term liability ratio have negative effects. To our knowledge, this is the first study to use the recurrent survival analysis technique in studying bondholders’ sequential conversion behaviors in a dynamic setting.

The paper is organized as follows: Section 2 gives the development of the empirical model of sequential conversions, and a review of the recurrent survival analysis technique is given; Section 3 gives our empirical study; and Section 4
concludes our study.

2. **Empirical Model of Instantaneous Rate of Conversions**

For simplicity, the literature often implicitly or explicitly assumes that all bond holders completely convert their bonds at the same time or not at all, i.e., the bond holders convert the bonds in a block. Ingersoll (1977) showed that the optimal timing of a block conversion is at the maturity or the call in a perfect market with no dividend payments and constant conversion terms. If dividends are paid and adverse changes in the conversion terms are not allowed, Brennan and Schwartz (1977, 1980) showed that the optimal timing for a block conversion is either immediately prior to a dividend payment date or at maturity.

Nevertheless, strategic conversions that occur before the dividend payment date or the maturity date are possible due to various frictions such as information asymmetry between bond and equity holders. Asquith and Mullins (1991) argued that when the conversion value was sufficiently high and the downside protection premium offered by the bond was sufficiently low, then the cash flow differential of dividends over interest could be great enough to engender strategic conversions that occur before the dividend payment date or maturity. In Sirbu and Shreve (2006), the conversion and the call of a bond are considered as a two-person, zero-sum game. By assuming equity owners receive dividend paid continuously over time at a rate $\delta S$, where $S$ is the total equity value and $\delta S$ is the dividend yield, Sirbu and Shreve (2006) showed that if the coupon rate was below $\delta K$, where $K$ is the firm’s call value, then a block conversion should precede a call. Conversely, if $\delta K$ is below the coupon rate, a call should precede a block conversion.

Lewis, Rogalski, and Seward (2003) derived the probability of a block conversion at maturity, which depends on the initial design features of the bond, including the
stock price $S$, the conversion price $X$, the risk-free interest rate $r$ at the time of bond’s issue, and the firm’s dividend yield $div$ the year before the bond’s issue date. This probability of conversion is derived using Black-Scholes formula as $N(d_2)$, where is the cumulative probability of a standard normal distribution, and

$$d_2 = \frac{\ln(S/X) + (r-div - \sigma^2/2)T}{\sigma\sqrt{T}}$$

(1)

where $T$ is the time to maturity, $\sigma$ is the standard deviation of the equity return. Based on the probability of a block conversion at maturity (1), Lee et al. (2009) found that firms with higher separation of control and ownership rights were prone to issue more debt-like convertibles with lower probability of a block conversion at maturity to avoid expropriation of minority shareholder wealth by controlling shareholders. Francois et al. (2006) showed that conversion had a greater likelihood to occur when the timing of an investment opportunity was close, thus the CB holders could collude with the shareholders to shift the risk. Constantinides (1984) was one of the few studies that considered the paths of sequential conversions, which depend on the dividends and the risk-free interest rate.

This study consider the following $q=14$ time-dependent covariates: (i) The dividend yield ($X_1$); (ii) The percentage of spread between the highest stock price and the conversion price ($X_2$); (iii) The dummies of credit rating categories 4-9 ($X_3, X_4, X_5, X_6, X_7, X_8$); (iv) The risk-free interest rate ($X_9$); (v) The current ratio ($X_{10}$); (vi) The operating cash flow ratio ($X_{11}$); (vii) The long-term debt ratio ($X_{12}$); (viii) The net worth ($X_{13}$); (ix) Total asset ($X_{14}$). Note the coupon rates of the sample bonds are all zeros and thus are not included.

The development of an empirical model for the time-varying instantaneous rate of conversions over the lifespan of a bond based on the aforementioned 14 covariates is given in the following. The model is estimated using the recurrent survival analysis technique, which is reviewed below.
2.1 Recurrent Survival Analysis Technique

The traditional Cox proportional hazard model (Cox, 1972) failed to cope with the problems to estimate the hazard rate or the instantaneous rate of conversions that occur recurrently over time. Instead, an extension of Cox proportional hazard model, namely, the recurrent survival analysis technique is considered in this study (Prentice et al., 1981; Andersen and Gill, 1982; Wei et al., 1989, 1997; Clayton, 1994; Kelly and Lim, 2000; Cook and Lawless, 2002; Duchateau et. al., 2003; Box-Steffensmeier et al., 2006).

Depending on the ways events’ categories and dependency within an individual are modeled, two types of recurrent survival analysis were considered in literature, they are the Andersen-Gill (AG) recurrent survival model (Andersen and Gill, 1982) and the conditional risk set model (PWP) by Prentice, Williams, Peterson (1981). The AG model is based on a non-homogeneous Poisson counting process, in which all the recurrent events within an individual are treated as identical and independent. For this reason, the order of the recurrent events is not taken into consideration in the AG model and the advantage lies in its efficiency and precision to give the most reliable estimates of covariate effects (Therneau and Grambsch, 2000). In the AG model, the instantaneous rate of experiencing the \( k^{th} \) conversion at time \( t \) for the \( i^{th} \) bond, \( 1 \leq i \leq N \), is a parametric function \( \lambda_{ik}(t) \) in the form

\[
\lambda_{ik}(t) = \lambda_0(t) \exp\left[ \beta_1 X_{i1}(t) + \beta_2 X_{i2}(t) + \ldots + \beta_q X_{iq}(t) \right]
\]  

(2)

where \( \lambda_0(\cdot) \) is the baseline hazard function, \( q \) is the number of covariates. The parametric function (2) specifies the time-\( t \) relation between the instantaneous rate \( \dot{\lambda}_{ik}(t) \) of the \( k^{th} \) conversion and the \( q \) predictors. The corresponding partial likelihood can be expressed as
where $\beta=(\beta_1, \beta_2, \ldots, \beta_q)$ are the coefficients of the $q$ covariates in (2), $T_{ik}$ is the time of the $k^{th}$ conversion of the $i^{th}$ bond, $1 \leq k \leq K_i$, while $K_i$ is the number of conversions for the $i^{th}$ bond and $T_{i,k,i+1}$ is the termination time of the $i^{th}$ bond, $1 \leq i \leq N$. The indicator $I(\cdot)$ is one if $T_{ik} \in (\cdot)$ and zero otherwise, while the indicator $\delta_{ik}$ equals to 1 if bond $i$ is converted at $T_{ik}$ and zero otherwise.

In contrast to the AG recurrent survival model, if the recurrent events within the same individual involve different types of categories and the ordering of the events is important, then the conditional risk set (PWP) model (Prentice, Williams, Peterson, 1981) should be adopted. In the PWP recurrent survival model, the risk set of the $k^{th}$ recurrent event are restricted to the individuals who have experienced the first $k-1$ recurrent events. In our case, a bond is not at risk for the $k^{th}$ conversion until its $(k-1)^{th}$ conversion has been occurred, only the bonds whose $(k-1)^{th}$ and $k^{th}$ conversion occurs prior to and after $T_{ik}$, respectively, are considered to be at risk, where $T_{ik}$ is the $k^{th}$ conversion time for the $i^{th}$ bond, where $1 \leq i \leq N$, $1 \leq k \leq C$. Here $C$ is the maximum number of conversions defined by

$$C=max\{K_1,\ldots,K_N\}$$

where $K_i$ is the number of conversions for the $i^{th}$ bond, $1 \leq i \leq N$. With different risk set, the baseline hazard functions are allowed to vary with $k$ for $1 \leq k \leq C$. Specifically, the instantaneous rate of experiencing the $k^{th}$ conversion at time $t$ for the $i^{th}$ bond, $1 \leq i \leq N$, is a parametric function $\lambda_{ik}(t)$ in the form

$$\lambda_{ik}(t)=\lambda_{ik}(t)\exp[\beta_1X_{i1}(t)+\beta_2X_{i2}(t)+\ldots+\beta_qX_{iq}(t)]$$

(4)

where $\lambda_{ik}(t)$ be the baseline hazard at time $t$ for the $k^{th}$ stratum, $1 \leq k \leq C$. The corresponding partial likelihood can be formulated as
3. Empirical Analysis

The estimation of the parametric functions (2) and (4) for the instantaneous rate of conversions using the AG and PWP recurrent survival techniques, respectively, is given in this Section. A total of 14 covariates $X_1 \sim X_{14}$ defined in (i)-(ix) of Section 2 are considered. The coupon rate is excluded as the coupon rates of all the sample bonds are all zeros.

3.1. Data

By excluding companies in the financial sector due to the comparability of financial indicators, a total of 170 convertible bonds on the list of Taiwan Stock Exchange that were outstanding from January 2004 and terminated before December, 2009 are eligible for the study. Note if two or more convertible bonds are issued by the same company, then only one of them is randomly selected. Among the 170 convertible bonds, 18 bond issues were traded with more shares for call, while 22 were traded with more shares for put. The bond issues that were traded with more shares for call or put are excluded from our sample. After removing them, there were 130 bonds traded mainly for conversion. Among the 130 eligible convertible bonds, observations with their differences between the highest stock price and the conversion price exceeding $200 were considered as outliers. By removing these outliers, our final sample consists of $N=128$ bonds.

The history records of the 128 sample convertible bonds during January 2004 to December 2009 were collected from Taiwan Gre-Tai Securities Market. Each record contains one or more of the following events: (1) A conversion by the bondholder; (2) The buy-back of the bonds by the firm; (3) The exercise of put options by the bondholder.
bondholders; and (4) The maturity of the bond. A total of 4521 history records were collected.

To facilitate our analysis, each record is in the counting process format (Therneau and Grambsch, 2000; Allison, 2010) given by \((id, \tau_1, \tau_2, \text{status}, X_1, X_2, X_3, \ldots, X_{14})\), where \(id\) was the identity number of the bond; \(\tau_1\) and \(\tau_2\) denoted the starting and terminating time points (monthly) of the record; \(\text{status}\) indicated whether conversion occurred at the interval specified by \([\tau_1, \tau_2]\); and \(X_1, \ldots, X_{14}\) denoted the 14 covariates, which are collected from Taiwan Economic Journal (TEJ). A total of 1646 conversion events are observed out of the 4521 records.

3.2. Results

Table 1 gives summary statistics of the 128 bonds. The time-to-maturity since issuance ranges between 10 and 60 months with an average of 35.15 months. The sizes of the bonds ranged between \(5 \times 10^4\) shares to \(1 \times 10^6\) shares at issuance. The credit ratings of the 128 bonds ranged from 4 to 9 with an average of 5.9. Here rating category “9” indicated the highest risk. Figures 1.1–1.6 illustrate the distributions of the 128 bonds’ time-to-maturity, size, credit rating, the expected probability of conversion at maturity given in (1), and the observed cumulative proportion of conversion at maturity, respectively.

The summary of the 128 bonds’ conversion history is given in Table 2. From Table 2, the earliest 1\(^{st}\) conversion among the 128 bonds takes place one month after the issuance date of the bond, while the last 1\(^{st}\) conversion takes place at the 24 months after the issuance date. In average, the 1\(^{st}\) conversion takes place at the 6.1 months after the issuance date. In Figure 2, the Kaplan-Meier estimate of the survival probabilities of the 1\(^{st}\) conversion is depicted. Figure 2 illustrates that the 1\(^{st}\) conversion takes place in less than 4 months after the issuance date among 50\% of the 128 bonds. More than 80\% of the 128 bonds have their 1\(^{st}\) conversion time before the
10th month after the issuance date. Table 2 shows that in average, the earliest time that the cumulative percentages of converted shares exceeding 25% is 14.57 months after the issuance date, while an average of 18.74 and 24.89 months are required for the cumulative percentages of converted shares exceeding 50% and 75%, respectively. Figure 2 also shows the Kaplan-Meier estimates of the survival probabilities that the cumulative percentages of converted shares exceeding 25%, 50% and 75%, respectively. Among the 128 bonds, the least number of conversions is 2 times and the largest is 25 times, with an average of 10.77 times per bond. Figure 1.4 shows the distribution of the number of conversion times for the 128 bonds.

Figure 1.5 illustrates the distribution of the expected probability of conversion at maturity given in (1) for the 128 bonds, while Figure 1.6 illustrates the observed distribution of the cumulative proportion of converted shares at the maturity. It can be seen more than 50% and 60% of the 128 bonds in Figures 1.5 and 1.6, respectively, were converted by percentages exceeding 99% of the outstanding issue.

In Table 3, the AG model is estimated. Among the aforementioned covariates, dividend ($X_1$), the percentage of spread between the highest stock price and the conversion price ($X_2$), the dummy of credit rating category 9 ($X_9$), current ratio ($X_{10}$), long-term debt ratio ($X_{12}$), and total asset ($X_{14}$) are significant at $\alpha=0.05$ level. Among the six covariates, the covariates $X_1$, $X_2$ and $X_{14}$ have positive effects on the hazard rate of conversion, while $X_9$, $X_{10}$, and $X_{12}$ have negative effects on the hazard rate of conversion. The relationships between the six covariates $X_1$, $X_2$, $X_9$, $X_{10}$, $X_{12}$ and $X_{14}$ and the predicted hazard rates by the AG model can also be found from Figures 3.1~3.6.

For the PWP model, as the maximum number of conversions within an individual bond is $C=25$, to avoid the problem of over stratification, we consider the stratification scheme that classifies observations up to the 5th conversion as the first
strata, observations after the 5\textsuperscript{th} conversion up to the 10\textsuperscript{th} conversion as the second strata, observations after the 10\textsuperscript{th} conversion up to the 15\textsuperscript{th} conversion as the third strata, observations after the 15\textsuperscript{th} conversion up to the 20\textsuperscript{th} conversion as the fourth strata, observations after the 20\textsuperscript{th} conversion as the fifth strata. In Table 4, the PWP model with five strata is estimated. The result of PWP model coincides with that of AG model except the dummy covariate $X_8$ of credit rating category 9, the other five covariates $X_1, X_2, X_{10}, X_{12}$ and $X_{14}$ are all significant at $\alpha=0.05$ level.

The result of positive dividend effect $\beta_1$ is in line with Asquith and Mullins (1991) and Sirbu and Shreve (2006). The positive $\beta_2$ of the percentage spread between the highest stock price and the conversion price coincides with those of Asquith & Mullins (1991), Lewis et al. (2003), and Sirbu and Shreve (2006). The positive effect $\beta_{14}$ of the total asset value on the hazard rate of sequential conversions might be due to the fact that assets represent value of ownership that can be converted into cash in the future and thus higher asset values increase the likelihood of conversion by the bondholders.

To the contrary, the negative effect $\beta_8$ of the dummy of credit rating category 9 might be due to the fact deteriorated credit rating represents decreasing future cash flows and thus reduce the likelihood of conversion by the bondholders. The negative effect $\beta_{10}$ of the current ratio specifies the phenomum that increased ability of paying back the short-term debt reduces the likelihood of conversions by the bondholders. At the same time, the negative effect $\beta_{12}$ of the long-term debt ratio specifies the phenomum that increased long-term debt reduces the likelihood of conversions by the bondholders. In addition to the above results, the effect of the risk-free interest rate on sequential conversion is insignificant in both the AG and the PWP models, which contradicts with the result that the path of sequential conversions depends on the risk-free interest rate by Constantinides (1984).
4. Conclusion

As the conversion strategy of CBs influences the number of outstanding stocks, their value, and the capital structure of the firm, the CBs values are dependent on the conversion strategy that is followed by the investors. Yet, the literature implicitly assumes a block conversion to evaluate the CBs due to its simplicity. This study develops an empirical model to explore the bondholders’ sequential conversion behavior so that the instantaneous rates of conversions can be predicted. Recurrent survival analysis techniques are used to estimate the model. The estimated instantaneous rates of conversions can be incorporated into the risk-discount rate of a reduced-form pricing model for CBs.
Table 1. Summary Statistics of 128 Convertible Bonds

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Min</th>
<th>Max</th>
<th>Average</th>
<th>St Dev.</th>
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<tr>
<td>Time-to-Maturity (months)</td>
<td>128</td>
<td>10.00</td>
<td>60.00</td>
<td>35.1562</td>
<td>15.7182</td>
</tr>
<tr>
<td>Number of Shares</td>
<td>128</td>
<td>5×10⁴</td>
<td>1×10⁶</td>
<td>5.3×10⁴</td>
<td>9.4×10⁵</td>
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<tr>
<td>Credit Rating at Issuance¹</td>
<td>128</td>
<td>4</td>
<td>9</td>
<td>5.9298</td>
<td>0.9896</td>
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</table>

Note. ¹ The credit rating is rated at issuance by TCRI.

Table 2. Summary of 128 Convertible Bonds’ Conversion

<table>
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<tr>
<th></th>
<th>N</th>
<th>Min</th>
<th>Max</th>
<th>Average</th>
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<tr>
<td>1st conversion time</td>
<td>128</td>
<td>1</td>
<td>24</td>
<td>6.125</td>
<td>4.934</td>
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<tr>
<td>T_{Q1}¹</td>
<td>128</td>
<td>2</td>
<td>48</td>
<td>14.570</td>
<td>9.866</td>
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<tr>
<td>T_{Q2}²</td>
<td>128</td>
<td>2</td>
<td>56</td>
<td>18.742</td>
<td>11.344</td>
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<td>T_{Q3}³</td>
<td>128</td>
<td>3</td>
<td>60</td>
<td>24.898</td>
<td>13.743</td>
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<tr>
<td>Conversion frequency⁴</td>
<td>128</td>
<td>2</td>
<td>25</td>
<td>10.773</td>
<td>4.789</td>
</tr>
</tbody>
</table>

¹ The earliest conversion time with more than 25% shared converted. ² The earliest conversion time with more than 50% shared converted. ³ The earliest conversion time with more than 75% shared converted. ⁴ The conversion frequency is counted as the number of shares converted of the bond is exceeding 10.
Table 3. Summary of the Fitted AG Model

|                | \(\beta_i\) | \(\exp(\beta_i)\) | SE(\(\beta_i\)) | Robust S.E. \(^3\) | z       | Pr(>|z|) |
|----------------|-------------|---------------------|------------------|---------------------|---------|----------|
| Dividend       | 0.170       | 1.185               | 0.031            | 0.030               | 5.626   | 0.0000   |
| Spread\(^1\)   | 0.079       | 1.082               | 0.012            | 0.038               | 2.057   | 0.0400   |
| Rating\(^4\)   |             |                     |                  |                     |         |          |
| 4              | 0.155       | 1.168               | 0.198            | 0.370               | 0.419   | 0.6700   |
| 5              | -0.134      | 0.875               | 0.198            | 0.332               | -0.402  | 0.6900   |
| 6              | 0.142       | 1.153               | 0.204            | 0.352               | 0.405   | 0.6900   |
| 7              | -0.073      | 0.930               | 0.215            | 0.368               | -0.197  | 0.8400   |
| 8              | -0.647      | 0.524               | 0.249            | 0.392               | -1.649  | 0.0990   |
| 9              | -1.085      | 0.338               | 0.386            | 0.529               | -2.049  | 0.0400   |
| Riskfree Rate  | -0.072      | 0.930               | 0.082            | 0.128               | -0.563  | 0.5700   |
| Current Ratio  | -0.007      | 0.993               | 0.002            | 0.003               | -2.495  | 0.0130   |
| Cash Flow Ratio| -0.001      | 0.999               | 0.001            | 0.001               | -0.474  | 0.6400   |
| Long-term Debt Ratio | -0.017 | 0.983               | 0.003            | 0.006               | -2.882  | 0.0040   |
| Net Worth      | 0.000       | 1.000               | 0.005            | 0.009               | -0.013  | 0.9900   |
| Total Asset    | 0.119       | 1.126               | 0.027            | 0.055               | 2.143   | 0.0320   |

H\(_0\): \(\beta_1=\beta_2=\beta_3=\beta_4=\beta_5=\beta_6=\beta_7=\beta_8=\beta_9=\beta_{10}=\beta_{11}=\beta_{12}=\beta_{13}=\beta_{14}=0\) vs. H\(_1\): Not all \(\beta\)s are zeros.

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<tr>
<th>Statistics</th>
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<td>14</td>
</tr>
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</tr>
</tbody>
</table>

Note. 1. “Spread” denotes the percentage of the spread between the stock and conversion prices. 2. \(\exp(\beta_i)\) denotes the instantaneous rates of conversions of the corresponding covariate. 3. Robust S.E. denotes the robust standard deviation of the estimator of the coefficient \(\beta_i\) based on AG model. 4. The 6 dummy variables of ratings categories 4–9.
Table 4. Summary of the Fitted PWP Model

|                | $\beta_j$ | $\exp(\beta_j)$ | SE($\beta_j$) | Robust S.E. $^3$ | z         | Pr(>|z|)  |
|----------------|-----------|------------------|---------------|------------------|-----------|-----------|
| Dividend       | 0.136     | 1.146            | 0.032         | 0.023            | 5.841     | 0.0000    |
| Spread$^1$     | 0.076     | 1.079            | 0.017         | 0.038            | 1.985     | 0.0470    |
| Rating$^4$     |           |                  |               |                  |           |           |
| 4              | 0.213     | 1.237            | 0.208         | 0.292            | 0.730     | 0.4700    |
| 5              | 0.041     | 1.042            | 0.209         | 0.276            | 0.150     | 0.8800    |
| 6              | 0.205     | 1.227            | 0.214         | 0.282            | 0.727     | 0.4700    |
| 7              | 0.082     | 1.086            | 0.225         | 0.294            | 0.280     | 0.7800    |
| 8              | -0.276    | 0.759            | 0.260         | 0.324            | -0.851    | 0.3900    |
| 9              | -0.592    | 0.553            | 0.392         | 0.489            | -1.210    | 0.2300    |
| Riskfree Rate  | -0.059    | 0.943            | 0.083         | 0.089            | -0.662    | 0.5100    |
| Current Ratio  | -0.004    | 0.996            | 0.002         | 0.002            | -2.383    | 0.0170    |
| Cash Flow Ratio| -0.001    | 0.999            | 0.001         | 0.001            | -0.720    | 0.4700    |
| Long-term Debt Ratio | -0.012 | 0.988            | 0.003         | 0.004            | -3.174    | 0.0015    |
| Net Worth      | -0.002    | 0.998            | 0.005         | 0.005            | -0.391    | 0.7000    |
| Total Asset    | 0.080     | 1.083            | 0.029         | 0.031            | 2.619     | 0.0088    |

H$_0$: $\beta_1=\beta_2=\beta_3=\beta_4=\beta_5=\beta_6=\beta_7=\beta_8=\beta_9=\beta_{10}=\beta_{11}=\beta_{12}=\beta_{13}=\beta_{14}=0$ vs. H$_1$: Not all $\beta$s are zeros.

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Figure 1.1. Distribution of 128 Bonds' Time-to-maturity
Figure 1.2. Distribution of 128 Bonds' Number of Shares Issued
Figure 1.3. Distribution of 128 Bonds’ Credit Rating at Issuance
Figure 1.4: Distribution of 128 Bonds' Conversion Frequency
Figure 1.5 Distribution of Expected Probability of Conversion at Maturity

Figure 1.6 Distribution of Cumulative Proportion of Conversion at Maturity
Figure 2 Kaplan-Meier Estimates of Survival Function

- Blue line: 1st Conversion
- Red line: >=25% shares converted
- Green line: >=50% shares converted
- Purple line: >=75% shares converted

Survival prob. vs. time (month)
Figure 3.5 Long-term Debt Ratio vs. Conversion Hazard Rate by Andersen-Gill Model

Figure 3.6 Total Asset vs. Conversion Hazard Rate by Andersen-Gill Model
References


Ho, T., and D. Pfefer (1996) Convertible bonds: Model, value attribution and analytics,
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